Leader Election

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Leader Election in Rings

- Under different models:
  - Synchronous vs. asynchronous
  - Anonymous vs. non-anonymous (knowledge of unique id)
  - Knowledge of "n" (non-uniform) vs. no knowledge (uniform)
- Impossibility result: there is no synchronous, non-uniform algorithm if the processors are anonymous.
  - Implies that there are no uniform algorithms as well
  - Implies that there are no asynchronous algorithms as well

Outline

- Leader election in asynchronous rings:
  - An O(n^2) messages algorithm
  - An O(n logn) messages algorithm
- Brief mention of a lower bound
- Synchronous model:
  - Breaking the O(n logn) barrier by abusing the synchronous model
  - For both uniform and non-uniform systems
- Leader election in arbitrary topologies
  - Using simultaneous DFS traversals

Asynchronous model: simple algorithm

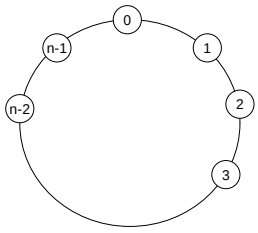
```
Upon receiving no message:
        send my_id in clockwise direction
```

```
Upon receiving "m":
        case
           m.id < my_id:  send m in clockwise direction
           m.id > my_id:  discard m
           m.id == my_id:
                    leader = my_id
                    send <terminate, my_id> in clockwise direction
                    terminate
```

```
Upon receiving <terminate, id>:
        leader = id;
        send <terminate, id> in clockwise direction
        terminate
```

Complexity

- Time complexity: O(n)
- Message complexity:
  - Clearly less than n^2 messages are sent
  - And Ω(n^2) is sent in the following worst case scenario



Hirschenberg-Sinclair Algorithm

- For bidirectional, asynchronous rings: achieve a O(n logn) message complexity
- Each node operates in phases:
  - In each phase, nodes that are still active send out their uid in both directions
  - In phase k, the tokens travel a distance of 2^k and return back to their points of origin
  - A token might not make it back if it encounters a node with lower uid
  - A node makes it to the next phase only if it receives its tokens back from the previous round

Detailed Description

```
Upon receiving no message
        if asleep then
                asleep = false
                phase = 0
                send [my-id, out, 1] to left and right
```

```
Upon receiving [id, out, h] from left:
Case
        id < my-id and h > 1:   send [id, out, h-1] to right
        id < my-id and h==1:    send [id, in, -] to left
        id = my-id:     leader = my-id
```

```
Upon receiving [my-id, in, -] from left and right:
        phase = phase + 1
        send [my-id, out, 2^phase] to left and right
Upon receiving [not-my-id, in, -] from left or right:
        send [not-my-id, in, -] to right or left
```

Correctness

- Messages from node with lower id is never discarded
- Messages from nodes with higher id eventually reach the node with the lowest id and gets discarded
- Therefore the correct leader is elected (safety)
- Liveness: eventually the node with the lowest id reaches phase log(n) and sends its id throughout the entire ring

Communication Complexity

- In phase 0, every processor sends a message:
  - Maximum of 4n messages
- In phase k+1:
  - Only processors that send tokens are those that "won" in the previous phase
  - There is at most one winner for every 2^k + 1 processors
  - Winners after phase k: $n/(2^k + 1)$
  - Tokens travel a distance in phase k+1 of: $2^{(k+1)}$
  - Total number of messages in phase k+1:
    $4*2^{(k+1)}*n/(2^k + 1) < 16n$
  - Total number of phases: $1 + \log n$
  - Number of messages: $O(n \log n)$

Question:

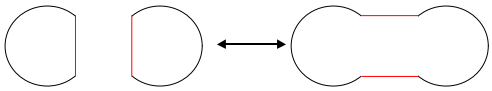
- What is the time complexity?

Lower bound

- AW has a lower bound proof:
  - Asynchronous networks require O(n log n) messages to perform leader election
- Proof sketch:
  - Provide a lower bound for a constrained leader election problem:
    - Elects the node with the minimum id
    - Everyone should know the identity of the winner
  - Construct an "open schedule" for a ring:
    - Open schedule is not complete; it is a prefix of an admissible execution
    - Open execution corresponds to taking a ring, blocking one of its channels, but allowing all other events to proceed as normal

Lower bound (contd.)

- AW prove the following:
  - Every ring of size n, has an open schedule that sends at least the following number of messages M(n)
  - When n=2, M(n) = 1 (easy to show)
  - For higher n, $M(n) = 2\,M(n/2) + \frac{1}{2}(n/2 - 1)$
    - Assume that an open schedule exists for n/2 sized rings
    - Then show that there is an open schedule for n-sized rings:
    - The two scenarios are not distinguishable



    - Wait for the two rings to reach a quiescent state
    - Show that a further $\frac{1}{2}(n/2 - 1)$ messages will be sent if one of the two channels is unblocked

Announcements

- Will post some “homework” questions on chapter 2 from AW
- Send me email if you are still looking for a partner

Synchronous rings

- Leader election with fewer than O(n logn) messages is possible
  - Can convey information by not sending a message:
    - “if you do not hear from me, then assume that …”
- Assume that:
  - Uids are positive integers
  - Can be manipulated using arbitrary arithmetic operations
- Two algorithms: TimeSlice, VariableSpeeds
- TimeSlice:
  - n is known to all processors (non-uniform)
  - Unidirectional communication is sufficient
  - O(n) messages

TimeSlice Algorithm

- Recall that a round in synchronous networks is:
  - Deliver all messages, have every processor take one compute step
- Define the notion of a phase
  - Each phase consists of “n” rounds
  - In phase k >= 0
    - If no one is elected yet
    - Processor with uid k:
      - Declares itself as the leader
      - Sends token with its uid around
- Message complexity: n
- Time complexity: n*(minimum uid value)

VariableSpeeds Algorithm

- Uniform algorithm: “n” is not known
- Unidirectional communication is sufficient
- Still achieves the O(n) messages bound
- Assumes there are two kinds of processors:
  - Those that are awake and participating in the leader election
  - Those that are non-participants and simply serve as relays
- Message life cycle:
  - A message is in phase one:
    - Until it is received by an awake processor
    - Forwarded immediately
  - A message is in phase two:
    - Once received by an awake processor
    - Forwarded after (2^message-uid – 1) rounds

Algorithm (contd.)

- When participant receives a message
  - If message.id > my-uid or the minimum message-id seen so far:
    - Swallow it
  - Else:
    - Delay for 2^(message.id – 1) rounds
- For a relay:
  - If message.id > minimum message-id seen so far, swallow it
  - Else, delay for 2^(message.id – 1) rounds
- If a processor gets its message back, it elects itself as the leader
- Correctness:
  - No processor will swallow the message with minimum uid
  - A message has to go through all processors before a leader is elected

Complexity

- By the time UID_{min} goes around the ring, the second smallest UID has gone only half way, third smallest a fourth of the way, etc.
- Forwarding the token carrying UID_{min} has caused more messages than all the other tokens combined
- Message complexity: O(n)
- Time complexity: $n*2^{UID\text{-}min}$

General Networks

- What if a network has arbitrary topology?
- Here is a simple algorithm based on DFS algorithm
- DFS algorithm from a specified root:
  - When a node first receives a message M
    - Send accept to sender
    - For each child:
      - Send M
      - Wait for accept or reject before considering next child
  - When a node later receives a message M
    - Send reject to sender

General networks (contd.)

- Start DFS spanning tree algorithm from all nodes
- In addition:
  - Send node's uid along with M
  - When two DFS traversals collide, the copy with the lower uid wins
  - The other DFS stalls – no response is sent to the sender
    - The sender waits forever
  - Only the processor that has the minimum uid gets a response from all of its children
- Message complexity: O(n * m)
- Time complexity: O(m)
- See text for details