

You are given three pairs of similar typing rules, and in each pair one rule is correct and the other incorrect. For each pair:

- Identify which rule is correct and which is incorrect.
- Say whether the incorrect rule affects soundness (allows unsafe programs to pass) or completeness (does not typecheck programs that are clearly safe), or both.
- Give an example program that exposes the problem with the incorrect rule.

For the final pair, assume that our type system has subtyping.

$$\frac{O \vdash e_1 : int \quad O \vdash e_2 : int}{O \vdash e_1 + e_2 : int}$$

$$\frac{O \vdash e_1 : int}{O \vdash e_1 + e_2 : int}$$

$$\frac{O \vdash e_0 : int \quad O[T_0/x] \vdash e_1 : T_1}{O \vdash T_0 \ x = e_0 ; e_1 : T_1}$$

$$\frac{O \vdash e_0 : T_0 \quad O[T_0/x] \vdash e_1 : T_1}{O \vdash T_0 \ x = e_0 ; e_1 : T_1}$$

$$\frac{\begin{array}{c} O, M \vdash e_0 : T_0 \\ O, M \vdash e_1 : T_1 \\ \dots \\ O, M \vdash e_n : T_n \\ M(T_0, f) = (T'_1, T'_2, \dots, T'_n, T_r) \\ T_i = T'_i \text{ (for } 1 \leq i \leq n) \end{array}}{O, M \vdash e_0.f(e_1, \dots, e_n) : T_r}$$

$$\frac{\begin{array}{c} O, M \vdash e_0 : T_0 \\ O, M \vdash e_1 : T_1 \\ \dots \\ O, M \vdash e_n : T_n \\ M(T_0, f) = (T'_1, T'_2, \dots, T'_n, T_r) \\ T_i \leq T'_i \text{ (for } 1 \leq i \leq n) \end{array}}{O, M \vdash e_0.f(e_1, \dots, e_n) : T_r}$$