Finite Differencing of Computable Expressions

AJ Shankar

It all comes down to laziness

- I’m lazy
- Say I’m making a PowerPoint presentation...
- I realize I need to cover a new topic
- What do I do?
  - A) Do the entire presentation over
  - B) ???
  - C) Profit!!!

Okay, what have I done?

- I used the work I had already done
- And *incrementally* constructed a new presentation

- That's pretty much all finite differencing is

THE END

Okay, okay...

- Things we still need to figure out:
  - What are the benefits of differencing?
  - When is it safe?
    - (and what do we mean by safe?)
    - Can I compute Go positions incrementally?
  - How does it work?
  - Can it be automated?
  - Does the content of this paper really justify its 50 page length?
  - And more...

First, a more relevant example

- Goal: compute the successive sums of each m-element window in an n-element array (with $m < n$)

2	1	4	2	7	0	3	5	2

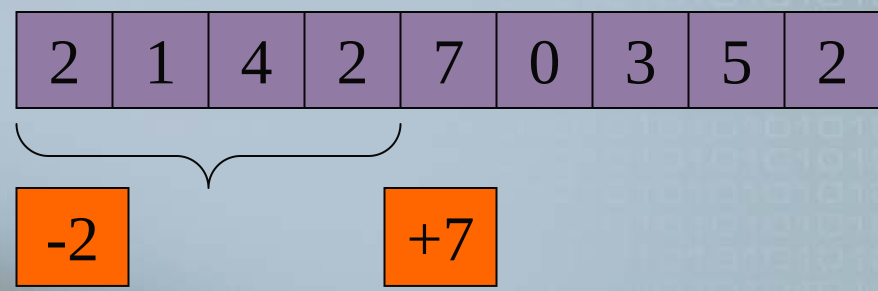
$n = 9; m = 4$

The simple way

```
for (int i = 0 ; i < n-m ; i++) {
  for (int j = i ; j < m ; j++) {
    sum[i] += ary[j];
  }
}
```

- Sum up each window independently

Using finite differencing



- Compute a running sum

```
for (int j = 0 ; j < m ; j++) {
  sum[0] += ary[j];
}
for (int i = 1 ; i < n-m ; i++) {
  sum[i] = sum[i-1] - ary[i-1] + ary[i+m-1];
}
```

Benefits of differencing

- Speedups
  - Possibly in asymptotic complexity
    - (What happened in the example?)
- If done automatically, simple code can become efficient
  - Can stick to “the simple way”
  - No need to uglify it yourself

Sounds *unbelievably* good

- This is going to revolutionize computing
- We took that $O(n^2)$ algo to $O(n)$...
- Let's difference that $O(n)$ algorithm to get an $O(1)$ algorithm!
- The fun never ends

- (Sanity check...)

When is it safe to difference?

- Must guarantee the transformation is *semantics preserving*
  - Just like any other optimization
  - Stay tuned...

Finite differencing overview

- Derivative
  - The building block of differencing
- Chain rule
  - Stringing differential expressions together
- Tricks for initialization

Computable derivatives

- "Differencing": figuring out the *difference* between f(x) and f(y)
- **Derivative:** how f changes with respect to x
- We extend this notion to code

Computable derivatives

- Let $E = f(x_1, ..., x_n)$
  - E is the incremental replacement for f
- Let dx_i be an update to x_i, e.g.

```
E = f(list) = length(list);
while (*) {
  list = item :: list;  // dx0
}
```

Derivative example

- Here, E = f(list) = length(list);
- dlist is list = item :: list;
- Then dE(dlist) = E += 1

- What should we expect of the derivative of E? dE(dx_0)
  - What properties must hold?

Derivative: formal definition

- Derivative is code blocks [B1, B2]:

$$\left.\begin{array}{l} B1 \\ dx_i \\ B2 \end{array}\right\} \quad \text{Differenced code}$$

- With properties:
  - B1 and B2 only modify locals and E
  - Semantics of dx_i are preserved
  - If $E = f(x_1, \ldots, x_n)$ before, then $E = f(x_1, \ldots, x_n)$ afterwards

Derivative questions

- Why is this definition semantics preserving?
- How broad is it?
  - Can it be applied to our sliding window example?
- Why do we need B1 *and* B2?
- Where do these derivs come from?

Differentiable code

- So we have derivatives of f… when can we apply them?
- E = f() is *differentiable* w.r.t a code block C if:
  - We know the derivatives of f w.r.t. for each x_i updated in C
  - f is known, or at least computable, at the start of C
- This should intuitively make sense

Doing the differencing

- To difference f w.r.t. C:
  - Replace all dx_i in C by $B1_i$; dx_i; $B2_i$;
  - Replace all uses of f with E
  - Initialize E properly
- Does this preserve the semantics of C?
- ... Works for C1; C2 as well

Example

```
a := {};
while eof = false
  read(i);
  a with:= i;
end while;
print({x \in a | x mod 2 == 0});
```

da (dx_0)

f(a)

Example

```
a := {}; E := {};                  dE(a := {})
while eof = false
  read(i);                         dE(da)
  if (i mod 2 == 0) then
    E with:= i;
  a with:= i;                      da
end while;                         E = f(a)
print(E);
```

Chaining

- So we’ve got the whole program differenced on f
- But what about g, h, etc?
- Just apply them in turn

Chaining, cont'd

- We have E_1 and E_2. Can chain if
  - $E_1 = f_1$ is differentiable w.r.t. B, transforming it to B'
  - And $E_2 = f_2$ is differentiable w.r.t. ???
  - B'
- How does this preserve semantics?
- What if all E_i were differentiable w.r.t. just B?

Computing speedups

- There’s a lot of stuff in the paper about figuring out when differencing will produce a speedup
- But: it’s pretty obvious stuff
  - Initial costs should be relatively low
  - Derivatives should be faster than recomputing f
  - ...

Initialization tricks

- We have a bunch of differenced code
- But need to initialize $E_i = f_i(\ldots)$ first
  - “setting up the invariant”
- Doing each E_i separately wasteful
- Jamming...

Vertical Jamming

- Want to initialize c_1, c_2

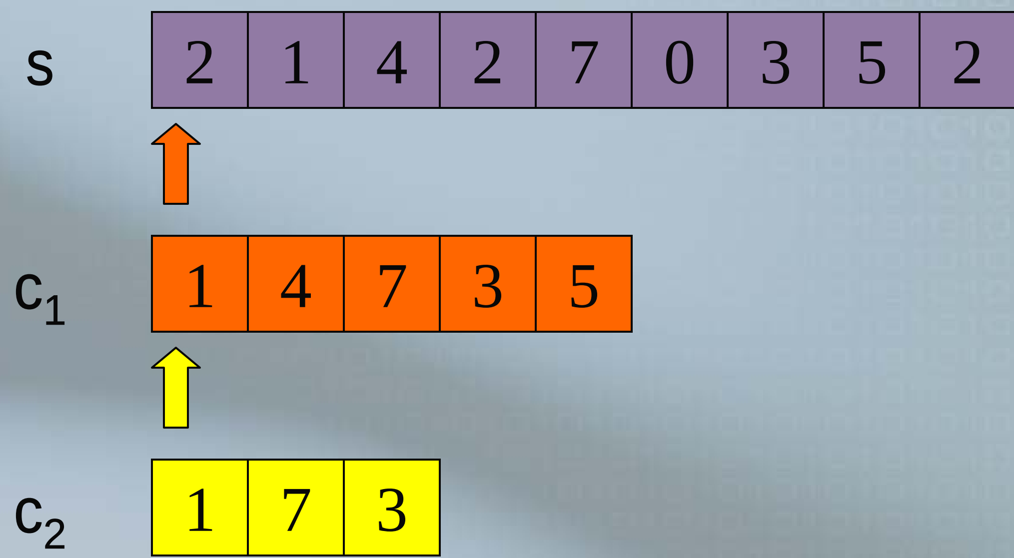
```
for (x in s) {
  if (k1(x))
    c1 with: x;
}

for (x in c1) {
  if (k2(x))
    c2 with: x;
}
```

Vertical Jamming, cont'd

- Remember, we're lazy.

Vertical Jamming, cont'd



- Wow, that saved a lot of effort.
  - Except for the effort it took me to make these slides.

The resulting code

```
for (x in s) {
  if (k1(x)) {
    c1 with: x;
    if (k2(x))
      c2 with: x;
  }
}
```

Horizontal jamming

- I'll spare you the animations

```
for (x in s) {
  if (k1(x))
    c1 with: x;
}

for (x in s) {
  if (k2(x))
    c2 with: x;
}
```

Horizontal jamming, cont'd

- Becomes

```
for (x in s) {
  if (k1(x))
    c1 with: x;
  if (k2(xi))
    c2 with: x;
}
```

Automating the process

- What are the hurdles?
  - Picking an f
  - Coming up with a derivative
    - How comprehensive is the list in the paper?
  - What else?
- “Already implemented a semiautomatic system”
  - “Results reported in near future”

Practical concerns

- Are there any language hurdles?
- What other problems are in the way?
  - (Why isn’t this system used now?)