Sketching

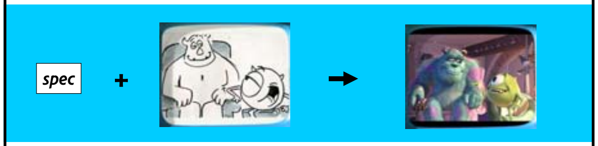
CS294 Spring 2006

Work by: Armando Solar-Lezama, Liviu Tancau, David Turner,
Rastislav Bodik, Sanjit Seshia UC Berkeley Vijay Saraswat IBM

Administrativia

- Thanks for all the pointers to papers
  - will be added to the master list tonight
- Tue homework:
  - select a paper you'd like to lead a discussion on
    - add comments such as "should be preceded by paper X"
  - email me summary for the Prospector paper
    - suggested format to be posted
  - if all goes well, "coffee service" will start
    - 10 cappuccinos at 9:30
- Thu: 10-minute presentations on challenge problems
  - sign up by Mon; auditors can present

The sketching experience



specification

sketch

implementation
(completed sketch)

Programming with StreamBit

- Specification
  - **executable:** easy to debug, serves as a prototype
  - **a reference implementation**: simple and sequential
  - **written by domain experts**: crypto, bio, MPEG committee
- Sketched implementation
  - **program with holes:** filled in by synthesizer
  - **programmer sketches strategy:** machine provides details
  - **written by performance experts:** vector wizard; SSE guru

Example: divide and conquer parallelization

- **Parallel algorithm:**
  - **Data rearrangement + parallel computation**
- **spec:**
  - sequential version of the program
- **sketch:**
  - parallel computation
- **automatically synthesized:**
  - Rearranging the data (dividing the data structure)

Benefits of sketching

- **productivity**
  - many tedious details synthesized automatically
  - focus on creative process
- **separation of roles**:
  - domain expert, performance expert collaborate
- **separation of aspects**: correctness vs. performance
  - rapidly develop high-quality implementations
  - without fear of introducing bugs

Verification, synthesis, sketching

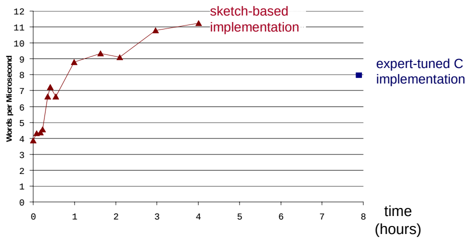
- Verification: does your program implement the spec?
  - user responsible for low-level implementation details
  - redundancy: implementation restates aspects of spec
- Synthesis: produce a program that implements the spec
  - say what not how; say it only once
  - hard to synthesize a good implementation
- Sketching: synthesis + partially described implementation

Sketching: uses and expectations

- What it can do:
  - synthesize hard-to-get-right expressions (masks, indices)
  - synthesize algebraic tricks when merging parallel results
  - synthesize data structures (re)layout for parallelization
  - and other parallelization machinery
- What it is not designed to do:
  - invent a good algorithm automatically (search space too large)
  - provide clever algorithm ideas (but helps in exploring them)
  - remove fun from programming (focus on clever ideas)

Sketching in StreamBit [PLDI'05]: Best Results

- Implementing a mini cipher, sketching vs. C:



Sketching in StreamBit [PLDI'05]: Worst Result

- sketching easy to explain but mastering took a while
  - sketches not really programs, but meta-level rewrite rules
  - baseline compiler, its rules overridden by rewrite rules
  - implementation broken into multiple, hierarchical sketches
  - dataflow programming model useful but unfamiliar
- sketching limited in expressibility
  - implementations had to use instructions that were semi-permutations (bit shift ok, addition no)

SKETCH

- A language that addresses all the limitations
  - like C without pointers
  - sketching support: two simple constructs
- restricted to finite programs:
  - input size known at compile time, terminates on all inputs
- most high-performance kernels are finite:
  - matrix multiply: **yes**
  - binary search tree: **no**

Example 1

- bitvector parallelism
  - exploited thanks to some algebra
- sketches help reinvent the trick
- sketches are reusable

Ex1: Isolate rightmost 0-bit. 1010 0111 → 0000 1000

```
bit[W] isolate0 (bit[W] x) {      // W: word size
    bit[W] ret = 0;
    for (int i = 0; i < W; i++)
          if (!x[i]) { ret[i] = 1; break; }
    return ret;
}

bit[W] isolate0Fast (bit[W] x) implements isolate0 {
    return ~x & (x+1);
}

bit[W] isolate0Sketched (bit[W] x) implements isolate0 {
    return ~(x + ??) & (x + ??);
}
```

Sketches are reusable

```
bit[W] expression (bit[W] x) {
   return ~(x + ??) & (x + ??);
}
bit[W] isolate0Sketched (bit[W] x) implements isolate0 {
   return expression(x);
}
bit[W] isolate1Sketched (bit[W] x) implements isolate1 {
   return expression(x);
}
```

Programmer's view of sketches

- the ?? operator replaced with a suitable chunk of bits
- as directed by the **implements** clause.

- the ?? operator introduces non-determinism
- the **implements** clause constrains it.

Meaning of sketches

- programs with ?? have many meanings

```
            ~(x + ??) & (x + ??);
      means:
            ~(x + 0) & (x + 1);
            ~(x - 1) & (x + 0);
            ...
```

- loops are unrolled:

```
            x = ??; loop (x) { y = y + ??; }
      means:
            x = 2;  y = y + 4; y = y + 0;
            x = 3;  y = y + 2; y = y + 4; y = y + 17;
            ...
```

- *f* implements *g*:
  - synthesizer "selects" the meaning of *f* that is functionally equivalent to *g*

Example 2

- divide and conquer parallelism
- SIMD parallelism, and how to emulate SIMD semantics
- sketching table-based implementations
- more on reusability
- rapidly prototyping multiple implementations

Ex 2: Population count. 0010 0110 → 3

```
int pop (bit[W] x)
{
   int count = 0;
   for (int i = 0; i < W; i++) {
      if (x[i]) count++;
   }
   return count;
}
```

Parallel pop, divide-and-conquer

- pop(word x) = pop (1st half of x) + pop(2nd half of x)
  - recurse until argument is single bit
- idea: execute simultaneously all operations of same size
  - store sums in the word itself, SIMD style
  - O(log W) steps
- tricky implementation
  - SIMD subword size different at each step
  - on non-SIMD, must "emulate" SIMD semantics with bitmasks
  - example, base case:

```
x = (x & 0x5555) + ((x>>1) & 0x5555)
```

Sketch of the parallel pop

```
int popSketched (bit[W] x) implements pop {
   loop (??) {
        x = (x & ??) + ((x >> ??) & ??);
   }
   return x;
}

int popSketched (bit[W] x) implements pop {
   x = (x & 0x5555) + ((x >> 1) & 0x5555);
   x = (x & 0x3333) + ((x >> 2) & 0x3333);
   x = (x & 0x0077) + ((x >> 8) & 0x0077);
   x = (x & 0x000F) + ((x >> 4) & 0x000F);
   return x;
}
```

Table-based implementation

```
int popTable (bit[8] in) implements pop {
   int[256] table = ??;
   return table[in];
}
```

Table-based implementation

```
int popTable (bit[W] in) implements pop {
   int[256] table = ??;
   int ret = 0;
   loop (??) {  ret += table[ in>>??  &  ?? ];  }
   return ret;
}
```

Implementation for sparse populations

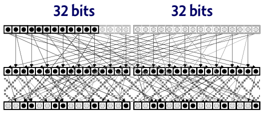
```
int popSparseSketched (bit[W] in) implements pop {
   int ret;
   for (ret = 0; in; ret++) {
        in &= ~expression(in);    // ~(x + ??) & (x + ??);
   }
   return ret;
}
```

Beyond synthesis of literals

- Synthesizing values of ?? already very useful
  - tricky expressions
  - parallelization machinery

- We can synthesize more than values
  - semi-permutations: functions that select and shuffle bits
  - polynomials: over one or more variables

Example 3: IP from DES.



```
bit[64] IPsketched (bit[64] x) implements IP {
   bit[64] result;
   bit[32] table[8][16] = ??;
   x = (x>>??) {|} (x<<??) {|} x;
   for (int i=0; i<8; ++i) {
      result[0:31]  |= table[i][x[i*4::4]];
      result[32:63]|= table[i][x[32+i*4::4]];
   }
   return result;                              table[i][permutation(x)];
}                                              table[i][permutation(x)];
```

Template for an arbitrary permutation

```
bit[N] permutation<int N>(bit[N] x) {
   bit[N] result;
   int i=0;
   loop (??) {
      result ^= x>>i & ??;
      result ^= x<<i & ??;
     ++i;
   }
   return result;
}
```

More higher-level synthesis

- **Synthesizing polynomials**

Synthesizing polynomials

```
int spec (int x) {
   return 2*x*x*x*x + 3*x*x*x + 7*x*x + 10;
}

int p (int x) implements spec {
   return (x+1)*(x+2)*poly(3,x);
}

int poly(int n, int x) {
   if (n==0) return ??;
   else return x * poly(n-1, x) + ??;
}
```

Karatsuba's multiplication

$x = x1*b + x0$ $y = y1*b + y0$ $b=2^k$

$x*y = b^2*x1*y1 + b*(x1*y0 + x0*y1) + x0*y0$

$x*y = poly(??,b) * x_1*y_1 +$
$+ poly(??,b) * poly(1,x_1,x_0,y_1,y_0)*poly(1,x_1, x_0, y_1, y_0)$
$+ poly(??,b) * x_0*y_0$

$x*y = (b^2 +b) * x_1*y_1$
$+ b * (x_1 - x_0)*(y_1 - y_0)$
$+ (b+1) * x_0*y_0$

$O(N^{1.5})$ vs. $O(N^2)$

Sketch of Karatsuba

```
bit[N*2] k<int N>(bit[N] x, bit[N] y) implements mult {
    if (N<=1) return x*y;

    bit[N/2] x1 = x[0:N/2-1];          bit[N/2+1] x2 = x[N/2:N-1];
    bit[N/2] y1 = y[0:N/2-1];          bit[N/2+1] y2 = y[N/2:N-1];

    bit[2*N]  t11 = x1 * y1;
    bit[2*N]  t12 = poly(1, x1, x2, y1, y2) * poly(1, x1, x2, y1, y2);
    bit[2*N]  t22 = x2 * y2;

    return multPolySparse<2*N>(2, N/2, t11)    // log b = N/2
          + multPolySparse<2*N>(2, N/2, t12)
          + multPolySparse<2*N>(2, N/2, t22);
}
bit[2*N] poly<int N>(int n, bit[N] x0, x1, x2, x3) {
    if (n<=0) return ??;
    else return  (??*x0 + ??*x1 + ??*x2 + ??*x3) * poly<N>(n-1, x0, x1, x2, x3);
}
bit[2*N] multPolySparse<int N>(int n, int x, bit[N] y) {
    if (n<=0) return 0;
    else return  y << x*?? + multPolySparse<N>(n-1, x, y);
}
```

Semantic view of sketches

- the **??** operator modeled as reading from an oracle

```
int f (int y) {
    x = ??;
    loop (x) {
        y = y + ??;
    }
    return y;
}
```

```
int f (int y, bit[][][K] oracle) {
        x = oracle[0][i0++];
        loop (x) {
            y = y + oracle[1][i1++];
        }
        return y;
}
```

- synthesizer finds oracle satisfying *f* **implements** *g*

Synthesis as generalized SAT

- The sketch synthesis problem is an instance of 2QBF:

$$\exists\ o \in \{0,1\}^k\ .\ \forall\ x \in \{0,1\}^m\ .\ P(x) = S(x,o)$$

- Counter-example driven solver:

```
I = {}
x = random()
do
  I = I ∪ {x}
  c = synthesizeForSomeInputs(I)
  if c = nil then exit(“buggy sketch'')
  x = verifyForAllInputs(c)          // x: counter-example
while x ≠ nil
return c
```

Scalability of the synthesizer

Program	Input size, W	Synthesis time (s)	SAT unknowns
AES MixCol	32	443	2602
DES.IP	64	693	178
Tblcrc	24 (48)	5	32
Tblcrc2	8	245	2048
Reverse	64	193	522
Parity	24 (48)	1	45
Log2	24 (28)	1268	409
Pop	8 (16)	4	109
Polynomial	16	617	96
Karatsuba	6 (8)	11	63

Finite programs

- finite programs:
  - input size known at compile time, terminates on all inputs
  - matrix multiply: **yes**   binary search tree: **no**

- complete:
  - specification can specify any finite program
  - sketch can describe any implementation over given instructions
  - synthesizer can resolve any sketch in theory; in practice, scales to real-world problems

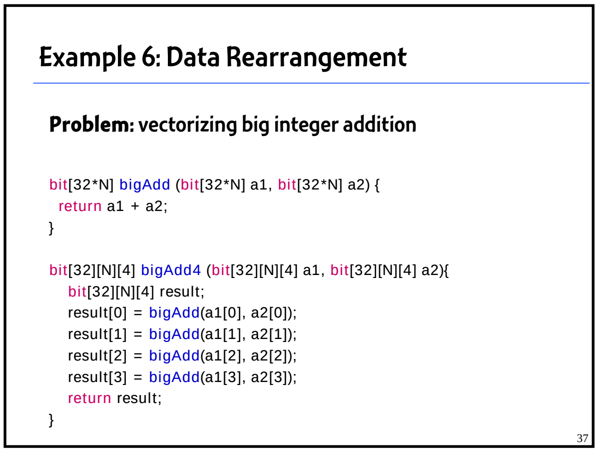
Beyond small finite programs

- Some finite programs are too large
  - Ex.: big-integer multiplication
  - here, our synthesis works only for word size of W=6
  - but synthesized result is same for all W
  - with this knowledge, programmer can already use our system
  - hope to develop static analysis proving synthesis independent of W

- Some programs are not finite
  - streaming computation
  - but the kernel applied on the stream is typically finite

Example 5: DCT

```
float[N] DCT<N>(float[N] x) implements DCTspec<N>{
    float[N] t;
    loop(log2(N)) {
        loop(N) {
            t[??] = ?? * x[??] + ?? * x[??];
        }
        x=t;
    }
    return x;
}
```

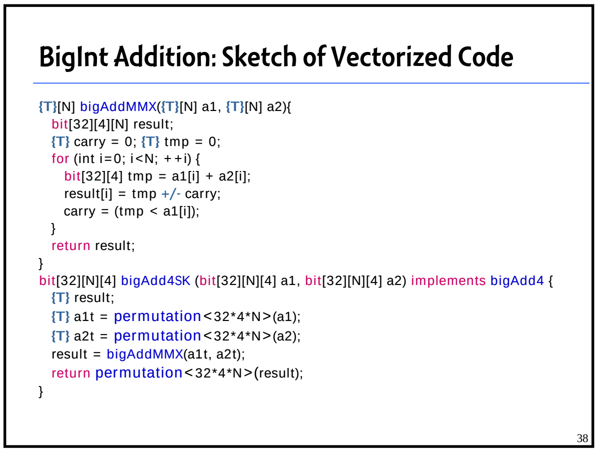


Example 6: Data Rearrangement

Problem: vectorizing big integer addition

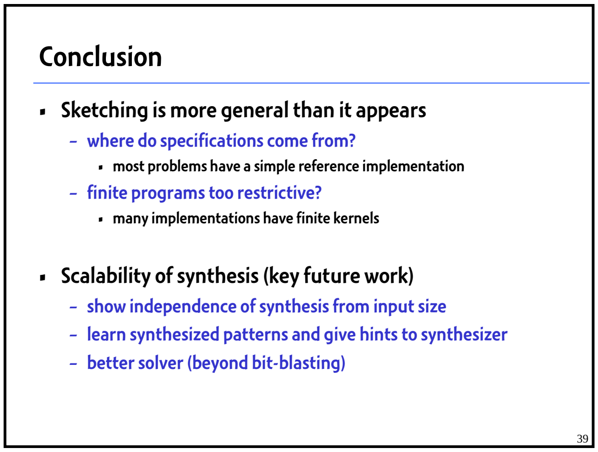
```
bit[32*N] bigAdd (bit[32*N] a1, bit[32*N] a2) {
  return a1 + a2;
}

bit[32][N][4] bigAdd4 (bit[32][N][4] a1, bit[32][N][4] a2){
    bit[32][N][4] result;
    result[0] = bigAdd(a1[0], a2[0]);
    result[1] = bigAdd(a1[1], a2[1]);
    result[2] = bigAdd(a1[2], a2[2]);
    result[3] = bigAdd(a1[3], a2[3]);
    return result;
}
```



BigInt Addition: Sketch of Vectorized Code

```
{T}[N] bigAddMMX({T}[N] a1, {T}[N] a2){
   bit[32][4][N] result;
   {T} carry = 0; {T} tmp = 0;
   for (int i=0; i<N; ++i) {
      bit[32][4] tmp = a1[i] + a2[i];
      result[i] = tmp +/- carry;
      carry = (tmp < a1[i]);
   }
   return result;
}
bit[32][N][4] bigAdd4SK (bit[32][N][4] a1, bit[32][N][4] a2) implements bigAdd4 {
   {T} result;
   {T} a1t = permutation<32*4*N>(a1);
   {T} a2t = permutation<32*4*N>(a2);
   result = bigAddMMX(a1t, a2t);
   return permutation<32*4*N>(result);
}
```



Conclusion

- Sketching is more general than it appears
  - where do specifications come from?
    - most problems have a simple reference implementation
  - finite programs too restrictive?
    - many implementations have finite kernels
- Scalability of synthesis (key future work)
  - show independence of synthesis from input size
  - learn synthesized patterns and give hints to synthesizer
  - better solver (beyond bit-blasting)