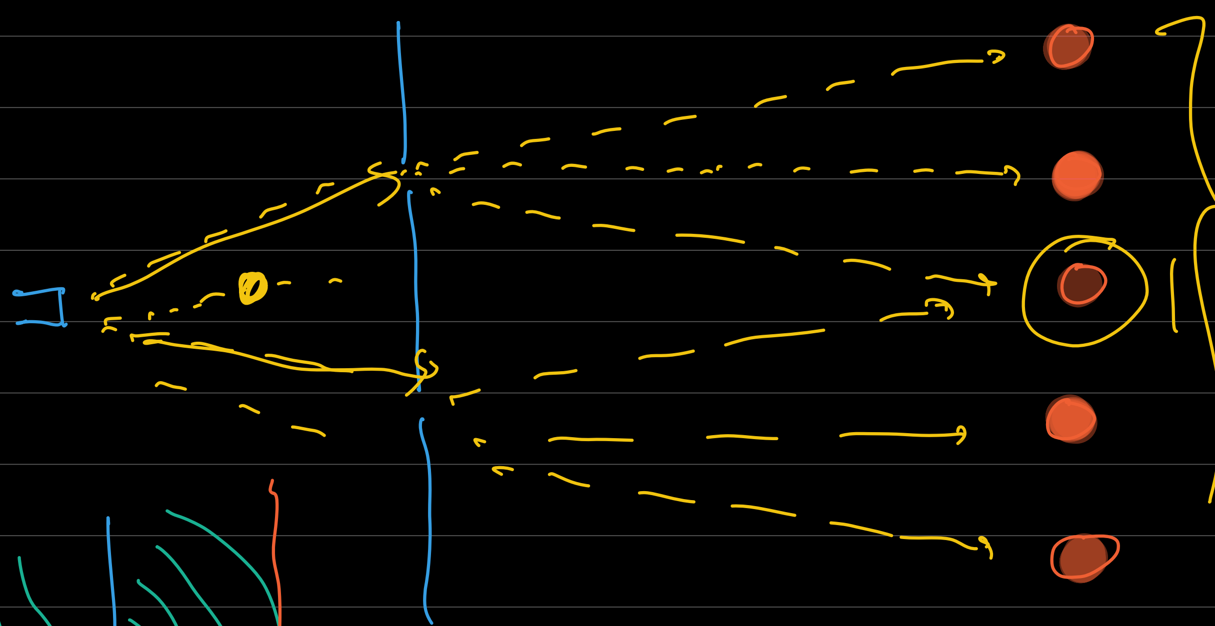
Qubits

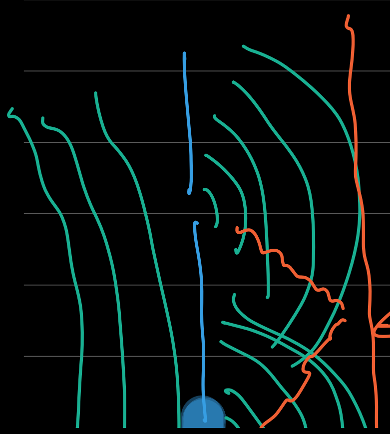
- HW#1 now posted (due: Jan 17 11:59pm)
- office hours: Special office hour on Fri for me

Logical bits: 0, 1

- high/low voltage
- magnetic domains
- presence or not of a photon on a fiber optic cable

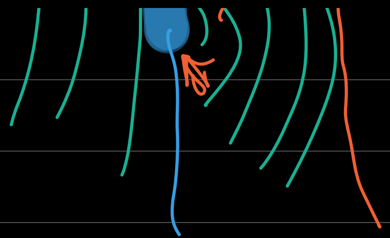
Double-slit experiment





reduce the intensity

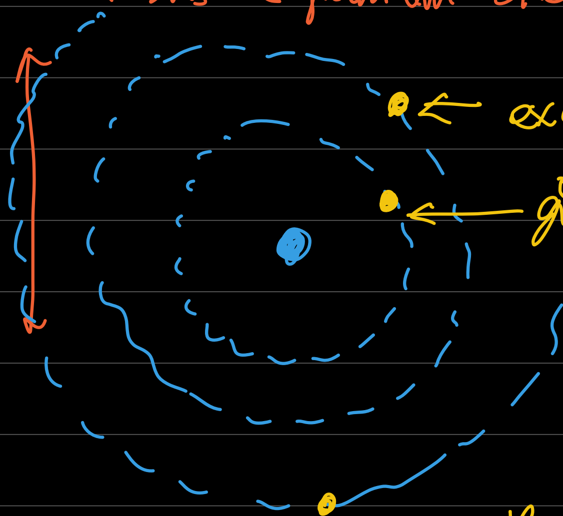
Done w/ molecules that



have $\approx$ 2000 atoms

$\alpha|0\rangle + \beta|1\rangle$ $\quad |\alpha|^2+|\beta|^2=1$

Qubits (quantum bits)



excited state $|1\rangle$

ground state $|0\rangle$

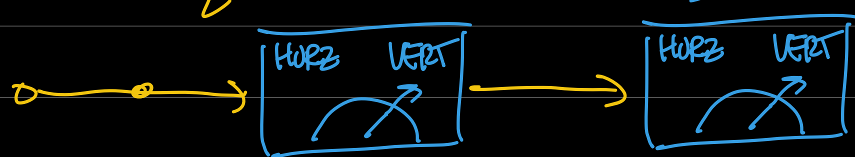
2nd excited state

Polarization of photons

$\leftrightarrow$ HORZ $|0\rangle$

$\updownarrow$ VERT $|1\rangle$

Same answer



Axiom #1 of QM: If a quantum system can be in one of two basic states, then it $\boxed{\text{can be}}$ in any superposition of those two states.

State of some qubit $\{ \alpha|0\rangle + \beta|1\rangle \}$

amplitudes

qubit

$$\alpha, \beta \in \mathbb{C}$$

$$|\alpha|^2 + |\beta|^2 = 1$$

$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$\alpha = a + bi, \quad a, b \in \mathbb{R}$

$|\alpha|^2 = a^2 + b^2$

$\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

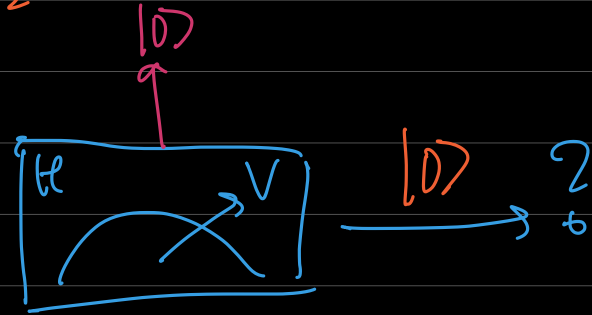
$0.8|0\rangle - 0.6|1\rangle$

$\boxed{0.8|0\rangle + 0.6|1\rangle}$

$\left.\begin{matrix} 0.8^2 + 0.6^2 = 1 \\ 0.64 \quad\quad 0.36 \end{matrix}\right\}$

$\left(\frac{1}{2} + \frac{1}{2}i\right)|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$

$\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$



Axiom #2 of QM:

$\alpha|0\rangle + \beta|1\rangle$ state

Measurement result is "$|0\rangle$" w/ prob. $|\alpha|^2$

"$|1\rangle$" w/ prob $|\beta|^2$

And after measuring, the state collapses to $|0\rangle$ or $|1\rangle$

(recall $|\alpha|^2 + |\beta|^2 = 1$)

$|1\rangle$
$|2\rangle$
$|3\rangle$
$|4\rangle$

$\underbrace{|1\rangle, |2\rangle, \ldots, |d\rangle}_{\text{qudit}}$

$|v\rangle \in \mathbb{C}^d$ $\qquad v = (\alpha_1, \alpha_2, \dots, \alpha_d)$

Axiom #1 General state is $\alpha_1|1\rangle + \alpha_2|2\rangle + \dots + \alpha_d|d\rangle$

with $|\alpha_1|^2 + |\alpha_2|^2 + \dots + |\alpha_d|^2 = 1$

Axiom #2 Result of measuring is $|j\rangle$ w. prob $|\alpha_j|^2$

State collapses to outcome $|j\rangle$.

measurement in the orthonormal $\{|1\rangle, |2\rangle, \dots, |d\rangle\}$ basis of $\mathbb{C}^d$

K qubits $\Rightarrow$ 2^k possible basic states

nature stores a state of this system as $d = 2^k$ complex numbers

$$\vec{v} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_d \end{bmatrix} \in \mathbb{C}^d \qquad \|\vec{v}\|^2 = |\alpha_1|^2 + \dots + |\alpha_d|^2 = 1$$

$$\|\vec{v}\|^2 := \langle \vec{v}, \vec{v} \rangle$$

$$\langle u, v \rangle := u_1^* v_1 + u_2^* v_2 + \dots + u_d^* v_d$$

$$(a + bi)^* = \overline{a + bi} = a - bi$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \dots \quad e_d = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

" " "

'' '' ... ''

$|1\rangle \qquad |2\rangle \qquad \cdots \qquad |d\rangle$

$$\alpha|0\rangle + \beta|1\rangle = \underbrace{\begin{bmatrix}\alpha\\ \beta\end{bmatrix}}_{\vec v}$$

$$\langle |0\rangle, \vec v\rangle^2 = |\alpha|^2$$
$$\langle |1\rangle, \vec v\rangle^2 = |\beta|^2$$

$|blah\rangle$ = a column vector ("ket")

$\langle blah| := |blah\rangle^*$ ("bra")

$$\langle u, v\rangle := u_1^* v_1 + u_2^* v_2 + \cdots + u_d^* v_d$$

$$= \begin{bmatrix}u_1\\ u_2\\ \vdots\\ u_d\end{bmatrix}^* \begin{bmatrix}v_1\\ v_2\\ \vdots\\ v_d\end{bmatrix}$$

$$= \begin{bmatrix}u_1^* & \cdots & u_d^*\end{bmatrix}\begin{bmatrix}v_1\\ \vdots\end{bmatrix}$$

$(1 \times d) \quad (d \times 1) \qquad \lfloor u \; d \rfloor$

$$\boxed{\langle u, v\rangle} = u^* v = \langle u| \cdot |v\rangle$$

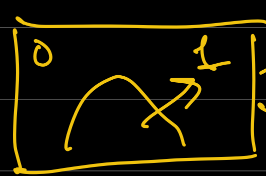
$$= \underbrace{\langle u|}_{\text{bra}}\underbrace{|v\rangle}_{\text{ket}}$$

$$\underbrace{\langle u|}_{\text{bra}} \quad \underbrace{|v\rangle}_{\text{ket}} = \boxed{\langle u|v\rangle}$$

$$|+\rangle := \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|-\rangle := \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$\{|0\rangle, |1\rangle\}$, $\{|+\rangle, |-\rangle\}$ two orthonormal bases

$|+\rangle \longrightarrow$  $\rightarrow |0\rangle$ w/ prob $\frac{1}{2}$

$\rightarrow |1\rangle$ w/ prob $\frac{1}{2}$

$|\psi\rangle \longrightarrow \boxed{|+\rangle \quad |-\rangle} \longrightarrow |+\rangle$ w.p. $\langle +|\psi\rangle^2$

$$\nearrow \; | \; \longrightarrow |-\rangle \text{ w.p } \langle -|\psi\rangle^2$$

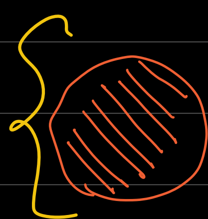
$$\boxed{|\psi\rangle = \underline{\langle +|\psi\rangle} |+\rangle + \underline{\langle -|\psi\rangle} |-\rangle}$$

$$|\psi\rangle = \underline{\langle 0|\psi\rangle} |0\rangle + \underline{\langle 1|\psi\rangle} |1\rangle$$

$$\langle |0\rangle, |\psi\rangle \rangle \qquad \langle |1\rangle, |\psi\rangle \rangle$$

only lets horizontally polarized photons through

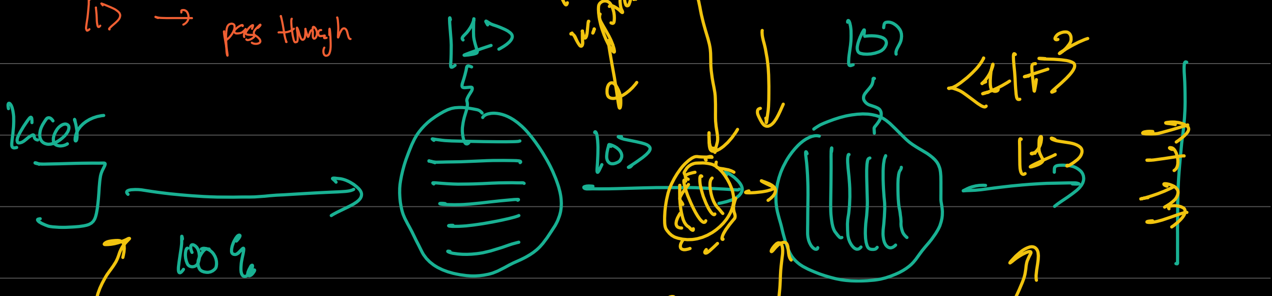
$|0\rangle \to$ pass through
$|1\rangle \to$ absorbed



$|+\rangle \to$ pass through
$|-\rangle \to$ absorbed

$|0\rangle \to$ absorbed
$|1\rangle \to$ pass through

$|+\rangle$ w/ prob. $\frac{1}{4}$ 0% brightness



$\boxed{\frac{1}{8}|+\rangle}$

12.5% original brightness

pass through prob:

$$\underbrace{\langle +|0\rangle}^2$$

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

1/4

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

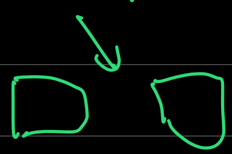
$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|+\rangle = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \qquad |-\rangle = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|0\rangle \rightarrow |+\rangle$$
$$|1\rangle \rightarrow |-\rangle$$

$\alpha|0\rangle + \beta|1\rangle$



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