- Monday no class (MLK)
- In person starting next Wed (CSE 2 G04)
- EdStem (coming today)

Review: Quantum state $\boxed{\underbrace{|\psi\rangle}_{\text{ket}} \in \mathbb{C}^d}$ } qudit

$$|\psi\rangle = \begin{bmatrix}\alpha_1\\ \alpha_2 \\ \vdots \\ \alpha_d\end{bmatrix} \quad \text{s.t.} \quad \boxed{|\alpha_1|^2 + \dots + |\alpha_d|^2 = 1}$$

(e.g. $|cat\rangle$)

$z = a+bi$, $\boxed{\bar{z} = a-bi}$

$|z|^2 = \bar{z} \cdot z = a^2+b^2$

$$\overset{e_1}{} \quad \overset{e_2}{} \quad \cdots \quad \overset{e_d}{}$$

$$\boxed{\{|1\rangle, |2\rangle, \dots, |d\rangle\}} \xleftarrow{d=2} \{|0\rangle, |1\rangle\}$$

computational basis

$$\begin{bmatrix}1\\0\\ \vdots \\ 0\end{bmatrix} \begin{bmatrix}0\\1\\0\\ \vdots \\ 0\end{bmatrix} \cdots \begin{bmatrix}0\\0\\ \vdots \\ 1\end{bmatrix}$$

$$|\psi\rangle = \langle 1|\psi\rangle |1\rangle + \langle 2|\psi\rangle |2\rangle + \dots + \langle d|\psi\rangle |d\rangle$$

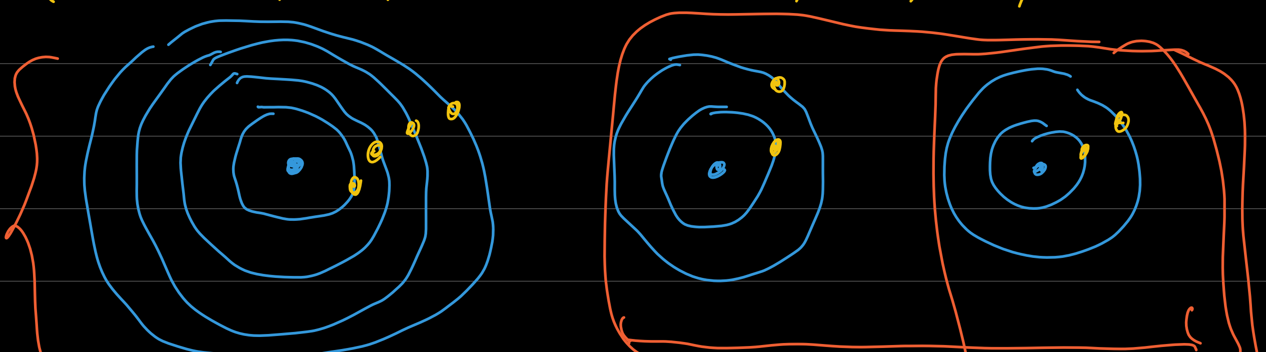
$$|\psi\rangle^* = [\bar{\alpha}_1 \ \bar{\alpha}_2 \cdots \bar{\alpha}_d] := \langle\psi| \text{ bra}$$

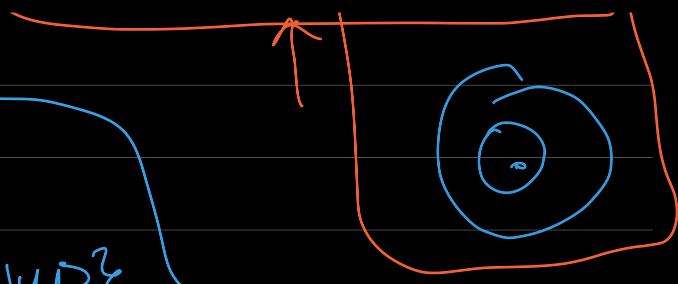
(e.g. $|u\rangle\langle v|$)

$$|u\rangle^* |v\rangle = \langle u||v\rangle = \langle u|v\rangle$$

$$\boxed{\mathbb{C}^4 \cong \mathbb{C}^2 \otimes \mathbb{C}^2}$$

$\{|1\rangle, |2\rangle, |3\rangle, |4\rangle\}$ $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$



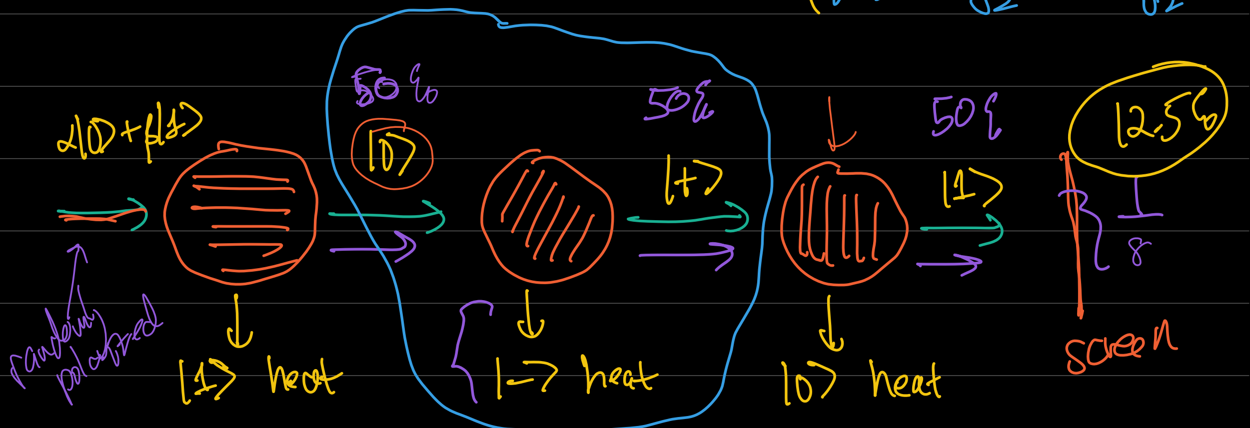


Measurement of $|\psi\rangle$
Orthonormal basis $\{|u_1\rangle, \dots, |u_d\rangle\}$

→ Measurement gives outcome "$|u_i\rangle$" w/ prob. $\boxed{|\langle u_i|\psi\rangle|^2}$ and the state $|\psi\rangle$ collapses to $|u_i\rangle$.

$$\sum_{i=1}^{d} |\langle u_i|\psi\rangle|^2 = \langle\psi|\psi\rangle^2 = 1.$$

$$\begin{cases} |+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \\ |-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \end{cases}$$

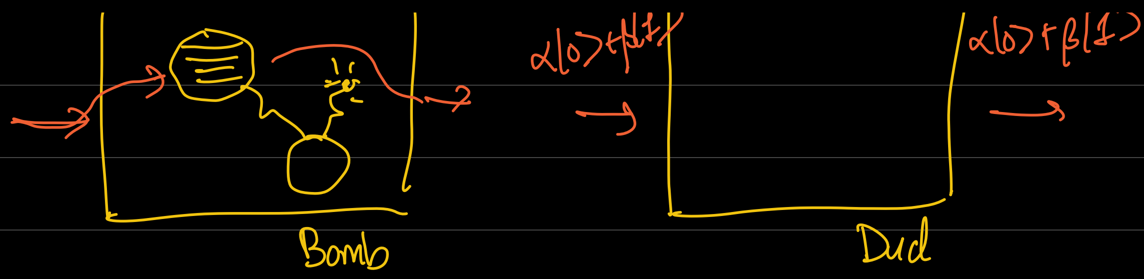


$\mathbb{P}[|0\rangle$ measure as $|+\rangle] = |\langle +|0\rangle|^2 = \frac{1}{2}$

$\mathbb{P}[$measure $|+\rangle$ as $|1\rangle] = |\langle 1|+\rangle|^2 = \frac{1}{2}$

$\mathbb{P}\left[\begin{array}{l}|0\rangle \text{ is measured as } |1\rangle \\ \text{by (||||)}\end{array}\right] = |\langle 1|0\rangle|^2 = 0$

Elitzur-Vaidman Bomb



Bomb: Measure $|0\rangle \to$ Output $|0\rangle$

Measure $|1\rangle \to$ heat

$\to$ Boom!

Dud: Does nothing

Goal: Figure out if it's a bomb w/out setting it off.

Classically: Send in $|0\rangle \to$ learn nothing

Send in $|1\rangle \to$

If it's a dud, we see that

If it's a bomb, Boom!

Quantum: Send in $|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

Dud: Get $\boxed{|+\rangle}$ as the output

Bomb: Measure $|+\rangle$ in the $\{|0\rangle, |1\rangle\}$ basis

50% chance of explosion (measure $|1\rangle$)

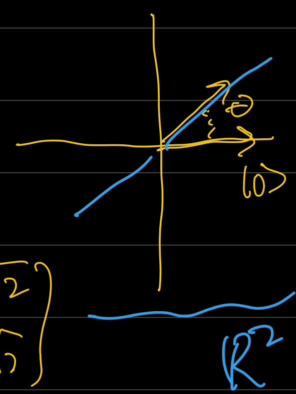
50% chance no explosion, output $\boxed{|0\rangle}$

Measure in the $\{|+\rangle, |-\rangle\}$ basis

If measure outcome is $|+\rangle$, learn nothing
If measure outcome is $\boxed{|-\rangle}$, learn it was a bomb.

Bomb: 50% chance of BOOM!
25% chance learn nothing
25% chance learn it's a bomb.

Rotations: $\boxed{R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}}$



$$R_{45°} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} = |+\rangle$$

$$R_{45°}|0\rangle = |+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \quad |+\rangle$$

$$R_{45°}|1\rangle = \frac{-1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \quad |\gamma\rangle$$

$\{|+\rangle, |\gamma\rangle\}$

$\{|+\rangle, |-\rangle\}$?

Want to measure $|\psi\rangle$ in the $\{|+\rangle, |\gamma\rangle\}$ basis
1) Measure $R_{-45°}|\psi\rangle$ in the $\{|0\rangle, |1\rangle\}$
2) Apply $R_{45°}$ to the outcome

$\boxed{|\psi\rangle = \alpha|+\rangle + \beta|\gamma\rangle}$

output $|0\rangle$ w.p. $|\alpha|^2$
Output $|1\rangle$ w.p. $|\beta|^2$

$R_{-45^\circ}|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \downarrow \qquad \downarrow R_{45^\circ}$

output $|+\rangle$ w.p. $|\alpha|^2$

$|\gamma\rangle$ w.p. $|\beta|^2$

<u>Reflection:</u> $\begin{bmatrix}\alpha\\ \beta\end{bmatrix} \to \begin{bmatrix}\beta\\ \alpha\end{bmatrix}$

$$X = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \begin{bmatrix}\beta\\ \alpha\end{bmatrix}$$

"quantum NOT"

"Swap"

$$S = \begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \begin{bmatrix}\alpha\\ i\beta\end{bmatrix}$$

<u>3^{rd} postulate of QM:</u> Any qudit $|\psi\rangle \in \mathbb{C}^d$ state can be changed by a $\boxed{\text{linear transformation}}$ that preserves the lengths of vectors

$$|\psi\rangle = \begin{bmatrix}\alpha_1\\ \vdots\\ \alpha_d\end{bmatrix}$$

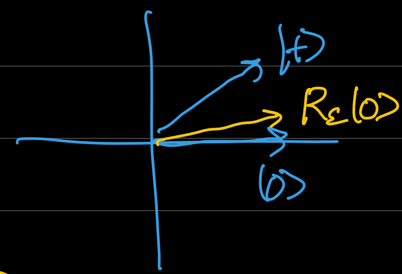
length is $\| |\psi\rangle \|^2 = \langle\psi|\psi\rangle$

$= |\alpha_1|^2 + \dots + |\alpha_d|^2$

$= 1$

$$\| |\psi\rangle \| = \| |\psi\rangle \|_2 = \sqrt{|\alpha_1|^2 + \dots + |\alpha_d|^2}$$

unitary transformations



- Start with $|0\rangle$
- Apply $\boxed{R_\varepsilon}$ ← $R_\varepsilon|0\rangle$
- Send it into the box

$$\begin{bmatrix} \cos\varepsilon & -\sin\varepsilon \\ \sin\varepsilon & \cos\varepsilon \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\varepsilon \\ \sin\varepsilon \end{bmatrix} = \cos(\varepsilon)|0\rangle + \sin(\varepsilon)|1\rangle$$

If Dud, $R_\varepsilon|0\rangle$ exits

If Bomb,

- $\mathbb{P}[\text{measure } |0\rangle] = (\cos\varepsilon)^2$ and $|0\rangle$ exits
- $\mathbb{P}[\text{measure } |1\rangle] = \boxed{(\sin\varepsilon)^2 \le \varepsilon^2}$ and $|1\rangle$ exits

for ε suff. small

$\varepsilon - \varepsilon^3/6 + \varepsilon^5/120 - \dots$

Repeat this $\boxed{n = \frac{90^\circ}{\varepsilon}} = \frac{\pi}{2\varepsilon}$ times

$\left(\varepsilon := \frac{\pi}{2n}\right)$

If Dud: $|0\rangle \to R_\varepsilon|0\rangle$

$\to R_{2\varepsilon}|0\rangle$

$\to \dots \to R_{90^\circ}|0\rangle = \boxed{|1\rangle}$ ←

If Bomb: $\mathbb{P}[\text{explosion}] \le \varepsilon^2 n$

If no explosion, output $\boxed{|0\rangle}$

weird

in $\{107, 187)$

$$\mathbb{P}[\text{explosion}] \le \varepsilon^2 n = \frac{\pi^2}{4n^2} \cdot n$$

$$= \frac{\pi^2}{4n} \le \frac{2.5}{n}$$

$n = 1000 \Longrightarrow \mathbb{P}[\text{explosion}] \le 0.0025$

If it doesn't explode, we know whether a dud or a bomb