CHSH game(s):

Ref.

$x, y \in \{0,1\}$ unif. at random

Alice $\xleftarrow{x}$ Ref. $\xrightarrow{y}$ Bob

$A(x) \in \{0,1\}$ $\qquad$ $B(y) \in \{0,1\}$

$\approx 84\%$ $\quad$ 85.3%

$$A(x) \oplus B(y) = x \wedge y$$

Lemma: No classical strategy for Alice/Bob, even used shared randomness can win w/ prob more than 3/4. $\leftarrow$ 75%

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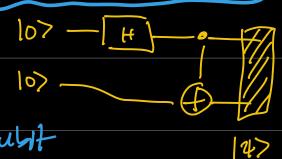
$A(x) \in \{0,1\}$ $\qquad$ $B(y) \in \{0,1\}$

$$A(x) \oplus B(y) = x \wedge y$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$



Alice has 1st qubit
Bob has 2nd qubit

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$|a_0\rangle = (\cos\frac{\pi}{8})|0\rangle + (\sin\frac{\pi}{8})|1\rangle$

$|a_1\rangle = (-\sin\frac{\pi}{8})|0\rangle + (\cos\frac{\pi}{8})|1\rangle$

$|b_0\rangle = (\cos\frac{\pi}{8})|0\rangle - (\sin\frac{\pi}{8})|1\rangle$

$|b_1\rangle = (\sin\frac{\pi}{8})|0\rangle + (\cos\frac{\pi}{8})|0\rangle$

Alice:

$x=0$: Measure in $\{|0\rangle, |1\rangle\}$ basis

$|0\rangle \to A=0$

$|1\rangle \to A=1$

$x=1$: Measure in $\{|+\rangle, |-\rangle\}$ basis

$|+\rangle \to A=0$

Bob:

$y=0$: Measure in $\{|a_0\rangle, |a_1\rangle\}$

$|a_0\rangle \to B=0$

$|a_1\rangle \to B=1$

$y=1$: Measure in $\{|b_0\rangle, |b_1\rangle\}$

$|-\rangle \rightarrow \boxed{A=1}$

$|a_0\rangle \rightarrow B=0$
$|a_1\rangle \rightarrow B=1$

Analysis: $x=y=0$ $\quad x \wedge y = 0$ $\quad$ win if $A=B$

$\Leftrightarrow$ outcomes are $|0\rangle, |a_0\rangle$ or $|1\rangle|a_1\rangle$

$\mathbb{P}[\text{Alice measures } |0\rangle] = \frac{1}{2}$

$\hookrightarrow$ Bob's qubit collapses to $|0\rangle$

$\mathbb{P}[\text{Bob measures } |a_0\rangle \mid \text{Alice measured } |0\rangle]$

$$= |\langle a_0 | 0\rangle|^2 = \cos\left(\frac{\pi}{8}\right)^2 \approx 0.853$$

$\Rightarrow \mathbb{P}[\text{measure } |0\rangle, |a_0\rangle] = \frac{1}{2}\cos\left(\frac{\pi}{8}\right)^2$

$\mathbb{P}[\text{Alice measures } |1\rangle] = \frac{1}{2}$

$\hookrightarrow$ Bob's state collapses to $|1\rangle$

$\mathbb{P}[\text{Bob meas } |a_1\rangle \mid \text{Alice meas } |1\rangle]$

$$= |\langle a_1 | 1\rangle|^2 = \underbrace{\frac{1}{2}\cos\left(\frac{\pi}{8}\right)^2}$$

$$\Rightarrow \mathbb{P}[A(x) \oplus B(y) = 0 \mid x=y=0] = \cos\left(\frac{\pi}{8}\right)^2$$

$|a_0\rangle\langle a_0|$ $\qquad \approx 0.853$

$$P_1 \otimes P_2 = (P_1 \otimes I)(I \otimes P_2)$$

$'l \qquad = (I_2 \otimes I)(P_1 \otimes I)$

$|0\rangle\langle 0|$

Ref.

$x, y \in \{0,1\}$

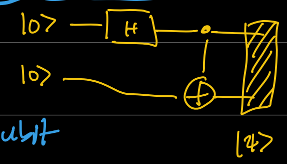
unif. at random

Alice $\xleftarrow{x}$ $\xrightarrow{y}$ Bob

$A(x) \in \{0,1\}$ $\quad$ $B(y) \in \{0,1\}$

$A(x) \oplus B(y) = x \wedge y$

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$



Alice has 1st qubit

Bob has 2nd qubit

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$|a_0\rangle = (\cos\frac{\pi}{8})|0\rangle + (\sin\frac{\pi}{8})|1\rangle$ $\qquad$ $|b_0\rangle = (\cos\frac{\pi}{8})|0\rangle - (\sin\frac{\pi}{8})|1\rangle$

$R_{\pi/8}$ $\quad |a_1\rangle = (-\sin\frac{\pi}{8})|0\rangle + (\cos\frac{\pi}{8})|1\rangle$ $\qquad$ $|b_1\rangle = (\sin\frac{\pi}{8})|0\rangle + (\cos\frac{\pi}{8})|0\rangle$

Alice:

$x=0$: Measure in $\{|0\rangle, |1\rangle\}$ basis

$|0\rangle \rightarrow A=0$

$|1\rangle \rightarrow A=1$

$x=1$: Measure in $\{|+\rangle, |-\rangle\}$ basis

$|+\rangle \rightarrow A=0$

$|-\rangle \rightarrow A=1$

Bob:

$y=0$: Measure in $\{|a_0\rangle, |a_1\rangle\}$

$|a_0\rangle \rightarrow B=0$

$|a_1\rangle \rightarrow B=1$

$y=1$: Measure in $\{|b_0\rangle, |b_1\rangle\}$

$|b_0\rangle \rightarrow B=0$

$|b_1\rangle \rightarrow B=1$

$x=y=1$: $\quad x \wedge y = 1$ $\quad$ win: $A \neq B$

$|+\rangle, |b_1\rangle$ $\qquad$ $|-\rangle, |b_0\rangle$

$A=0, B=1$ $\qquad$ $A=1, B=0$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|++\rangle + |--\rangle)$$

$|+\rangle$ $\qquad$ $|+\rangle$

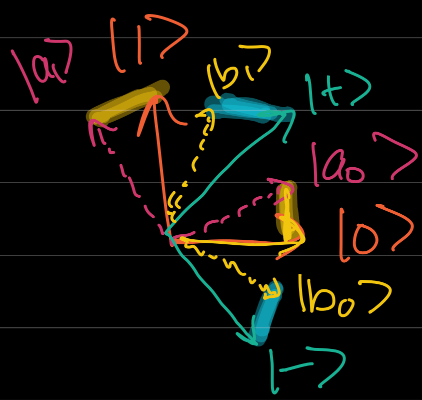
$$|++\rangle = \frac{(|0\rangle + |1\rangle)}{\sqrt{2}} \otimes \frac{(|0\rangle + |1\rangle)}{\sqrt{2}}$$

$\|$

$|+\rangle \otimes |+\rangle = \frac{1}{2}\left(|00\rangle + \cancel{|01\rangle} + \cancel{|10\rangle} + |11\rangle\right)$

$|--\rangle = \frac{(|0\rangle - |1\rangle)}{\sqrt{2}} \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$

$= \frac{1}{2}\left(|00\rangle + |11\rangle - \cancel{|01\rangle} - \cancel{|10\rangle}\right)$



No-Cloning Theorem

No quantum transformation that maps $|\psi\rangle \mapsto |\psi\rangle \otimes |\psi\rangle$

$\underbrace{|\psi\rangle |000\cdots0\rangle}$ ancillary qubits

$\not\mapsto |\psi\rangle \otimes |\psi\rangle \otimes |\text{garbage}\rangle$

$\text{CNOT } |0\rangle|0\rangle = |0\rangle|0\rangle$

$\text{CNOT } |1\rangle|0\rangle = |1\rangle|1\rangle$

$\text{CNOT } |+\rangle|0\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$ $\quad |++\rangle?$

$\qquad ||$ $\qquad\qquad\qquad\qquad ||$

$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $\qquad \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

No way to find out α and β.

$\begin{bmatrix}\alpha & \beta\end{bmatrix}\begin{bmatrix}0\end{bmatrix} = \alpha|0\rangle + \beta|1\rangle$

$|0\rangle$

Quantum tomography: $|\psi\rangle \otimes |\psi\rangle \otimes |\psi\rangle \otimes \cdots \otimes |\psi\rangle$