CSE P531: Computability and Complexity Theory (Spring, 2016)

Homework 4 Out: Thursday, 21-Apr. **Due**: Saturday, 30-Apr (9pm in the **Dropbox**)

Reading:

Sipser, 7.1—7.4

Instructions:

Your proofs and explanations should be clear, well-organized and as concise as possible.

You are allowed to discuss the problems with fellow students taking the class. However, you must write up your solutions completely on your own. Moreover, if you do discuss the problems with someone else, I am asking, on your honor, that you do not take any written material away from the discussion. In addition, for each problem on the homework, I ask that you acknowledge the people you discussed that problem with, if any.

Most of the problems require only one or two key ideas for their solution – spelling out these ideas should give you most of the credit for the problem even if you err in some finer details. So, make sure you clearly write down the main idea(s) behind your solution.

A final piece of advice: Begin work on the problem set early and don't wait until the deadline is only a few days away.

1. Graph isomorphism

Two undirected graphs H and G are called **isomorphic** if the nodes of G can be reordered so that it is identical to H. Define the language

$$ISO = \{\langle H, G\rangle : G \text{ and } H \text{ are isomorphic graphs}\}.$$

Show that $ISO \in \boldsymbol{NP}$.

2. TRIPLE SAT

Consider the language

TRIPLE-SAT = { $\langle \phi \rangle$: ϕ is a Boolean formula that has at least **three** satisfying assignments }

Show that TRIPLE-SAT is NP-complete using a reduction from SAT (you may assume that SAT is NP-complete).

3. Vertex Cover vs. Dominating Set

A **vertex cover** in an undirected graph $G = (V, E)$ is a subset of nodes $C \subseteq V$ such that every edge has at least one endpoint in C. A **dominating set** of G is a set of nodes $D \subseteq V$ such that every node in G is in D or has a neighbor in D. Consider the two problems:

VERTEX-COVER = { $\langle G, k\rangle$: G has a vertex cover of size at most k }

DOMINATING-SET = { $\langle G, k\rangle$: G has a dominating set of size at most k }

a) Prove that both problems are in NP

b) Prove that VERTEX-COVER $\leq_P$ DOMINATING-SET

EXTRA CREDIT [worth 3 regular problems]

Recall the Graph Isomorphism problem (from Problem #1 above): Two graphs H and G are isomorphic if the nodes of H can be reordered so that it is identical to G.

Show that if $P = NP$ then the following problem has a polynomial-time algorithm: Given input graphs H and G that are isomorphic, **find the reordering of H that yields G.**

Recall that P and NP involve **decision** problems (with yes/no answers). But your algorithm should actually output the correct reordering! (This is an example of the "search vs. decision" phenomenon we discussed in class.)