Checking Matrix Multiplication

A Randomized Algorithm

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Problem

- A, B, and C are matrices of size n x n.
- Question:

$$A \stackrel{?}{=} BC$$

Solution 1

$$A \stackrel{?}{=} BC$$

- Perform standard matrix multiplication for BxC and compare the results with A.
- $O(n^3)$ [n^2 output, each output requires n op]
- State of the art multiplication $O(n^{2.373})$ [Redefines the matrix to remove some redundant operations].
- Can we do better?

Solution 2

$$A \stackrel{?}{=} BC$$

- Choose a vector $v = (v_1, v_2, \ldots., v_n) \in \{-1, 1\}^n$ uniformly at random.
- Check $Av = (BC)v$.
- $(BC)v = B(Cv) \leftarrow O(n^2)$.
  [two matrix-vector multiplication]
- Pretty fast, but how accurate?

Solution 2

$A \stackrel{?}{=} BC$

$$Claim:\ If\ A \neq BC,\ then\ P[Av = BCv] \leq \frac{1}{2}$$

Solution 2

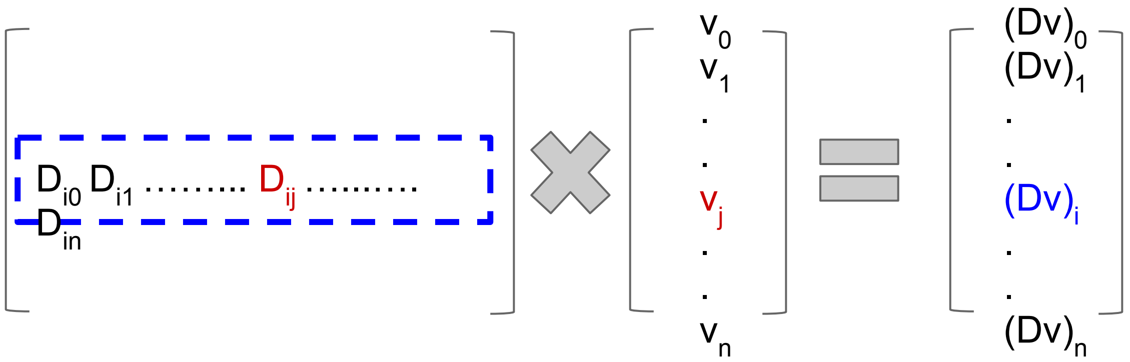
$$A \stackrel{?}{=} BC$$

- Proof: Let $D = A - BC$
- Since A □ BC, then D □ 0.
- Let i, j be indices such that D_{ij} □ 0.
- Then:

Solution 2

$$A \stackrel{?}{=} BC$$

$$D = A - BC$$



$$P[Av = BCv] = P[Dv = 0] \leq P[(Dv)i = 0]$$

$$(Dv)_i = \sum_{k=1}^{n} D_{ik} v_k \qquad (Dv)_i = D_{ij} v_j + \sum_{k \neq j}^{n} D_{ik} v_k$$

Questions?