Cellular Automata

The game of life or a new kind of science?

Richard Ladner

The Plan

- Automata
  - Von Neumann to Wolfram
- Demonstrations
  - Game of Life program
  - Developed by Jim Fix
  - Behaviors developed by high school students
  - Sophisticated behaviors implemented by Sam Coskey

What are Cellular Automata?

"Cellular automata have been invented many times under different names… In pure mathematics they can be recognized as a branch of topological dynamics, in electrical engineering they are sometimes called iterative arrays, and high school kids may know them as a sort of home-computer game. They have been used and abused by interdisciplinary scientists as well as interdisciplinary bumblers."

Toffoli and Margous
Cellular Automata Machines
1987

What are Cellular Automata?

"When I made my first discoveries about cellular automata in the early 1980s I suspected that I had seen the beginning of something important. But I had no idea just how important it would all ultimately turn out to be. And indeed over the past twenty years I have made more discoveries than I ever thought possible. And a new kind of science that I have spent so much effort building has seemed an ever more central and critical direction for future intellectual development."

Stephen Wolfram
A New Kind of Science
2002

Automata?

- Automata is the plural of automaton
- Simple computing device
- Properties
  - Finite set of states
  - Transitions from state to state
    - Sense the environment.
    - Possibly change the environment.
    - Go to a new state,

Automaton Example

Coke machine

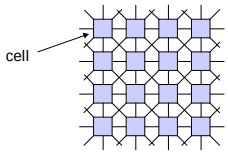
- Inputs:
  coins, bills, return button, choice buttons
- State:
  money entered so far,…
- Outputs:
  coke, sprite, dr. pepper, returned coins, …

Other Examples

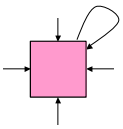
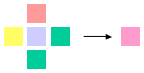



Cellular Automata

- Automata are arranged geometrically
- All automata are identical
- All automata change state simultaneously

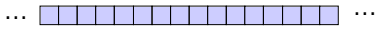


Communication

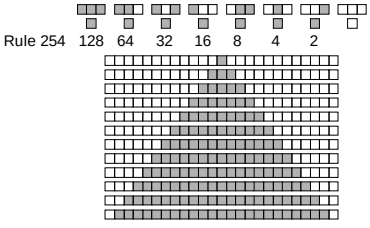
- Inputs are states of neighbors and self
- Output is the state (indicated by color)

One-Dimensional

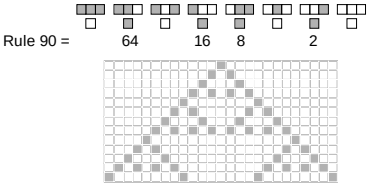


- Each cell has a left and right neighbor
- All cells identical
- Cell can be initialized to different states

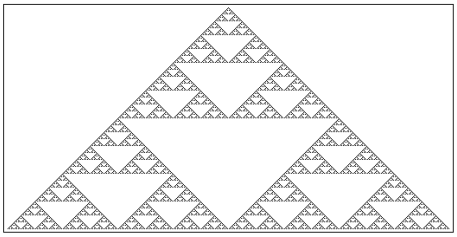
Two State Example



Rule 90



Rule 90



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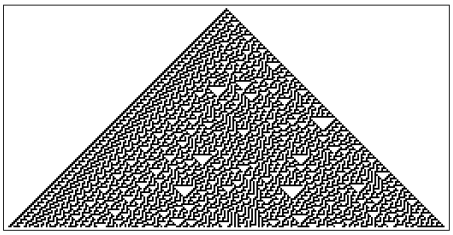
Rule 30



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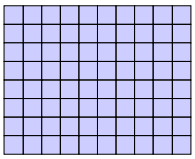
Two-Dimensional



- Each cell has 4 or 8 neighbors

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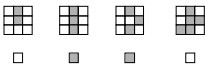
Game of Life

- Each cell is “live” or “dead”
- Transition rules
  - N = number of live neighbors among the 8
  - N ≤ 1 → death (loneliness)
  - N = 2 → no change
  - N = 3 → birth
  - N ≥ 4 → death (overcrowding)

examples 

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Game of Life

- The Glider
- The Glider gun and eater
  - Gosper 1970
- Alternative life games

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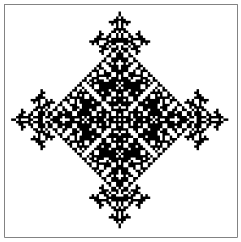
Code 494



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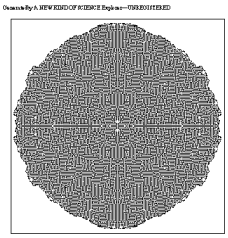
Code 746



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History

- John von Neuman & Stanislaw Ulam (1950)
  - Self reproducing Machines
- John Conway (1970)
  - The game of life
  - Popularized by Martin Gardner in *Scientific American* magazine
- Stephen Wolfram (2002)
  - "A New Kind of Science"

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Applications

- Biological systems
- Iterative arrays – parallel computer hardware
- Artificial societies
- Art and design
- Computer graphics
- Image processing
- Games

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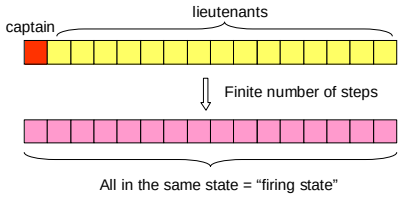
Firing Squad Problem

- One-dimensional cellular automaton
- Synchronous behavior possible



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Firing Squad Problem Solutions

- Proposed by Myhill (1957)
- Moore Solution (1962)
  - Called the "signal solution"
  - 13 states
  - 3n time
- Mazoyer Solution (1988)
  - Only 6 states
  - 2n time (minimal)
  - 4 states impossible
  - 5 states unknown

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Self-Reproducing Cellular Automaton

- Two-dimensional with 4 neighbors
- Initial configuration is exactly duplicated and spread throughout the plane
- Von Neumann Solution (1952)
  - 29 states, 200,000 cell initial configuration
- Langton Solution (1984)
  - 8 states, 125 cell initial configuration
- Byl Solution (1989)
  - 6 states, 16 cell initial configuration

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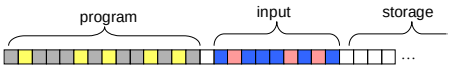
Universality

- There is a one-dimensional cellular automaton that is a general purpose computer.



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Life is Universal

- The Game of Life is universal (Gosper and Conway 1971)
  - Any computation can be done by setting up the initial configuration and letting it run.

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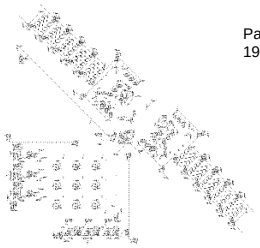
Rendell's Universal Life Machine



Paul Rendell
1980s

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Rule 110 is Universal

- One-dimensional
- Matthew Cook 1990s
- Rule 110

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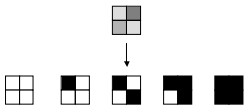
Image Processing Example

- Gray scale to black and white



Pick the 2x2 black and white block that
Best approximates the input block

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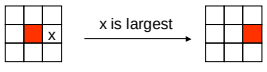
Follow the Scent Game

- Food is the highest number
- Numbers smaller farther from the food



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"A New Kind of Science"

- Wolfram's thesis
  - Complex behaviors are often the result of simple computational rules.
  - The proof: simple cellular automata and their variants produce such complex behavior.
- Corollary
  - Traditional mathematical approaches (continuous mathematics) to modeling complex behavior is not enough.

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Resources

- Books -
  - Martin Gardner - *Wheels, Life, and Other Mathematical Amusements*
  - Toffoli and Margolus - *Cellular Automata Machines*
  - Stephen Wolfram - *A New Kind of Science*
- Web Pages
  - http://nojava.cafaq.com/index.shtml
  - http://psoup.math.wisc.edu/
  - http://www.cs.washington.edu/homes/scoskey/ca/

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