NLTS Hamiltonians from good quantum codes

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Understanding classical proofs

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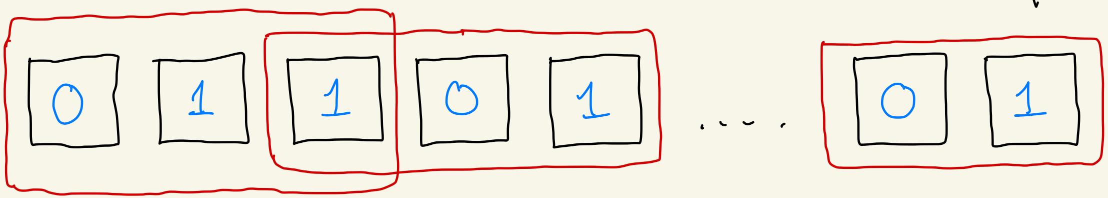

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local check $C_i = x_1 \oplus x_2 \oplus x_3 = 0$.

$C_i : \{0,1\}^3 \longrightarrow [0,1]$.

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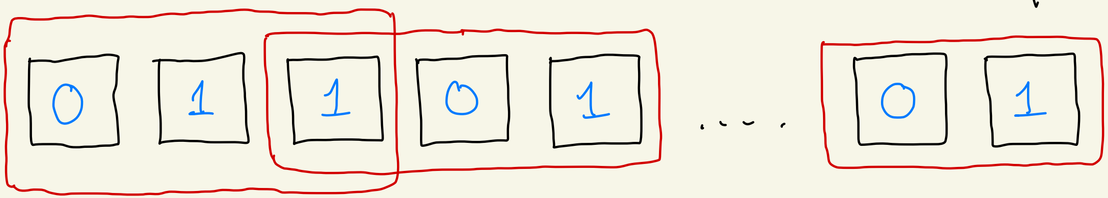

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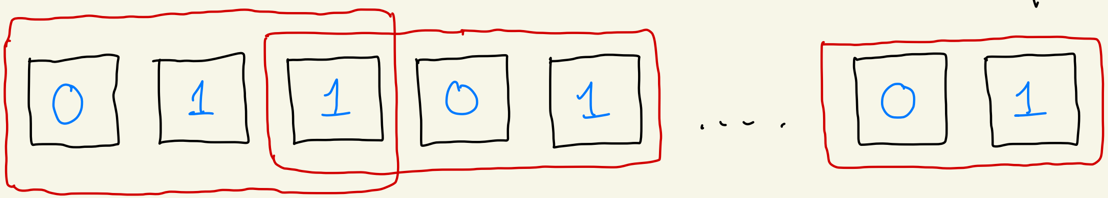

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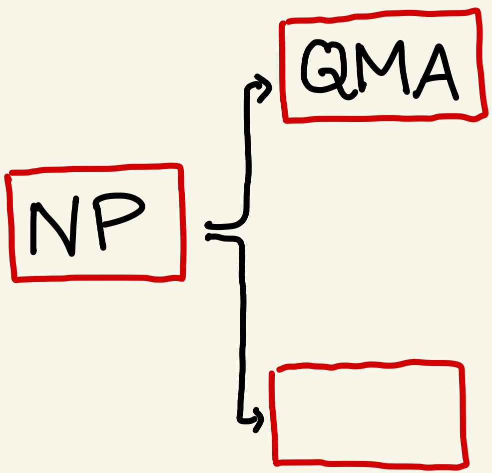
Decide if

① $\exists\, x,\ C(x) = 0$.

② $\forall\, x,\ C(x) \geq 1$.

Two extensions of the notion of proofs



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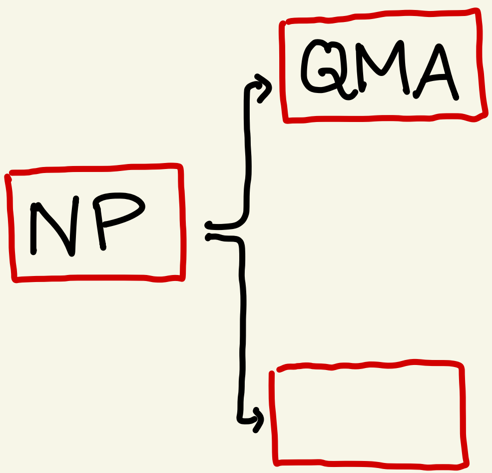

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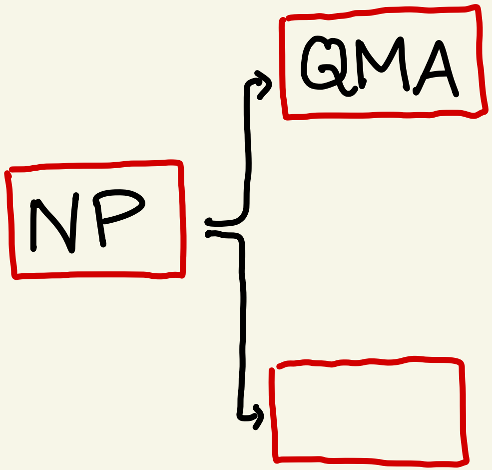

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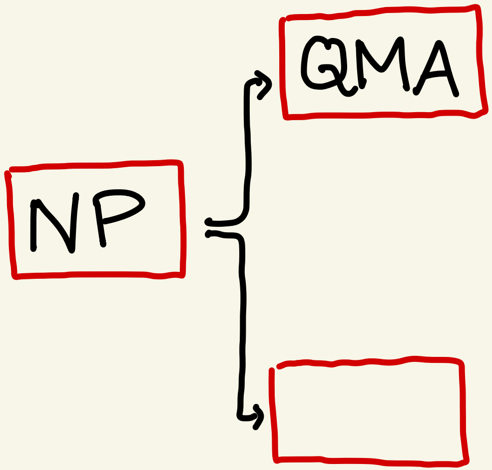

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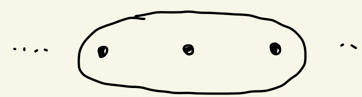

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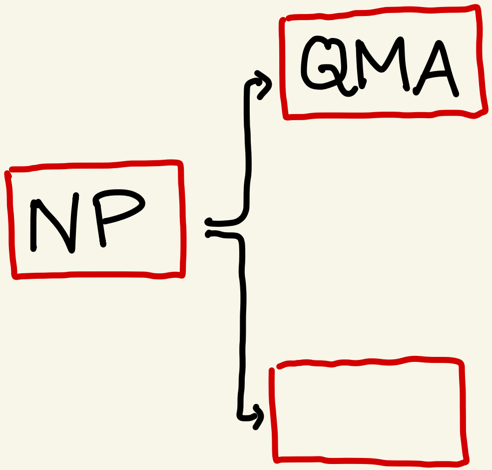

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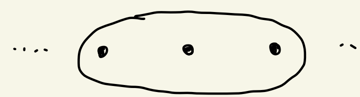

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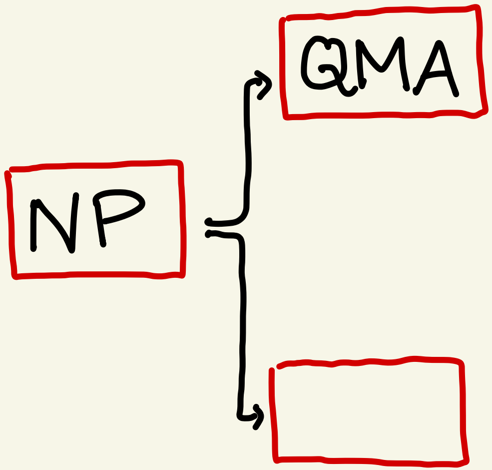

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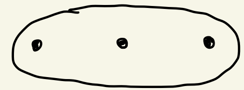
....  ... $h_i = |000\rangle\langle 000| + |111\rangle\langle 111|$

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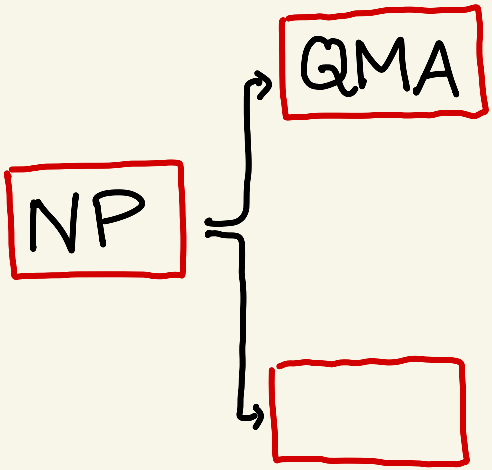

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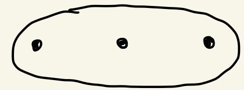
....  ... $h_i = |000\rangle\langle 000| + |111\rangle\langle 111|$

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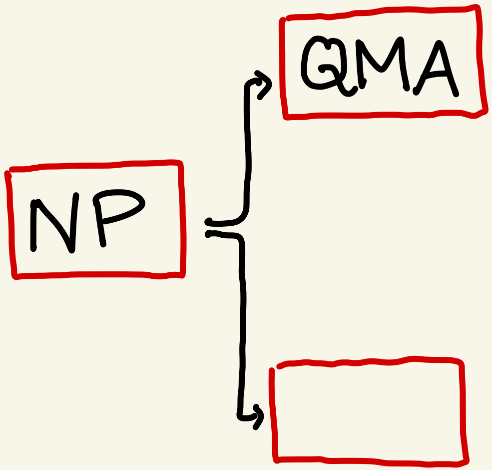

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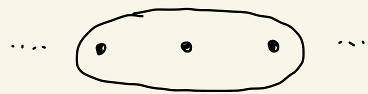

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QMA-hard to decide for $b-a = 1/\mathrm{poly}(m)$,

1. $\lambda_{min}(\mathbf{H}) \leq a \iff \exists |\psi\rangle, \langle\psi|\mathbf{H}|\psi\rangle \leq a$
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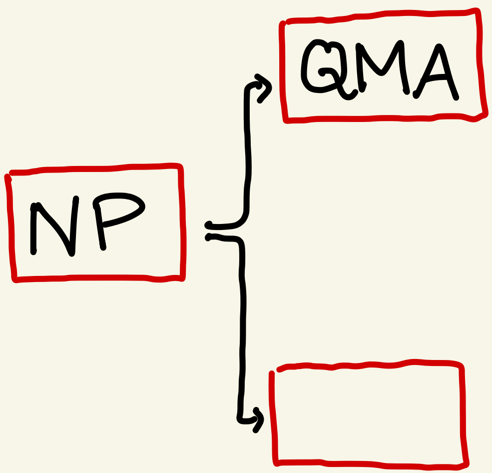

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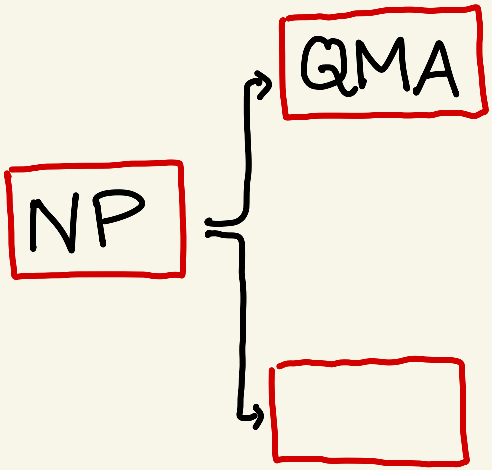

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$\Longrightarrow$ groundstates of local Hamiltonians are a "canonical" form for all q. pfs.

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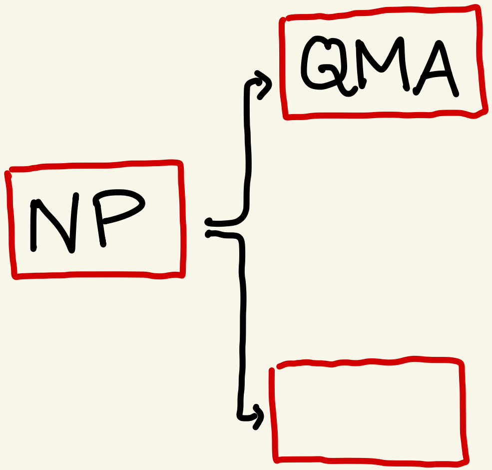

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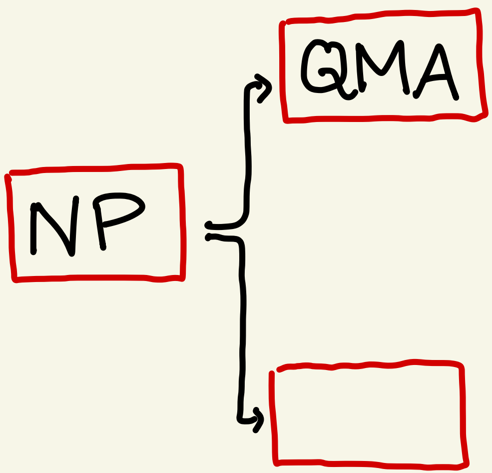

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It's widely believed that $NP \neq QMA$

> Therefore, not all groundstates of local Hamiltonians can be <u>classically</u> described (in an efficiently verifiable manner)

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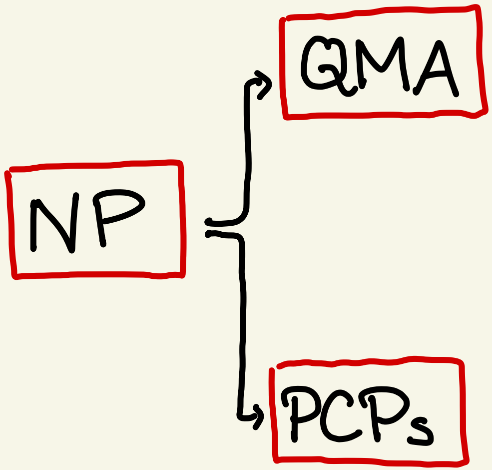

Two extensions of the notion of proofs

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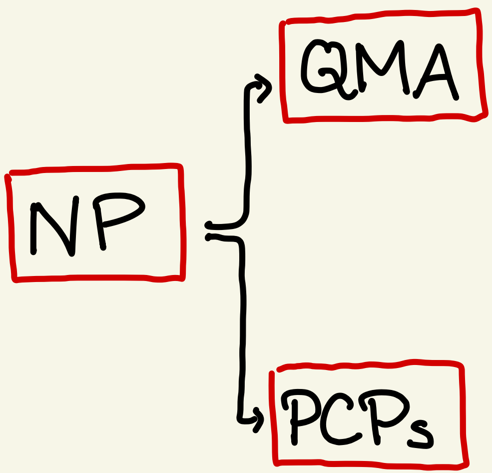

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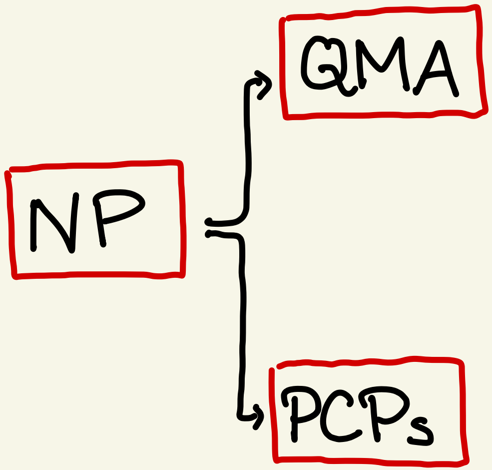

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Arora-Safra et.al 98, Dinur

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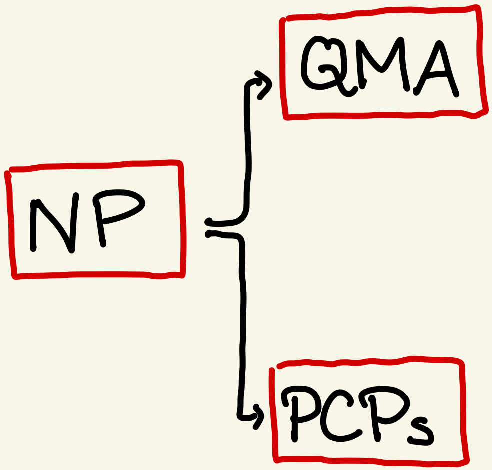

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[$C(x)$ = analog of $\langle \psi | \mathbf{H} | \psi \rangle$]

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NP → QMA

NP → PCPs

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Important consequence: $\boxed{\text{Noisy pfs suffice!}}$

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Any x s.t. $C(x) < \frac{m}{4}$ can be prob. verified with $O(1)$ queries.

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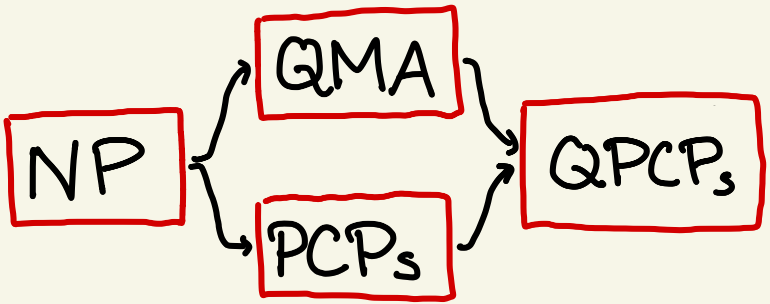
The Quantum Prob. Checkable Pfs. Conjecture



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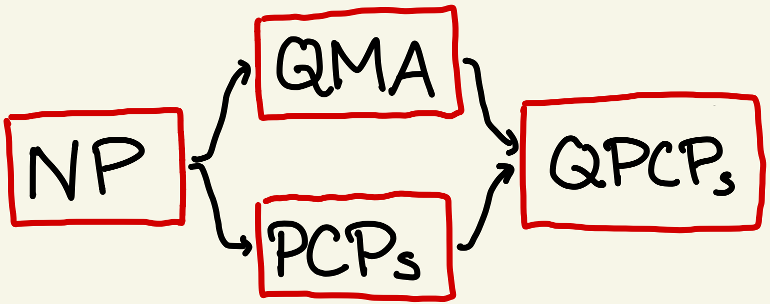

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Similar to PCP theorem, every state of energy $\leq \frac{\varepsilon}{2} m$ is a valid pf. for a QPCP local Hamiltonians.

Set of pfs is much larger!

An important consequence of QPCPs

(A) (if NP $\neq$ QMA) quantum pfs. cannot be classically described (in any efficiently checkable manner)

(B) low. energy states of QPCP local Hamiltonians are also valid pfs (since they are noisy pfs.)

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Local Hamiltonians corresponding to most* linear-rate and -distance QLDPC error-correcting codes are NLTS Hamiltonians.

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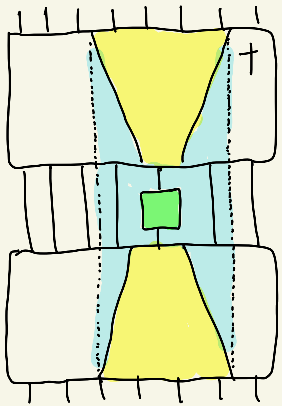
$\exists\, \varepsilon > 0$, and Hamiltonian family $\mathbf{H}$ s.t. every state ψ of energy $\leq \varepsilon n$, the minimum depth circuit to generate ψ is $\Omega(\log n)$.

Proof sketch of the NLTS theorem

① Trivial states ⇒ Local Hamiltonians

⇒ Circuit depth lower bounds



Lightcones for low depth circuits

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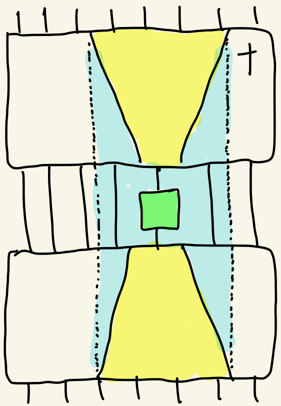
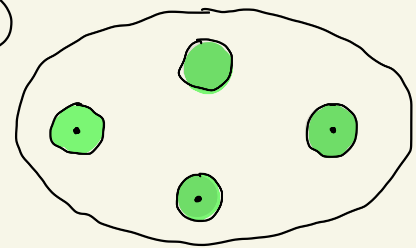


Lightcones for low depth circuits

Error Correction Codes (ECC)

② 

low energy subspace of expanding codes.

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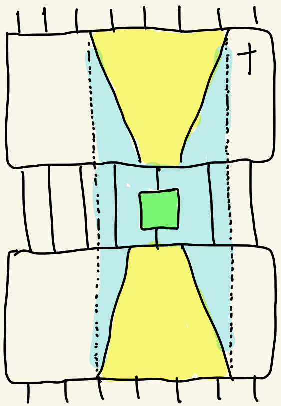

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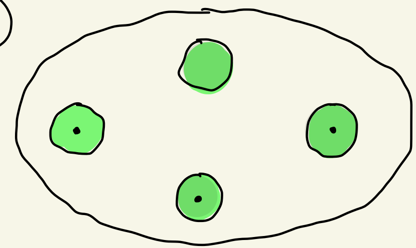
② 

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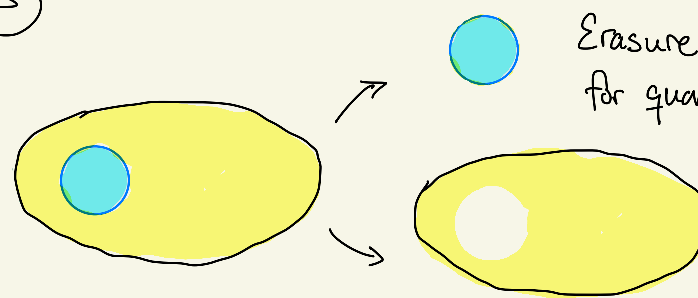
③ 

Erasure errors for quantum codes

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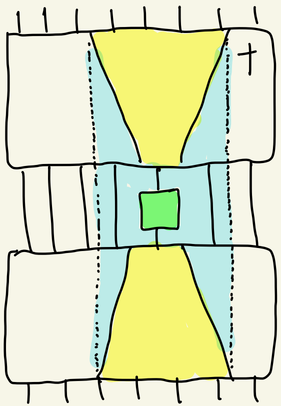

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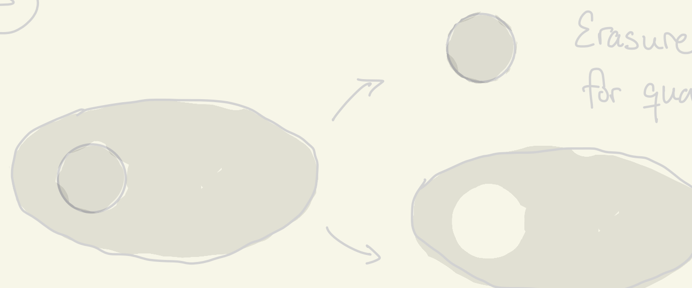

Erasure errors for quantum codes

Lightcones and quantum circuits

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$$\boxed{\text{Low-depth states are classical witnesses for energy}}$$

Lightcones and quantum circuits

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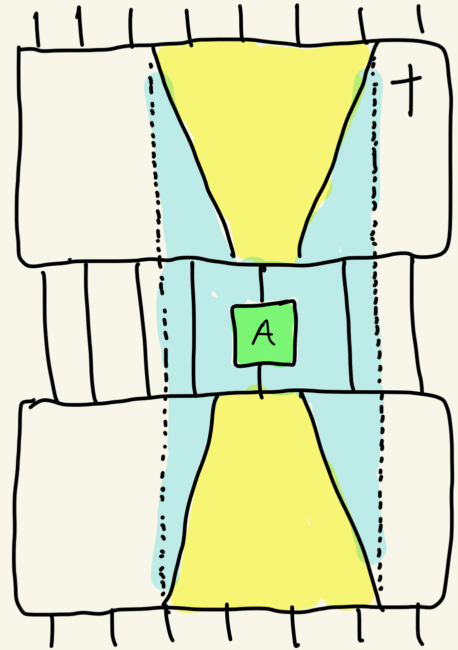

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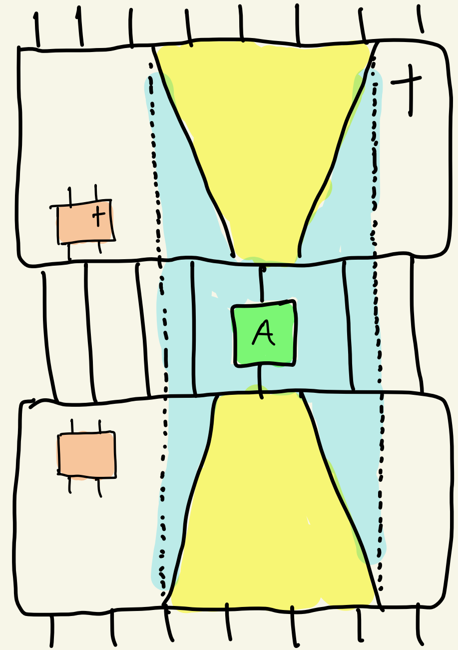

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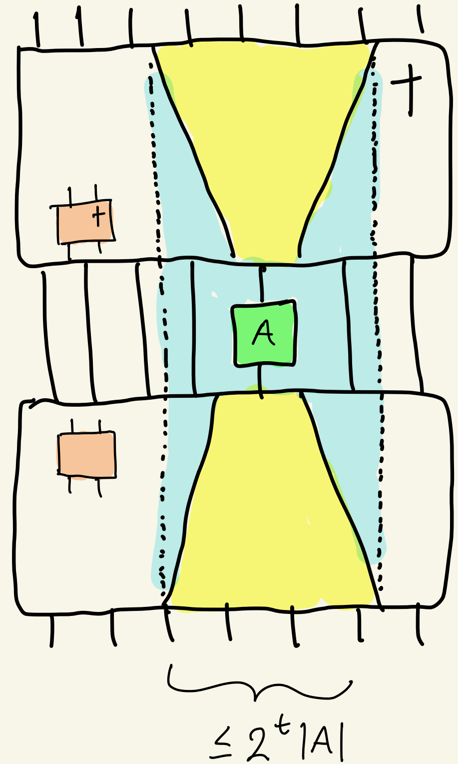

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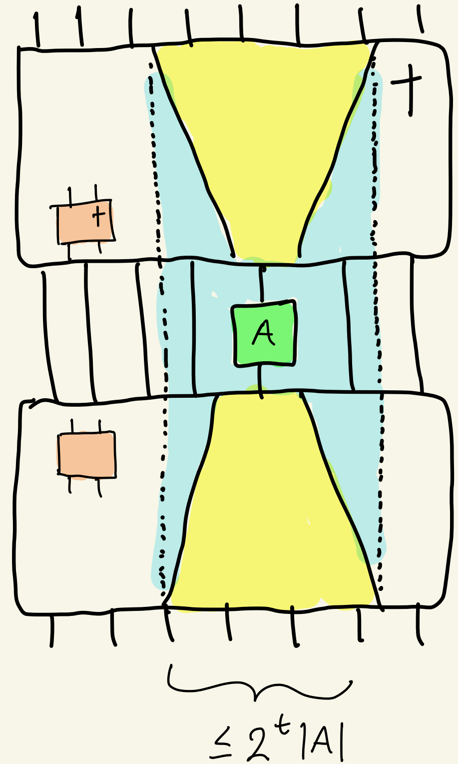

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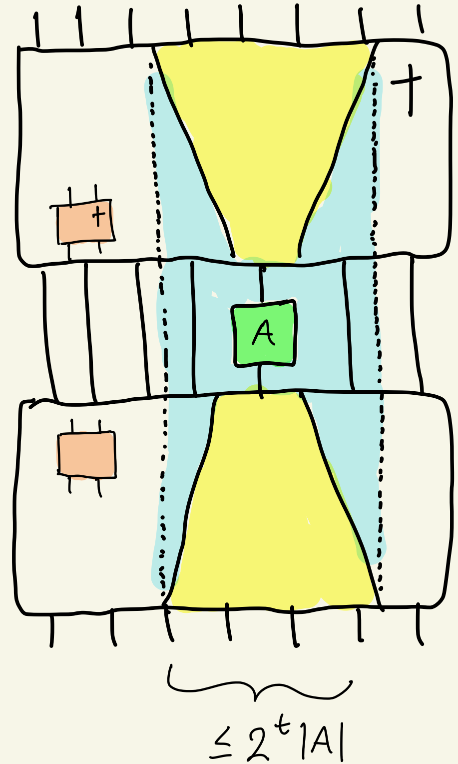

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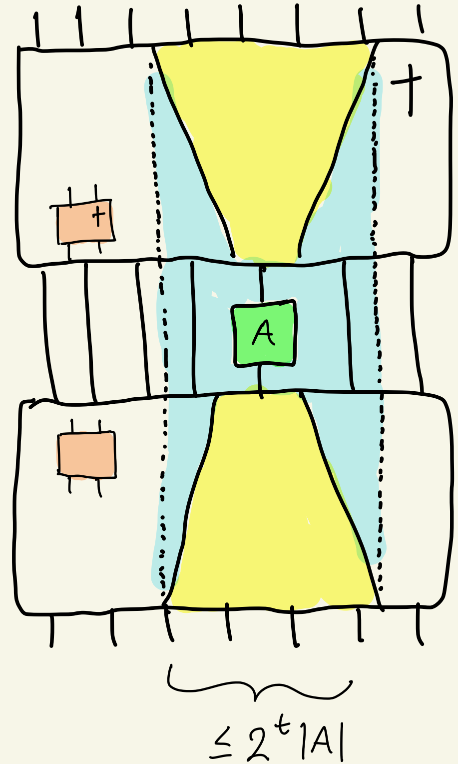

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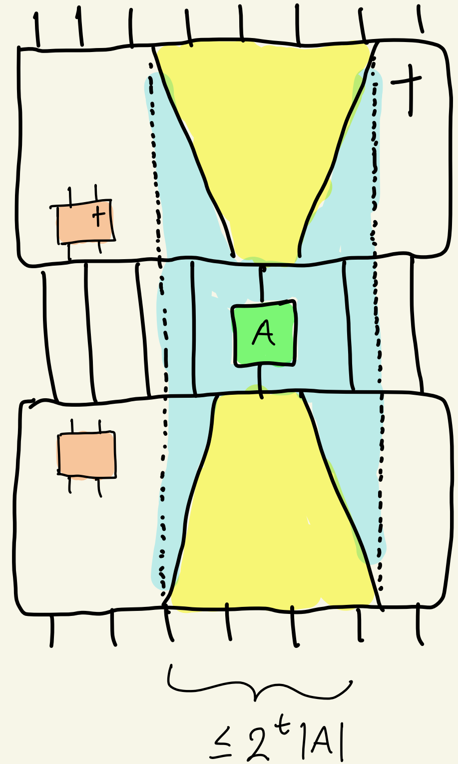

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Trivial states $\Rightarrow$ Local Hamiltonians

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And $\mathbf{H}_\mathcal{U}$ is a 2^t-local Hamiltonian.

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Any strict reduced density matrix equals

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But groundstate $|\psi\rangle$ is unique! $\Rightarrow$ $|\psi\rangle = |\psi'\rangle$, a contradiction!

Local indistinguishability

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Using mathematics from Chebyshev polynomials, we can make l.b. robust.

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$$\exists\, p : \mathbb{R} \to \mathbb{R} \text{ of deg } O_\mu(\sqrt{n}) \text{ s.t. } \left\| p(\mathbf{H}_u) - |\psi\rangle\langle\psi| \right\|_\infty \leq \mu$$

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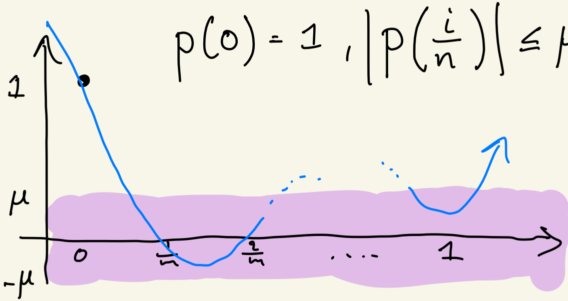
$1-p$ is the Chebyshev poly. approx. of the OR function.

$p(0) = 1$, $\left|p\left(\frac{i}{n}\right)\right| \leq \mu$



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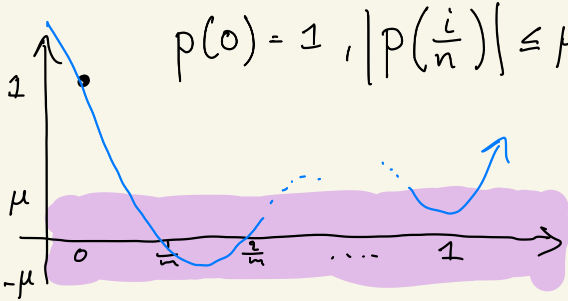

$p(\mathbf{H}_u)$ is a local Hamiltonian of

$$\text{locality } O(\underbrace{2^t}_{\text{locality of } \mathbf{H}_u} \cdot \underbrace{\sqrt{n}}_{\text{poly degree}})$$

Robust local indistinguishability

$p(\mathbf{H}_u)$ is a $L := O(2^t \cdot \sqrt{n})$ local Ham. s.t.

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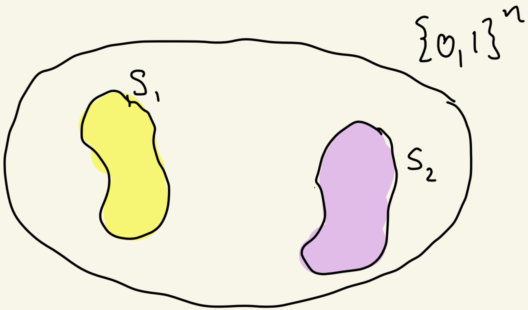
Let D be the dist. on $\{0,1\}^n$ formed by measuring $|\psi\rangle$.



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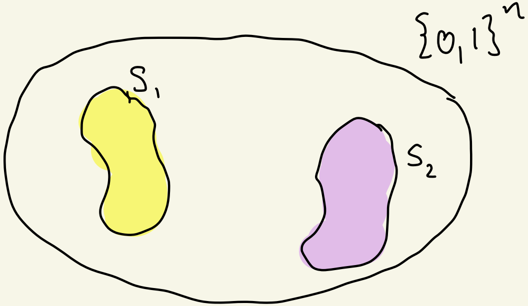

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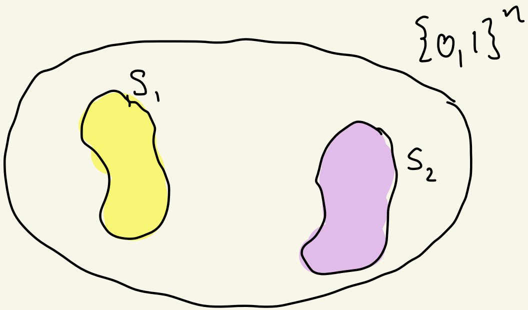

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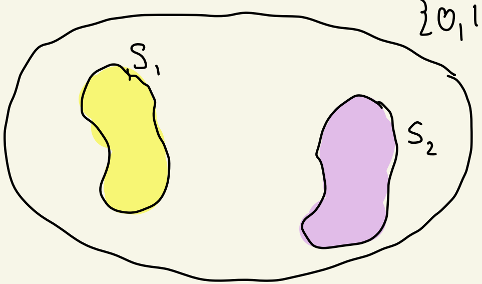

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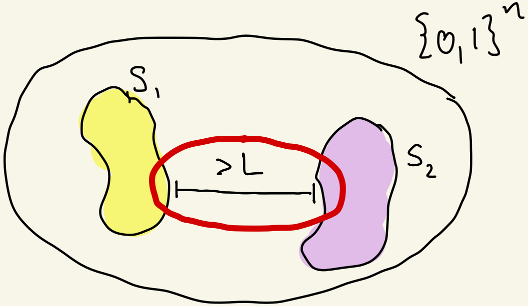

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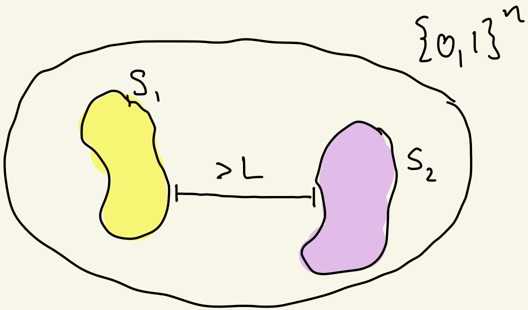

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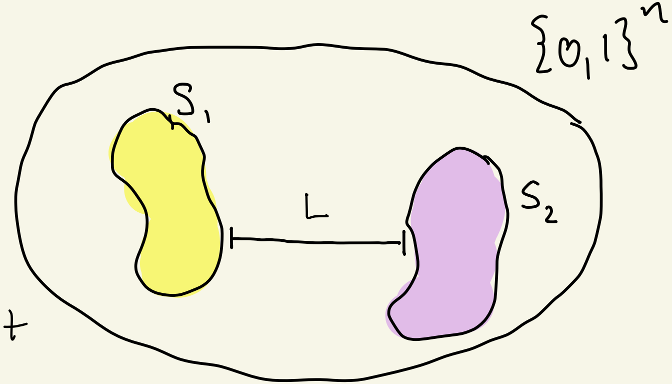
②
$$\| \Pi_{S_1} p(\mathbf{H}_u) \Pi_{S_2} \|_\infty = 0$$

due to locality of $p(\mathbf{H}_u)$ being small.

Robust local indistinguishability



Thm Any dist. D s.t. $D(S_1), D(S_2) > \mu$ cannot be generated by a quantum circuit of depth $\leq \Omega\left(\log\left(\frac{L^2\mu}{n}\right)\right)$.

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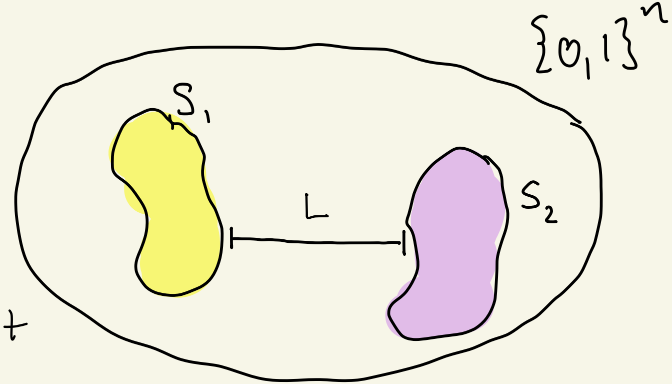

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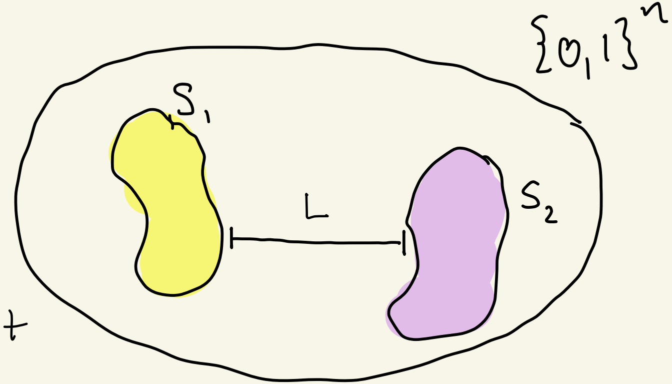

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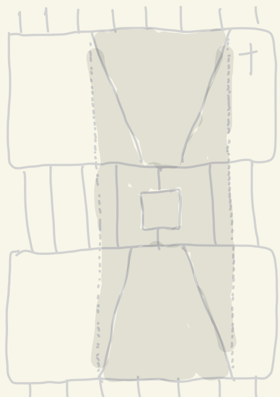
If $L \geq \omega(\sqrt{n})$ and $\mu \geq \Omega(1)$, call D a "well-spread" dist.

well-spread dist. is a signature of quantum depth.

Proof sketch of the NLTS theorem

1 Trivial states ⇒ Local Hamiltonians

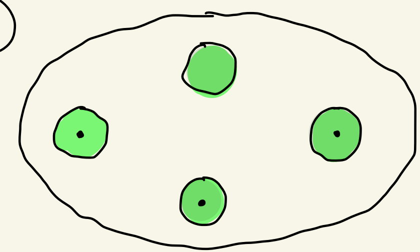
⇒ Circuit depth lower bounds



Lightcones for low depth circuits

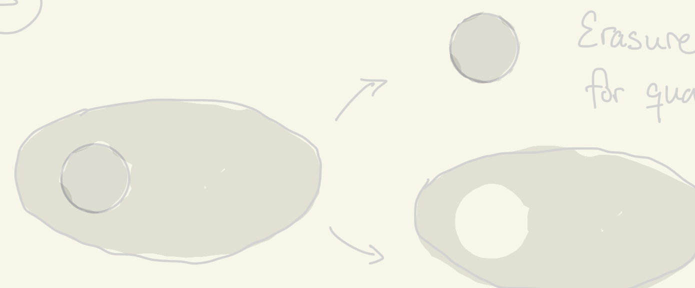
Error Correction Codes (ECC)

2



low energy subspace of expanding codes.

3



Erasure errors for quantum codes

Expanding codes & Tanner codes

A linear code $\subseteq \{0,1\}^n$ can be expressed as $\ker H$ for $H \in \mathbb{F}_2^{m \times n}$

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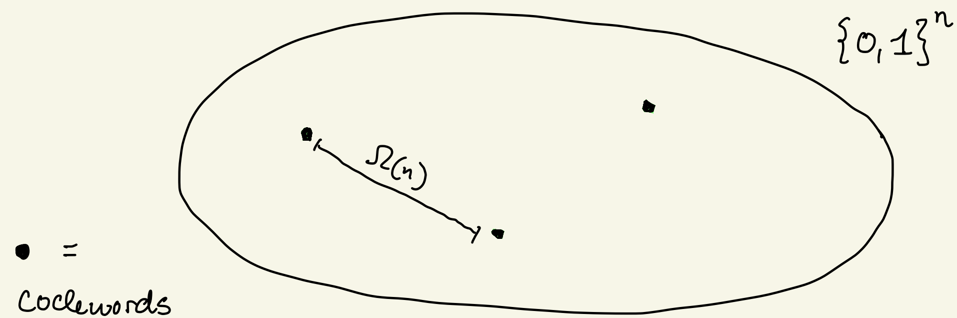

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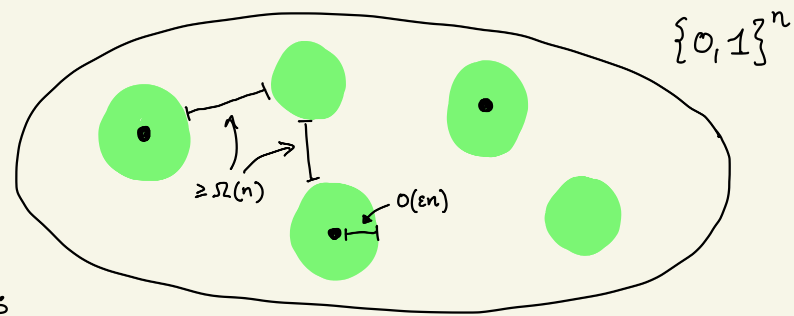
= states that violate $\leq \varepsilon m$ checks

• = codewords



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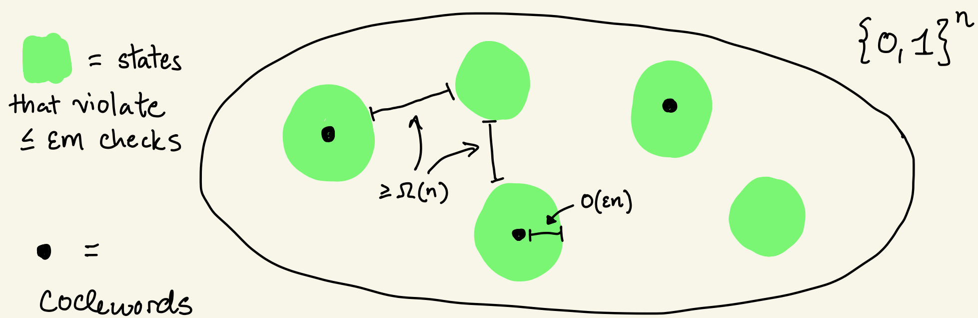

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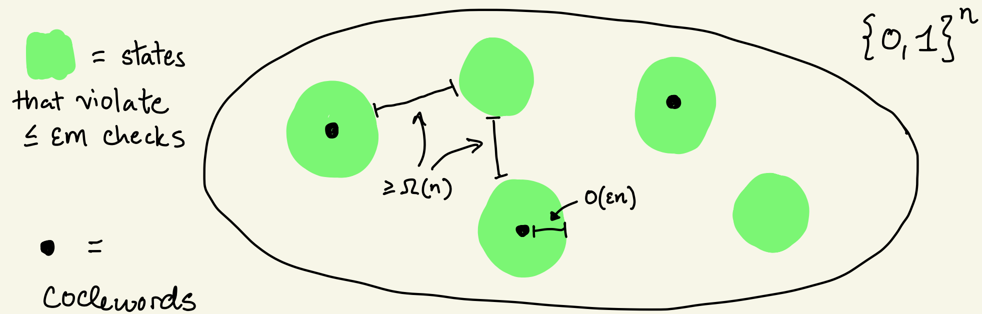
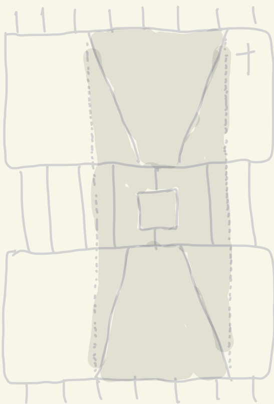


Only question is how to construct Hamiltonian with such property?

Proof sketch of the NLTS theorem

① Trivial states ⇒ Local Hamiltonians

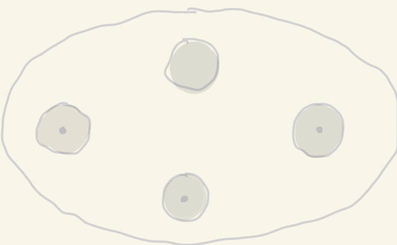
⇒ Circuit depth lower bounds



Lightcones for low depth circuits

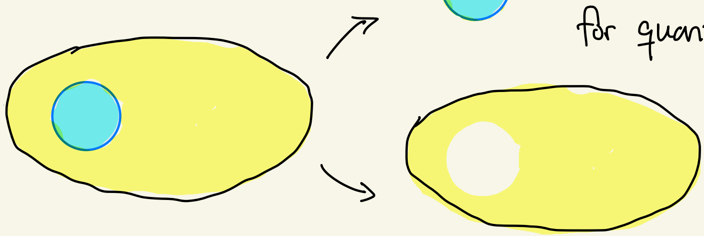
Error Correction Codes (ECC)

②



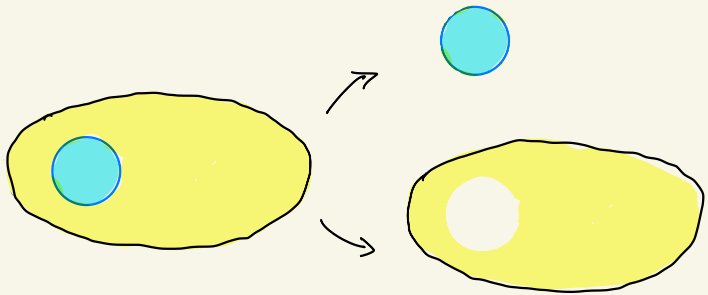
low energy subspace of expanding codes.

③



Erasure errors for quantum codes

Quantum error correcting codes



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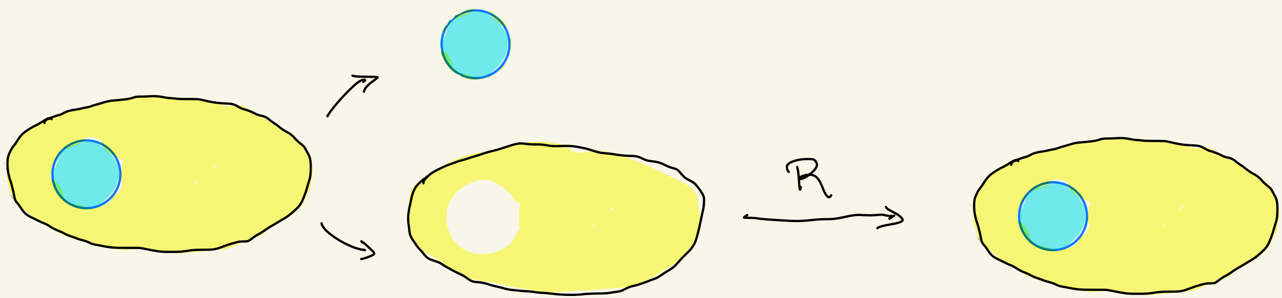

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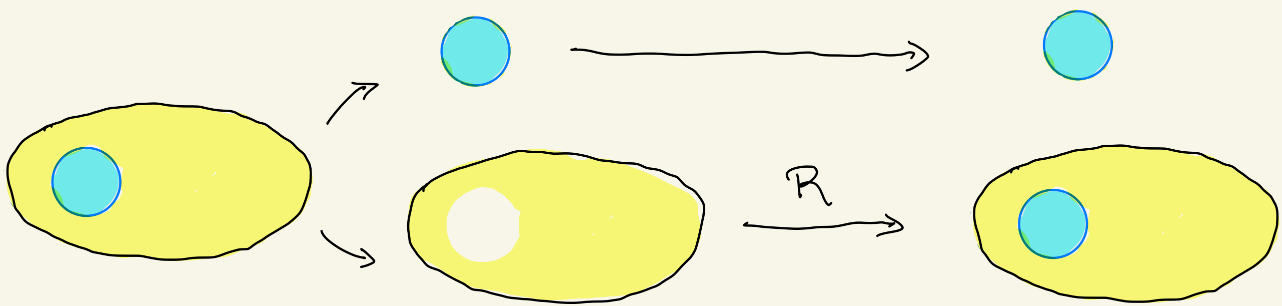

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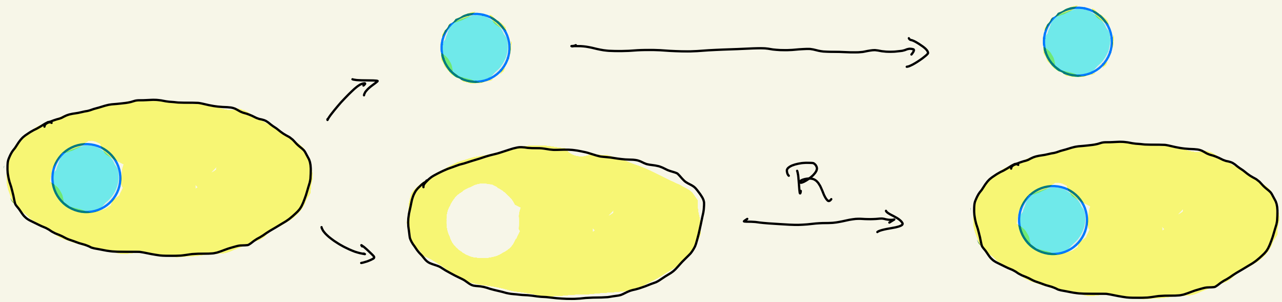

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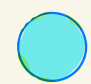
If we could recover the original state then unless  contains no information about the original state, this violates the no-cloning theorem.

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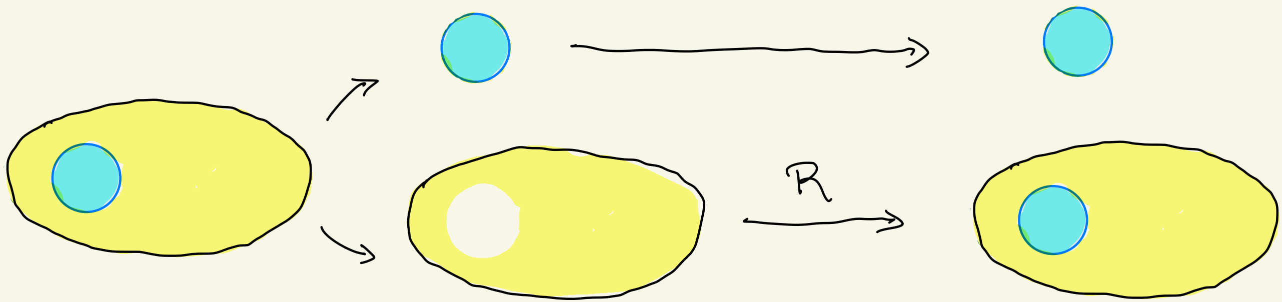

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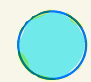
If we could recover the original state then unless  contains no information about the original state, this violates the no-cloning theorem.

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How do we prove circuit depth lower bounds for the low-energy subspace of these code Hamiltonians?

Optimal-parameter CSS codes

There is a class of q. codes called Calderbank-Shor-Steane codes that correct for X-type (bit-flip) and Z-type (phase-flip) errors separately.

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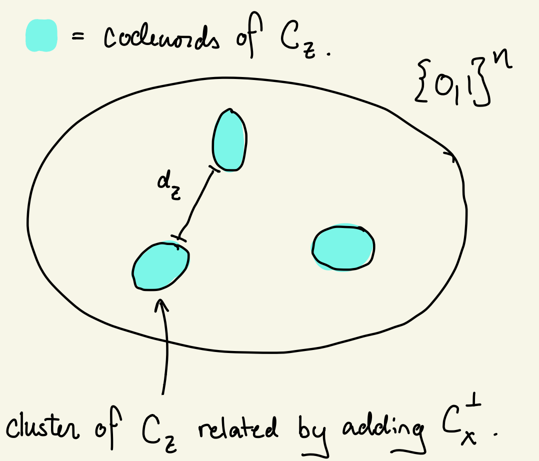

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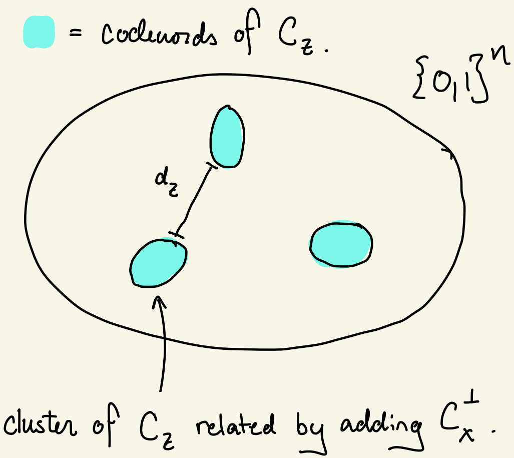

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Similar to classical example, we consider codes that have the property that if $|H_z y| \leq \varepsilon m$ then either

① $|y|_{C_x^\perp} \leq c_1 \varepsilon n$ or

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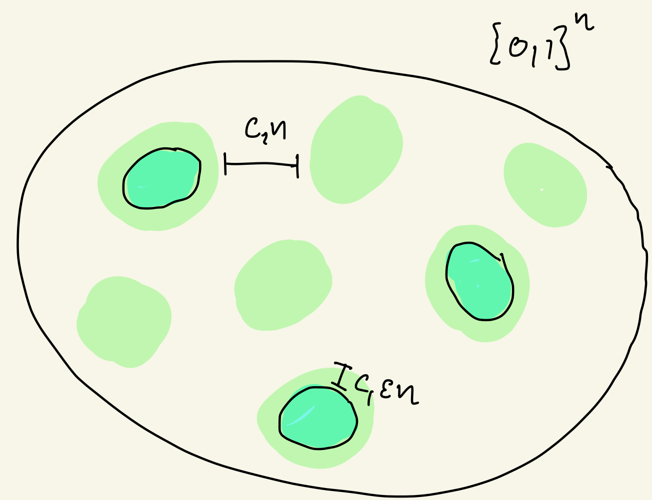

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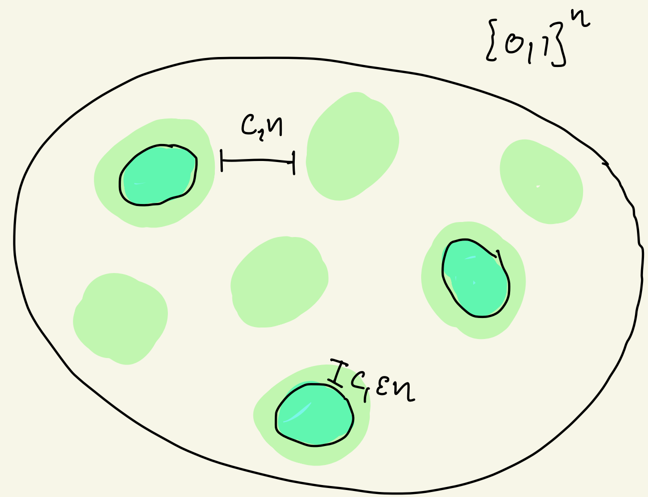
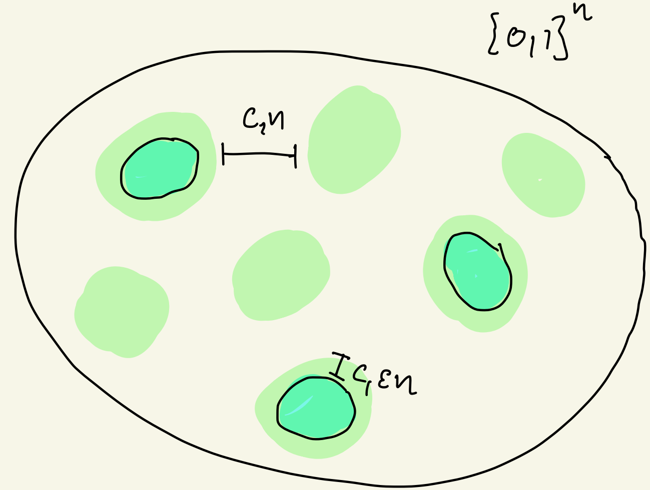


And, if we consider a $\frac{\varepsilon}{200}$-low-energy state of the code's local Hamiltonian, measuring in the Z-basis yields a dist. 99.5% supported on .

The uncertainty principle



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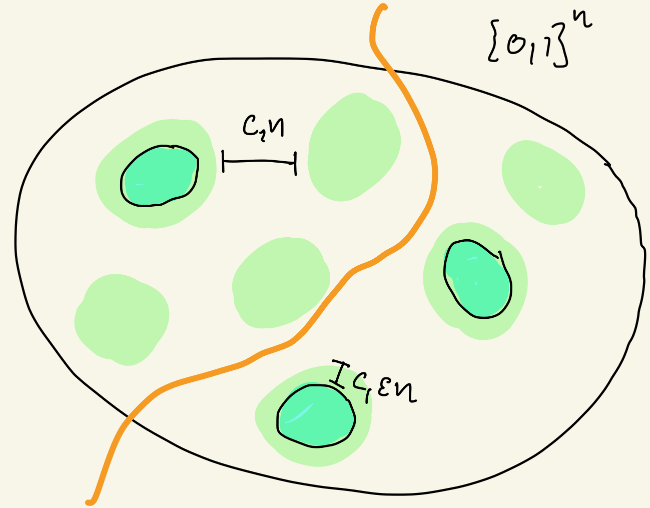

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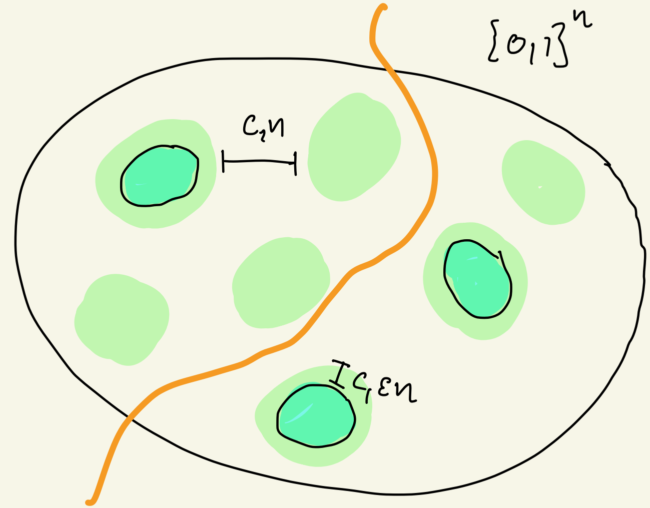

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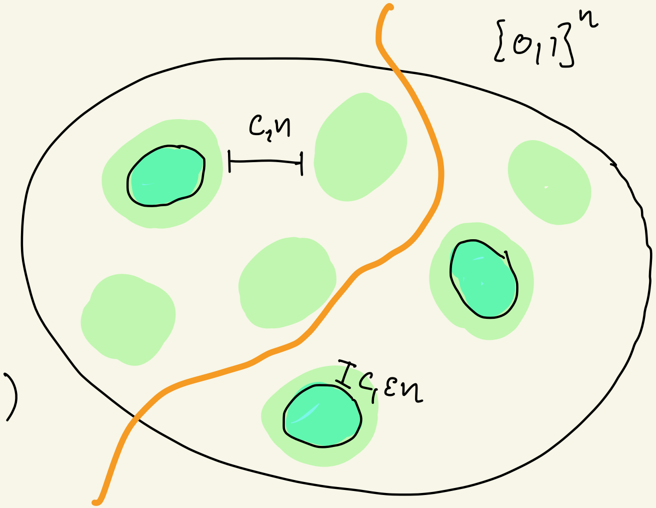

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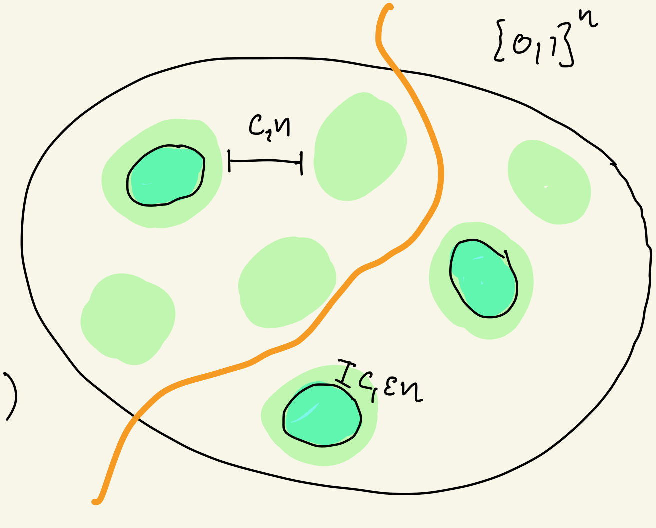

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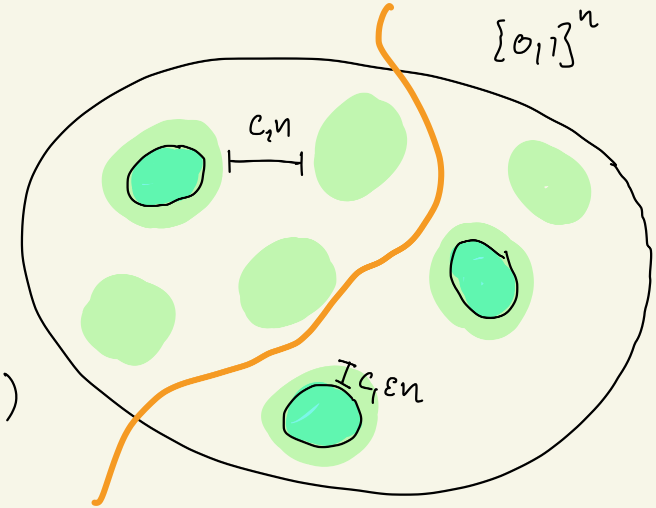

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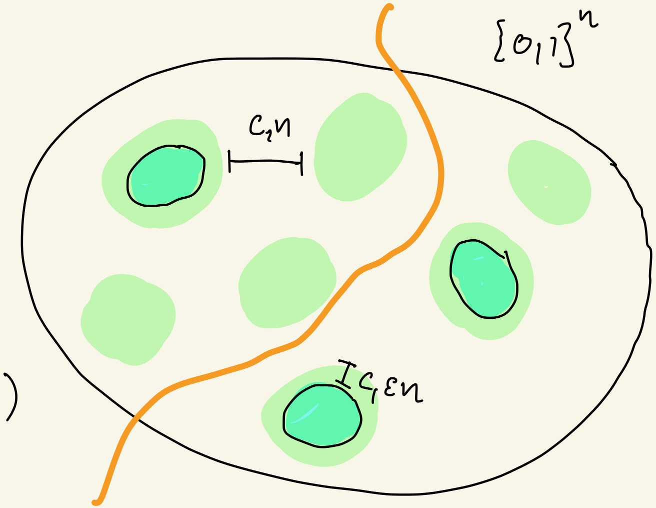

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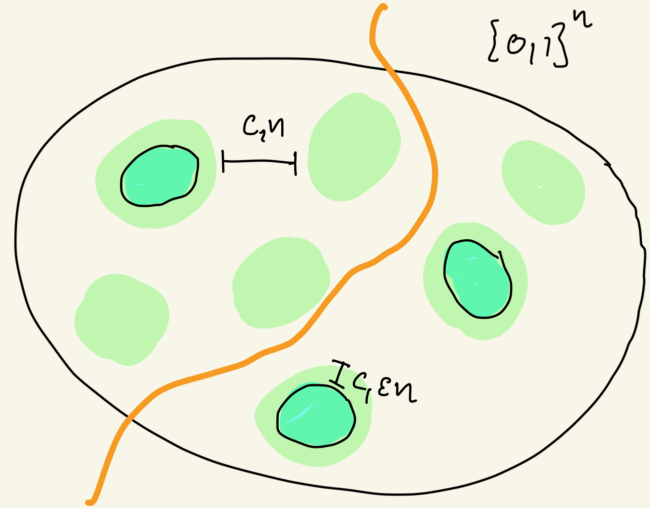

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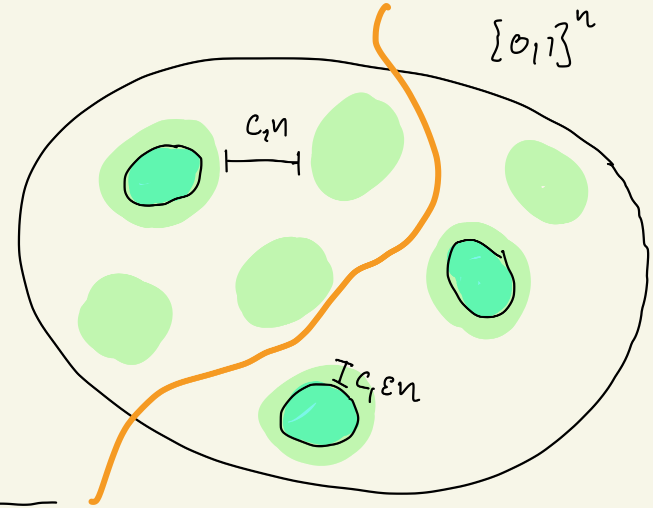

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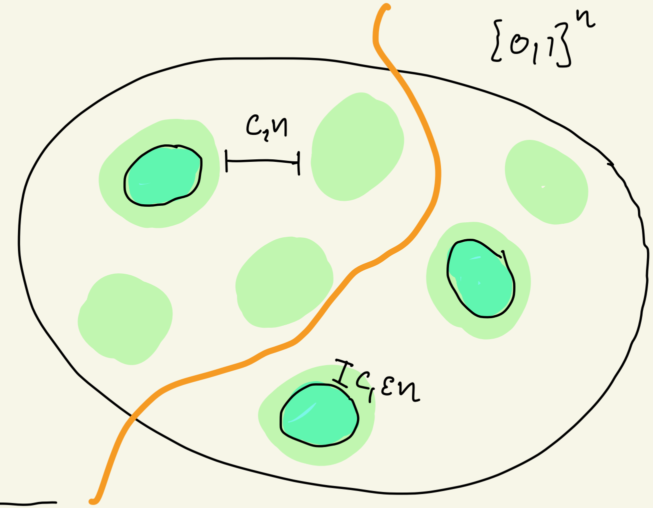

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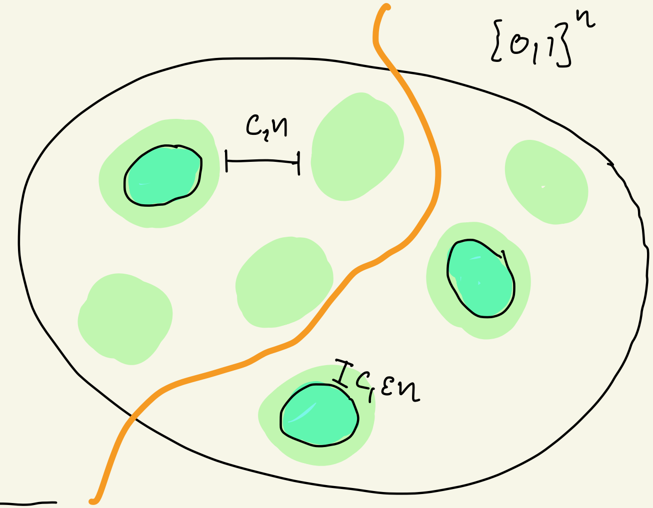

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$\uparrow$ code rate

So if $\varepsilon < O\left(\frac{k^2}{n^2}\right)$, then $D_X(T) < 0.99$.

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In progress: All linear-rate and -distance codes are NLTS.

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Next step: introduce computation, find NLTS Hamiltonians that capture NP (or MA) computations

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I think we need to prove lower bounds for the following ansatz: