Good approximate quantum LDPC codes from spacetime circuit Hamiltonians

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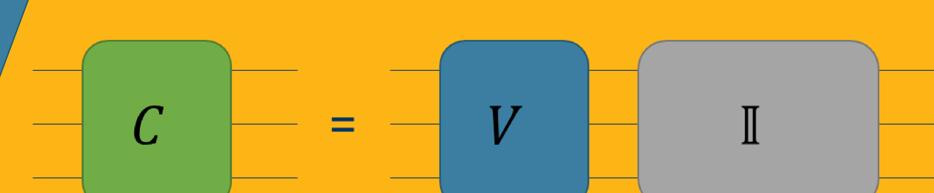
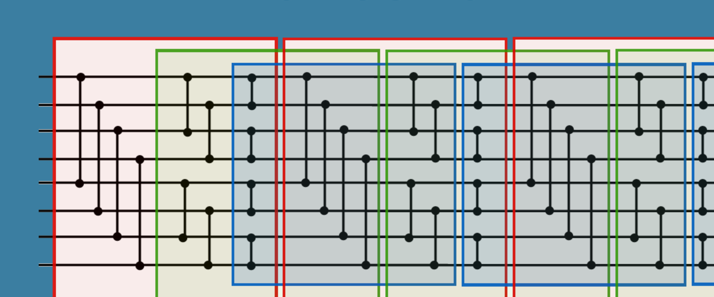
Optimal rate, distance and locality parameter **quantum error-correcting codes** *are* possible (modulo polylog corrections) if we go beyond stabilizer codes to **non-commuting and approximate codes**

A code with linear rate and distance and $O(\log^3 n)$ depth encoding circuit [Brown-Fawzi[13]]

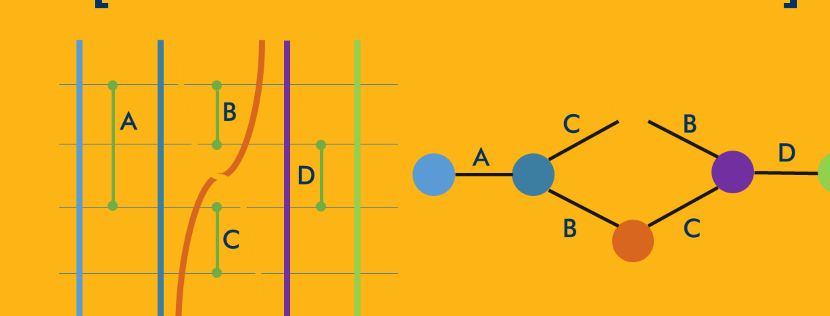
Pad the encoding circuit with identity gates



Uniformize the connectivity of the circuit using bitonic sorting circuits



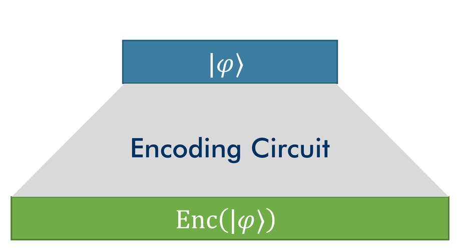
Build spacetime Hamiltonian of resulting circuit [Breuckmann-Terhal[14]]



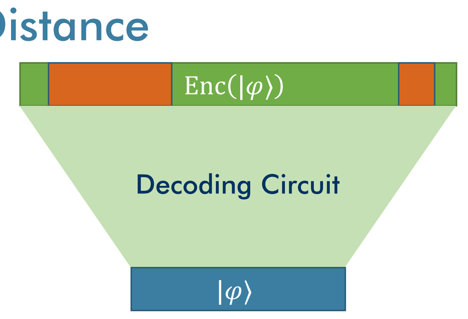
Scan for link to paper!

What makes a code good?

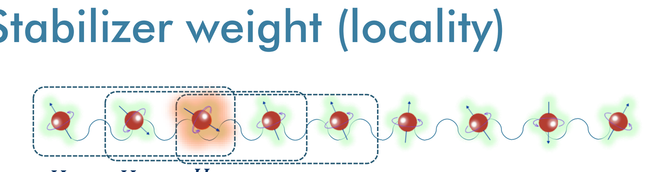
Rate



Distance

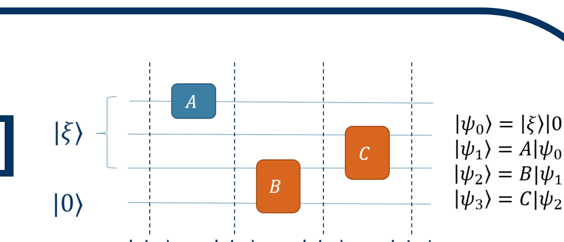


Stabilizer weight (locality)



Rate: $\frac{k}{n} = \Omega(1)$
Distance: $d = \Omega(n)$
Locality: $O(1)$

First attempt [N-Vazirani-Y[18]]



Create a Hamiltonian whose ground-space is almost exactly that of a CSS code but is locally checkable.

Express a code-state as the ground-state of a 5-local Hamiltonian (Feynman-Kitaev clock Hamiltonian) [Kitaev[99]]

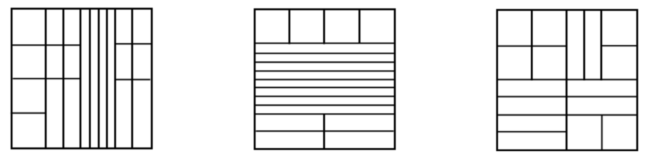
The groundspace of H is ≈ the groundspace of a CSS code tensored with junk.

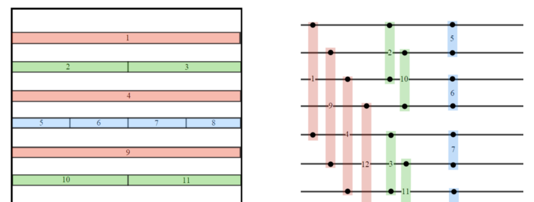
$$\mathcal{G}_C = \left\{ \frac{1}{\sqrt{T_C + 1}} \sum_{t=0}^{T} |t\rangle|\psi_t\rangle : \begin{array}{c} |\psi_t\rangle = C_t C_{t-1} \dots C_1 |\psi_0\rangle, \\ |\psi_0\rangle = |\xi\rangle |0\rangle^{\otimes(n-k)} \end{array} \right\}$$
$$\approx |\text{junk}\rangle \otimes \left\{ V|\psi_0\rangle : |\psi_0\rangle = |\xi\rangle|0\rangle^{\otimes(n-k)} \right\}.$$

Spectral gap analysis

Spectral gap of the code is based on the mixing time of valid configurations of a bitonic block

Bitonic blocks look similar to a structure called dyadic tilings studied in [Cannon-Levin-Stauffer[17]]





Dyadic tilings are ways of covering the unit square by 2^d rectangles with corner coordinates at multiples of 2^{-d}

The spectral gap of the Hamiltonian is $\widetilde{\Omega}(n^{-3.09})$ due to mixing time of dyadic tilings.