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"Quantum Supremacy" and the Complexity of Random Circuit Sampling

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Quantum supremacy

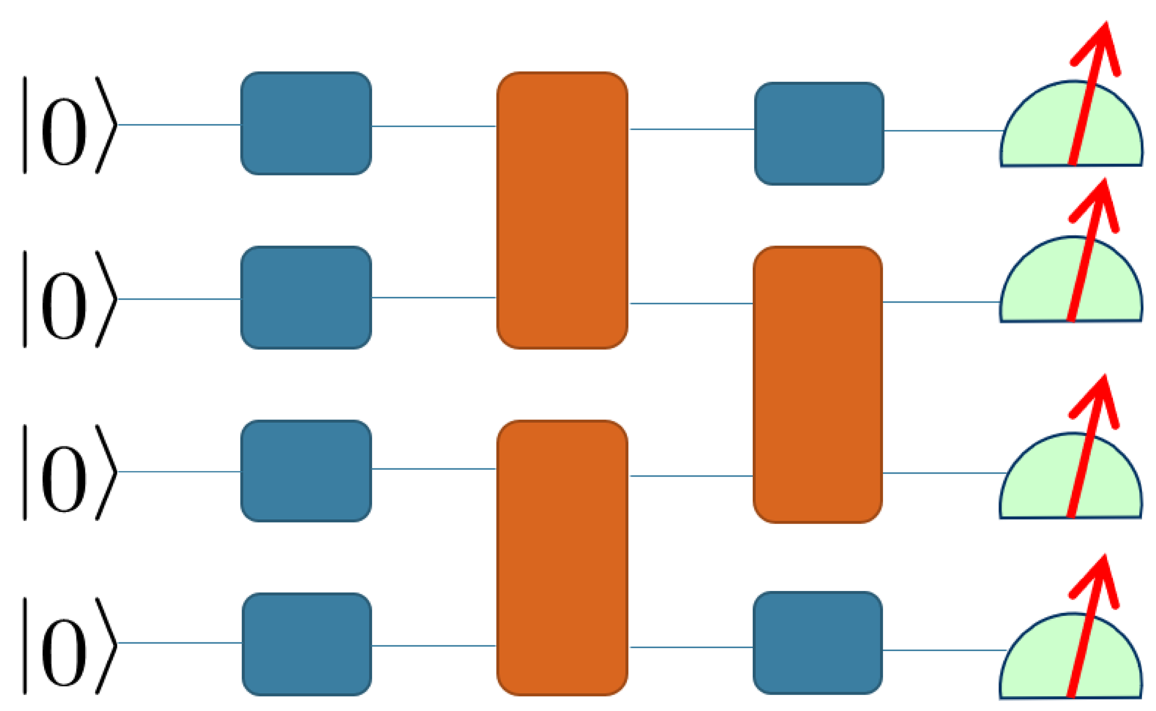
- Quantum supremacy is a demonstration of any quantum computation that is prohibitively hard for classical computers.
- It is both a necessary milestone on the path to useful quantum computers as well as a test of quantum theory in the realm of high complexity.
- A physical refutation of the extended Church-Turing thesis.

How did we get here?

- Complexity theoretic results of the 90's (e.g. Bernstein-Vazirani, Simons, and Shor) give evidence (oracle separation) of the power of quantum computers over classical.
- Sampling-problems are proposed as tasks for quantum supremacy such as BosonSampling [1] and IQP [3].
  - Satisfy #P average-case hardness via worst-to-average-case reductions.
  - Experimentally difficult to verify.
- Vast improvements are made in "noisy intermediate-scale quantum" (NISQ) devices [5] especially in the realm of super-conducting qubits [2].
- Google/UCSB proposes the 'Random Circuit Sampling' problem as the task with which they will demonstrate supremacy [4].

This leads to a need for complexity-theoretic evidence for the Random Circuit Sampling tasks.

Random Circuit Sampling



Random Circuit Sampling task: Given a circuit C from the architecture, sample from a distribution close to the distribution induced by C:

$$\Pr(y) = |\langle y|C|0^n\rangle|^2, \text{ for } y \in \{0,1\}^n.$$

Requirements for a proposal

The computational task is to sample from the output distribution D of some experimentally feasible quantum process or algorithm. To establish quantum supremacy we must show

Hardness No efficient classical algorithm can sample from any distribution close to D, and

Verification an algorithm can check that the experimental device sampled from an output distribution close to D.

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Average-case hardness

Verifiability imposes a robustness condition of the difficulty of sampling. In any reasonable noise model, a single outcome x has exponentially small occurrence probability $D(x)$ (a #P-hard quantity) — therefore, is extremely difficult to verify. Any *convincing* proof of supremacy must establish that D is actually uniformly difficult to sample from. This is a *worst-to-average-case* reduction.

Theorem 1 (Simplified)

It is #P-hard to compute $|\langle 0|C'|0\rangle|^2$ with probability > 0.76 over the choice of C', where C' is drawn from any one of a family of discretizations of the Haar measure.

Establishing

Verification

- The leading statistical measure proposed for verification is the "cross-entropy" [2]
$$\sum_x p^U_{\rm dev}(x) \log\left(\frac{1}{p^U_{\rm id}(x)}\right).$$
- It is designed so that it can be estimated with a few samples $x_1, \ldots, x_k$ and computing the average value $\mathbb{E}_i \log(1/p^U_{id}(x_i))$ using a classical supercomputer.
- Without assumptions as to how the quantum device operates, cross-entropy does not certify closeness in total variation distance.
- However, there is a natural assumption under which cross-entropy measure certifies closeness in total variation distance.

Theorem 2. *If $H(p_{dev}) > H(p_U)$, then achieving a cross-entropy score ϵ-close to ideal implies that $||p_{dev} - p_U|| \leq \sqrt{\epsilon/2}$.*
Proof. Pinsker's inequality.

The leading proposals

Proposal	*Worst-case hardness*	*Exact average-case hardness*	*Approximate average-case hardness*	*Anti-concentration*	*Feasible experiment?*
BosonSampling	✓	✓			
FourierSampling	✓	✓			
IQP	✓			✓	
Random Circuit Sampling	✓	✓		✓	✓

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