A hierarchical state-space model with Gaussian process dynamics for functional connectivity estimation

Understanding the dynamics of neural activity is a fundamental problem in neuroscience which could elucidate brain function. We take a systems neuroscience approach to understanding dynamics via the concept of functional connectivity, the temporal dependence of activity between different brain regions that indicates how information flows between regions. Specifically, we develop a technique for estimating functional connectivity from magnetoencephalography (MEG) data, and apply it to data from subjects performing an auditory attention task. Studying functional connectivity networks underlying auditory attention could improve our understanding of conditions such as central auditory processing disorder, motivating novel diagnosis and treatment.

Estimating functional connectivity from MEG data brings multiple challenges that need to be addressed to properly model the underlying structure. One challenge is that functional connectivity can change over short timescales, and should therefore be modeled as a dynamic process. Another is handling data from multiple subjects performing the same task, where we want to leverage all subjects' data to estimate shared connectivity structure while also accounting for meaningful subject-specific variations. To address both of these issues, we develop a hierarchical Bayesian model using Gaussian processes for functional connectivity estimation and a computationally efficient inference algorithm to learn the model parameters.

Since we are interested in dynamic functional connectivity, we model MEG recordings from a single subject as a time-varying linear dynamical system

$$x_{t+1} = A(t)x_t + \epsilon_t,\, \epsilon_t \sim \mathcal{N}(0, Q) \qquad y_t = Cx_t + \eta_t,\, \eta_t \sim \mathcal{N}(0, R)$$

Where t indexes time, y_t is a vector of MEG sensor values, and x_t is a vector of activity across multiple brain regions of interest (ROIs). The ROI activity x_t follows a linear autoregressive process, where each $A(t)$ is a matrix of interaction terms such that the entry $A_{ij}(t)$ encodes the functional connectivity from region j to region i at time t. For our application, C is a known matrix that describes the forward transformation from ROI activity to MEG sensor readings. We capture connectivity shared across subjects along with subject-specific variation by placing a hierarchical structure on the $A(t)$ matrices across S subjects

$$A_{ij}^{(\text{global})}(\cdot) \sim \mathcal{GP}(I_{ij}, k_0) \qquad A_{ij}^{(s)}(\cdot) \sim \mathcal{GP}(A_{ij}^{(\text{global})}(\cdot), k_1),\, s = 1, \ldots, S$$

Where $\mathcal{GP}(m, k)$ is a Gaussian process with mean function m and kernel function k (we use the squared exponential kernel $k(t_i, t_j) = \sigma^2 \exp(d(t_i - t_j)^2)$ to describe connectivity varying smoothly over time). The global dynamics $A^{(\text{global})}(\cdot)$ capture the shared network across subjects, while each $A^{(s)}(\cdot)$ is a subject's deviation from the global mean.

To estimate the dynamics, we use a Gibbs sampling inference algorithm; however, the dimensionality of the problem makes standard Gibbs sampling computationally infeasible. To address this, we incorporate iterative approximations via the Lanczos and conjugate gradient algorithms to efficiently sample from high-dimensional Gaussians and allow the inference to scale to our large MEG dataset. The dataset consists of MEG recordings collected from subjects performing a task where they exercise different aspects of auditory attention across multiple experimental conditions. Initial results demonstrate our model's ability to capture plausible functional connections at the group and subject-specific levels within each experimental condition as well as meaningful differences between conditions. Though further examination of results and future computational experiments are forthcoming, this hierarchical Gaussian process structure is a promising tool for modeling continuously time-varying dynamics from MEG data collected from multiple subjects.