A Hierarchical State-Space Model with Gaussian Process Dynamics for Functional Connectivity Estimation

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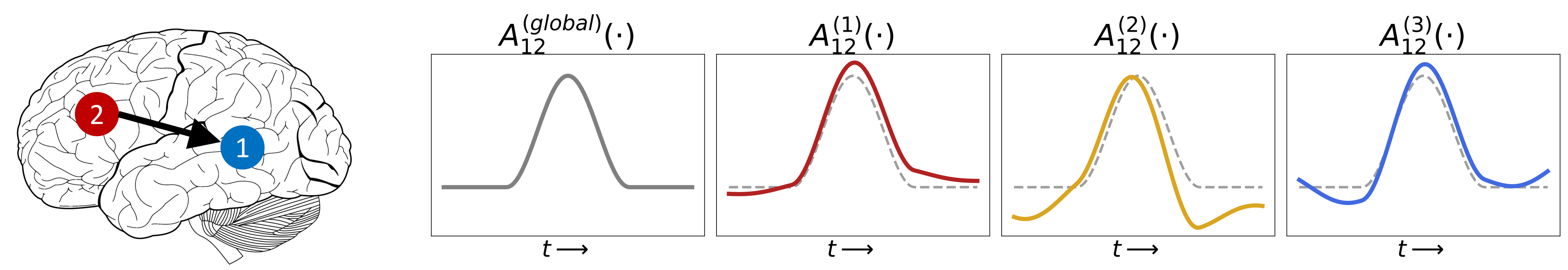
Goal

- We present an approach for learning **directed, dynamic functional interactions** between brain regions across **multiple subjects** from **magnetoencephalography (MEG) data**
- We pool subjects' data to learn a **group-level connectivity structure** while also modeling interesting **subject-specific differences**
- Our method incorporates **Gaussian processes** to model functional connections that are **smoothly time-varying** while capturing **estimation uncertainty**

Description of model



Example of global time-varying connection with subject-specific variation

$$A_{ij}^{(\text{global})}(\cdot) \sim \mathcal{GP}(I_{ij}, k_0)$$

$$A_{ij}^{(s)}(\cdot) \sim \mathcal{GP}(A_{ij}^{(\text{global})}(\cdot), k_1),\ s = 1, \ldots, \#\text{ subjects}$$

$$x_{t+1}^{(s)} = A^{(s)}(t)\, x_t^{(s)} + \epsilon_t^{(s)},\ \epsilon_t^{(s)} \sim \mathcal{N}(0, Q) \leftarrow \text{ROIs}$$

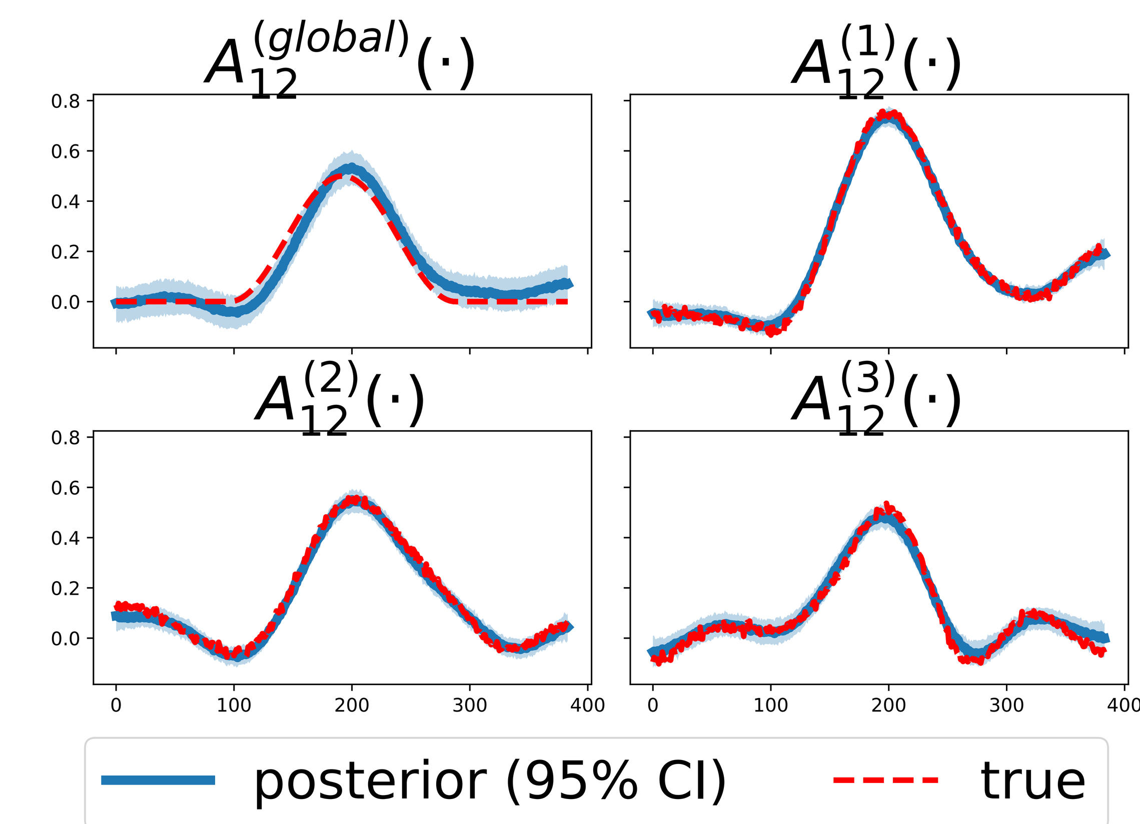
$$y_t^{(s)} = C^{(s)} x_t^{(s)} + \eta_t^{(s)},\ \eta_t^{(s)} \sim \mathcal{N}(0, R^{(s)}) \leftarrow \text{sensors}$$

$\mathcal{GP}(m, k)$ is a Gaussian process with mean function m and covariance function k. We use the squared exponential kernel:

$$k(t_i, t_j) = \sigma^2 \exp(-d(t_i - t_j)^2)$$

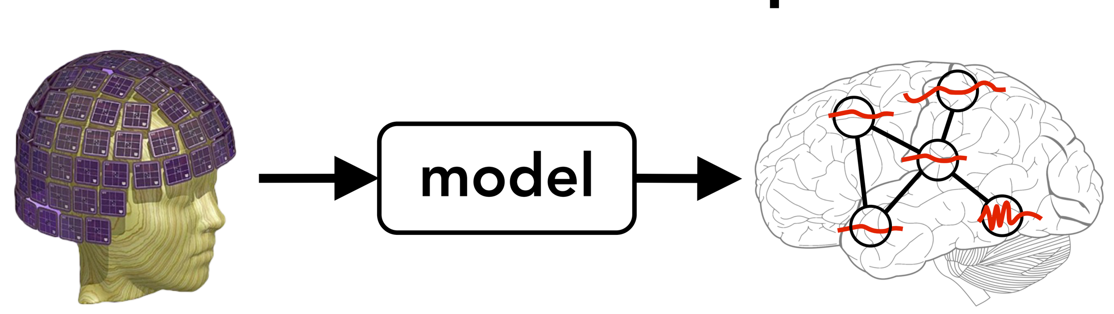
- Sensor values $y_t^{(s)}$ and observation model $C^{(s)}$, $R^{(s)}$ known for each subject
- ROI dynamics $A^{(s)}(t)$, $A^{(global)}(t)$ and noise Q unknown, inferred via Gibbs sampling
- Entries of $A(t)$ indicate strength of directed interaction between regions over time

Results



- Hierarchical model is able to learn **global dynamic connectivity structure (top left)** as well as **subject-specific variation (other subplots)**
- Bayesian inference allows the model to **capture uncertainty** in the time-varying connectivity (blue band shows 95% CI)

Problem setup

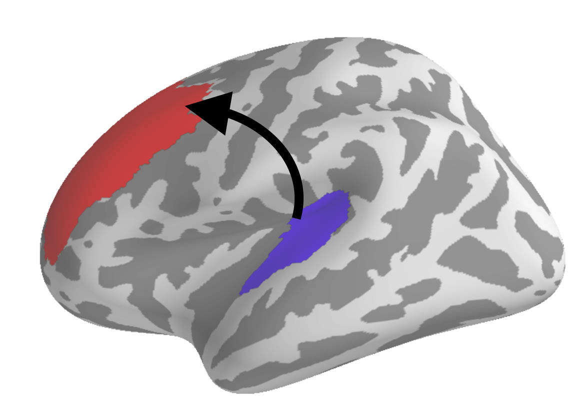


- Obtain MEG sensor recordings y from multiple subjects performing a task
- Observation model – which includes forward model (lead field matrix) C and sensor noise covariance R – is known (i.e., can be calculated) for each subject
- Goal is to infer time-varying, directed functional interactions between a specified set of **brain regions of interest (ROIs)** using sensor recordings and observation model for each subject

Inference

- **Gibbs sampling** with **iterative approximation methods** for efficient sampling from posterior of $A^{(s)}(t)$
  - **Conjugate gradient (CG)**: $\Sigma, \mathrm{x} \rightarrow$ approximation to $\Sigma^{-1}\mathrm{x}$ used for $\mu_A = \mathbb{E}[A_{ij}^{(s)}(\cdot) \mid rest]$
  - **Lanczos** algorithm: $\Sigma^{-1}, \mathrm{x} \rightarrow$ approximation to $\Sigma^{1/2}\mathrm{x}$ Used to sample from $\mathcal{N}(0, \Sigma)$
  - Given $\Sigma_A^{-1} = \left[\mathrm{Cov}\left(A_{ij}^{(s)}(\cdot) \mid rest\right)\right]^{-1}$:
    CG to construct μ_A
    Lanczos to generate $\epsilon_A \sim \mathcal{N}(0, \Sigma_A)$
    Construct sample $\mu_A + \epsilon_A$

Simulation experiments



- Sensor data simulated from hierarchical model for 3 subjects and 2 ROIs (above), using real MEG forward model $C^{(s)}$ and sensor noise covariance $R^{(s)}$ for each subject
- Simulated dynamics $A^{(global)}(t)$ constructed to include a time-varying interaction from one ROI (blue) directed towards the other (red)

Future directions

- Application to real MEG data recorded from multiple subjects performing an auditory attention task (analysis ongoing)
- Incorporating subject-specific auxiliary data (e.g., behavioral and cognitive assessments, audiological test results, subjective questionnaire answers)

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