Today

More on Abstract Interpretation
- Set, semilattice, lattice
- Galois connection
- Transfer function vs. lub (vs. glb)
- Exercise: concrete examples

Next week
- Wrap up Abstract Interpretation
- AI (CheckerFramework) tutorial
- Hands-on applications
- Move on to dynamic and hybrid techniques

Set, semilattice, lattice

Set
- unordered collection of distinct objects
Set, semilattice, lattice

Set
- unordered collection of distinct objects

Partially ordered set
- Reflexive: \( x \leq x \)
- Anti-symmetric: \( x \leq y \land y \leq x \Rightarrow x = y \)
- Transitive: \( x \leq y \land y \leq z \Rightarrow x \leq z \)

Join semilattice
- Partially ordered set with least upper bound (join)

Meet semilattice
- Partially ordered set with greatest lower bound (meet)

Concrete and abstract domains

Concrete (\( P(\mathbb{N}) \))
- \{..., 4, 6, 8, ...\}
- \{\}
- \{1\}  \{4\}  \{8\}

Abstract
- \( T \)
- \( E \)
- \( O \)
- \( \bot \)

What is the abstraction (\( \alpha \)) of {}?
Concrete and abstract domains

Concrete ($P(\mathbb{N})$)     Abstract

What properties should $\alpha$ and $\gamma$ satisfy?

Galois connection

• $\alpha: C \rightarrow A$
• $\gamma: A \rightarrow C$
• $\forall c \in C: c \leq \gamma(\alpha(c))$
• $\gamma$ and $\alpha$ are order preserving

Transfer function vs. lub
Transfer function vs. lub

Transfer function
- $f: A \times A \rightarrow A$
- must be monotonic
- may not be extensive

Abstract

Lub
- $lub: A \times A \rightarrow A$
- must be monotonic

Small-group exercise
- Three groups
- Work through three examples:
  - Reading question 1 (inadequacy of simple domains)
  - Join vs. meet operation ($f(int\ a, int\ b, int\ c): int$)
  - Termination/fix point iteration

int\ x = 2;
while \ (x < 10) \ {
  x = x + 2;
}
return(x);

Which parameters ($a, b, c$) will definitely be used?
Which parameters ($a, b, c$) may be used?
(cond is independent of the parameters)

Is the value of $x$ after the loop an even number? Use an abstract domain with \{odd, 2, even$_2$, and even$_4$\}

Report to class (random call)