CSE 503

Software Engineering

Winter 2021

Intro to Abstract Interpretation

January 13, 2021

Recap: static vs. dynamic analysis

Static analysis

- Reason about the program without executing it.
- Build an abstraction of run-time states.
- Reason over abstract domain.
- Prove a property of the program.
- Sound* but conservative.

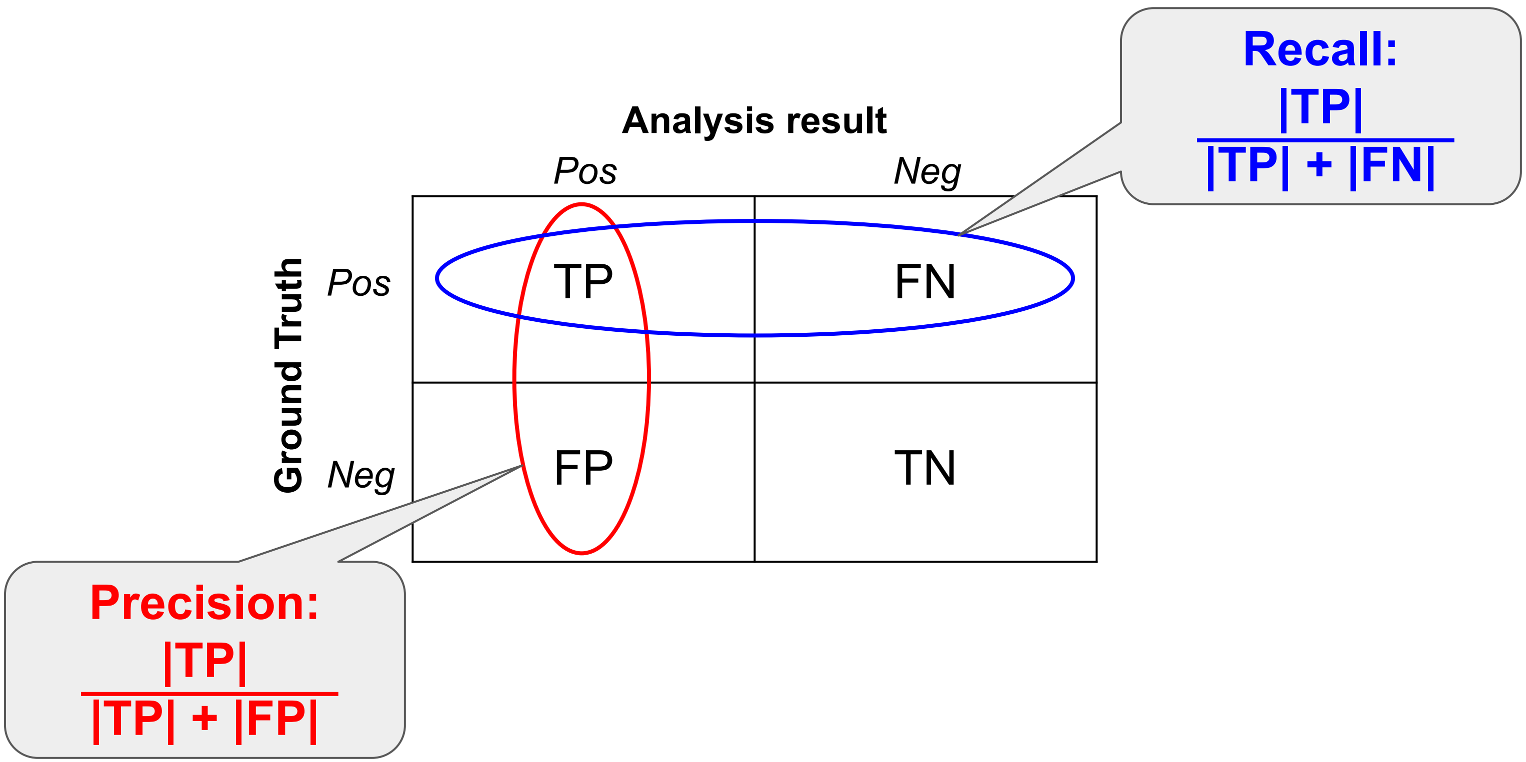
Dynamic analysis

- Reason about the program based on some program executions.
- Observe concrete behavior at run time.
- Improve confidence in correctness.
- Unsound* but precise.

* Some static analyses are unsound; dynamic analyses can be sound.

Recap: Terminology and important concepts

1. Precision vs. Recall (and FP/FN/TP/TN)



Recap: Terminology and important concepts

1. Precision vs. Recall (and FP/FN/TP/TN)
2. Soundness vs. Completeness

Soundness: no FNs

		Analysis result	
		Pos	*Neg*
Ground Truth	*Pos*	TP	FN
	Neg	FP	TN

Completeness: no FPs

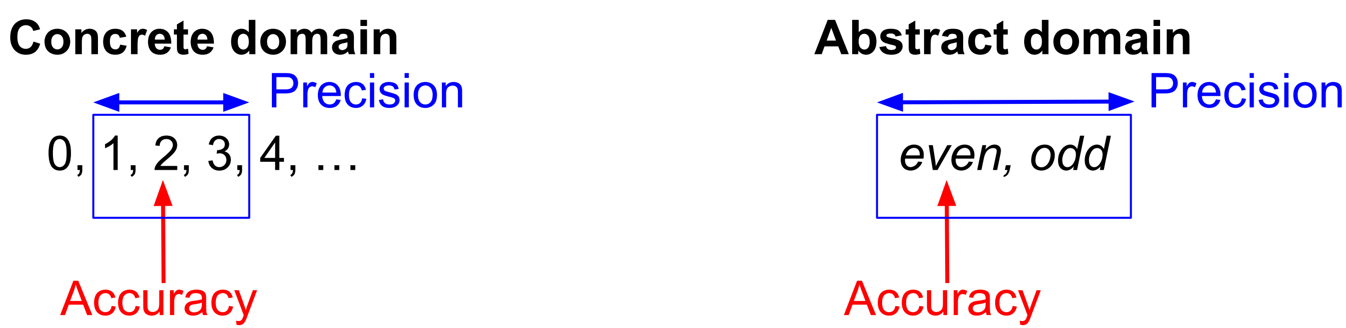
Recap: Terminology and important concepts

1. Precision vs. Recall (and FP/FN/TP/TN)
2. Soundness vs. Completeness
3. Concrete domain vs. Abstract domain

Concrete domain	**Abstract domain**
0, 1, 2, 3, 4, …	*even, odd*

Recap: Terminology and important concepts

1. Precision vs. Recall (and FP/FN/TP/TN)
2. Soundness vs. Completeness
3. Concrete domain vs. Abstract domain
4. Accuracy vs. Precision



Today

- Abstract interpretation
  - Introduction
  - Abstraction functions
  - Concretization functions
  - Transfer functions
  - Lattices

Properties of an ideal program analysis

- Soundness
- Completeness
- Termination

```
…
int x = 0;
while (!isDone()) {
  x = x + 1;
}
…
```

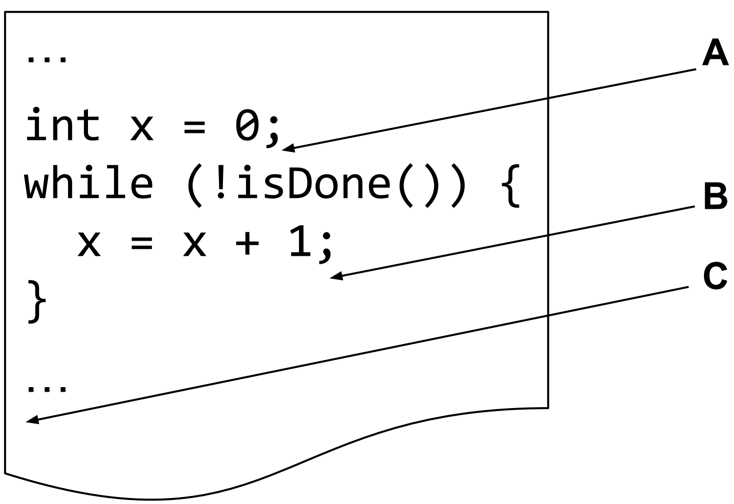
A

B

C

Properties of an ideal program analysis

- Soundness
- Completeness
- Termination



Abstract interpretation sacrifices completeness (precision)

Abstract interpretation: applications

Compiler checks and optimizations

- Liveness analysis (register reallocation)
- Reachability analysis (dead code elimination)
- Code motion (`while(cond){x = comp(); ...}`)

Abstract interpretation: code examples

Liveness

```
public class Liveness {
  public void liveness() {
    int a;
    if (alwaysTrue()) {
      a = 1;
    }
    System.out.println(a);
  }
}
```

Reachability

```
public void deadCode() {

  return;

  System.out.println("Here!");
}
```

Abstract interpretation: example

Program

```
x = 0;
y = read_even();
x = y + 1;
y = 2 * x;
x = y - 2;
y = x / 2;
```

Are all statements necessary?

Abstract interpretation: example

Program

```
x = 0;
y = read_even();
x = y + 1;
y = 2 * x;
x = y - 2;
y = x / 2;
```

SSA form

x_1 = 0;
y_1 = read_even();
$x_2 = y_1 + 1$;
$y_2 = 2 * x_2$;
$x_3 = y_2 - 2$;
$y_3 = x_3 / 2$;

X_1 is never read.

Abstract interpretation: example

Program

```
x = 0;
y = read_even();
x = y + 1;
y = 2 * x;
x = y - 2;
y = x / 2;
```

SSA form

x_1 = 0;
y_1 = read_even();
$x_2 = y_1 + 1;$
$y_2 = 2 * x_2;$
$x_3 = y_2 - 2;$
$y_3 = x_3 / 2;$

$y_3 = x_3 / 2$
$y_3 = (y_2 - 2) / 2$
$y_3 = (2 * x_2 - 2) / 2$
$y_3 = (2 * (y_1 + 1) - 2) / 2$
$y_3 = (2 * y_1 + 2 - 2) / 2$
$y_3 = y_1$
$x_3 = y_1 * 2$

Symbolic reasoning shows simplification potential.

Abstraction function

Program

```
x = 0;
y = read_even();
x = y + 1;
y = 2 * x;
x = y - 2;
y = x / 2;
```

Concrete execution

```
{x=0;  y=undef}
{x=0;  y=8}
{x=9;  y=8}
{x=9;  y=18}
{x=16; y=18}
{x=16; y=8}
```

Abstraction function

Program

```
x = 0;
y = read_even();
x = y + 1;
y = 2 * x;
x = y - 2;
y = x / 2;
```

Concrete execution

```
{x=0;  y=undef}
{x=0;  y=8}
{x=9;  y=8}
{x=9;  y=18}
{x=16; y=18}
{x=16; y=8}
```

Mapping to abstract domain (even, odd)

```
{x=e; y=e}
{x=o; y=e}
{x=o; y=e}
{x=e; y=e}
{x=e; y=e}
```

Abstraction function

Program

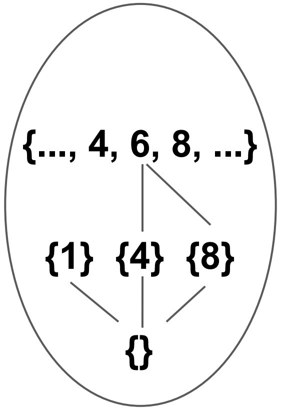
```
x = 0;
y = read_even();
x = y + 1;
y = 2 * x;
x = y - 2;
y = x / 2;
```

Concrete execution		Mapping to abstract domain (even, odd)
{x=0; y=undef}		
{x=0; y=8}	→	{x=e; y=e}
{x=9; y=8}	→	{x=o; y=e}
{x=9; y=18}	→	{x=o; y=e}
{x=16; y=18}	→	{x=e; y=e}
{x=16; y=8}	→	{x=e; y=e}

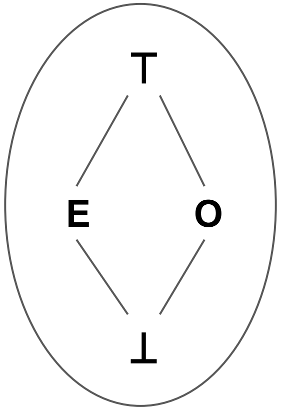
But, what's the purpose of the abstraction function?

Abstraction function

Concrete ($P(\mathbb{N})$)



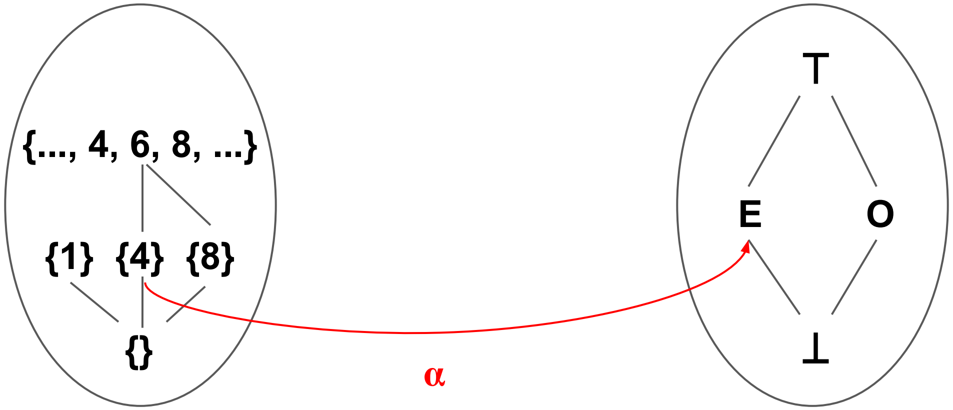
Abstract



What is the abstraction (α) of {4}?

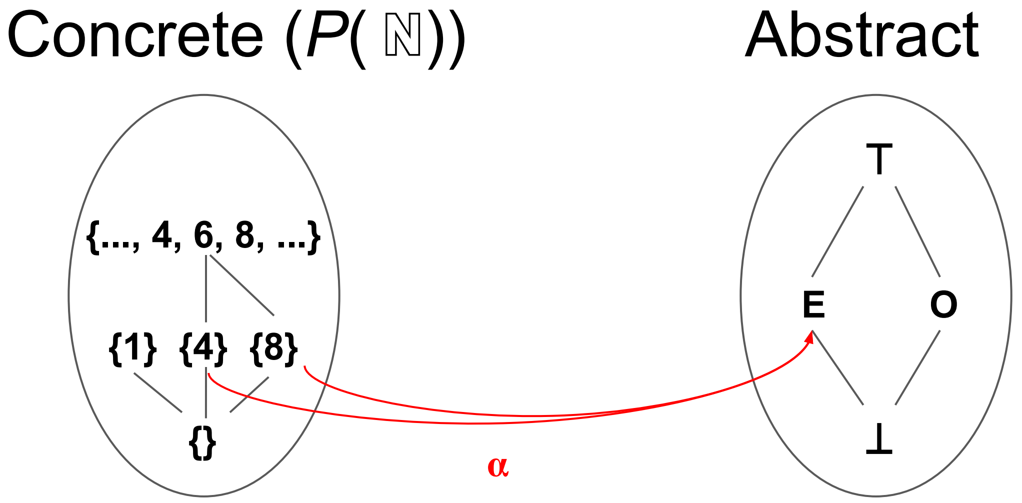
Abstraction function

Concrete ($P(\mathbb{N})$) Abstract



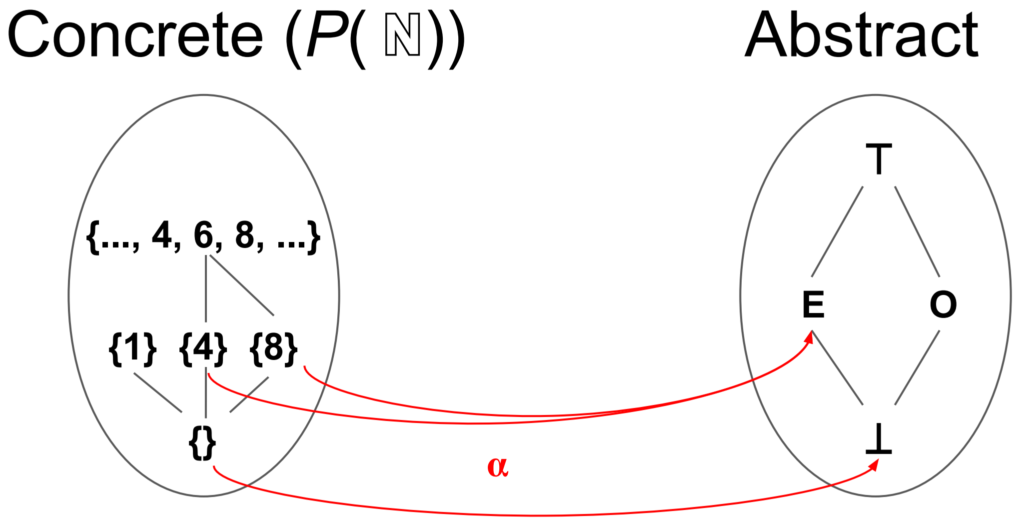
What is the abstraction (α) of {8}?

Abstraction function

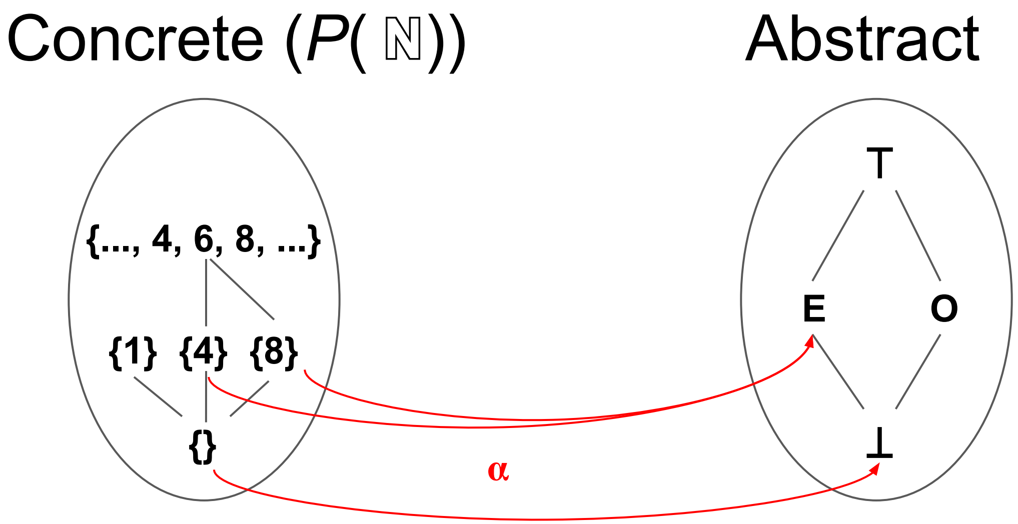


What is the abstraction (α) of {}?

Abstraction function

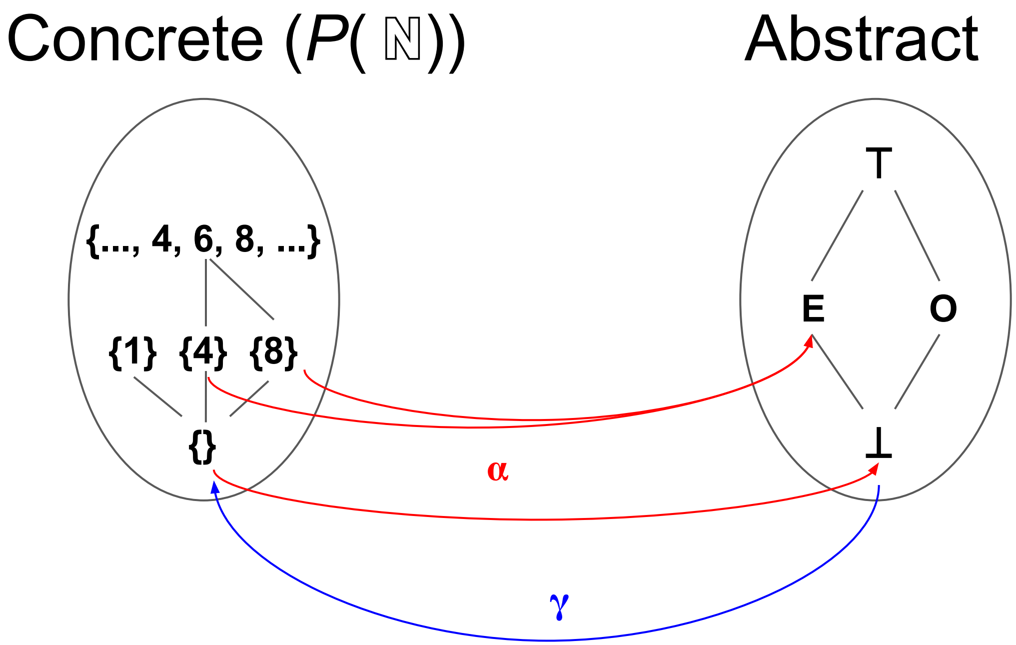


Concretization function



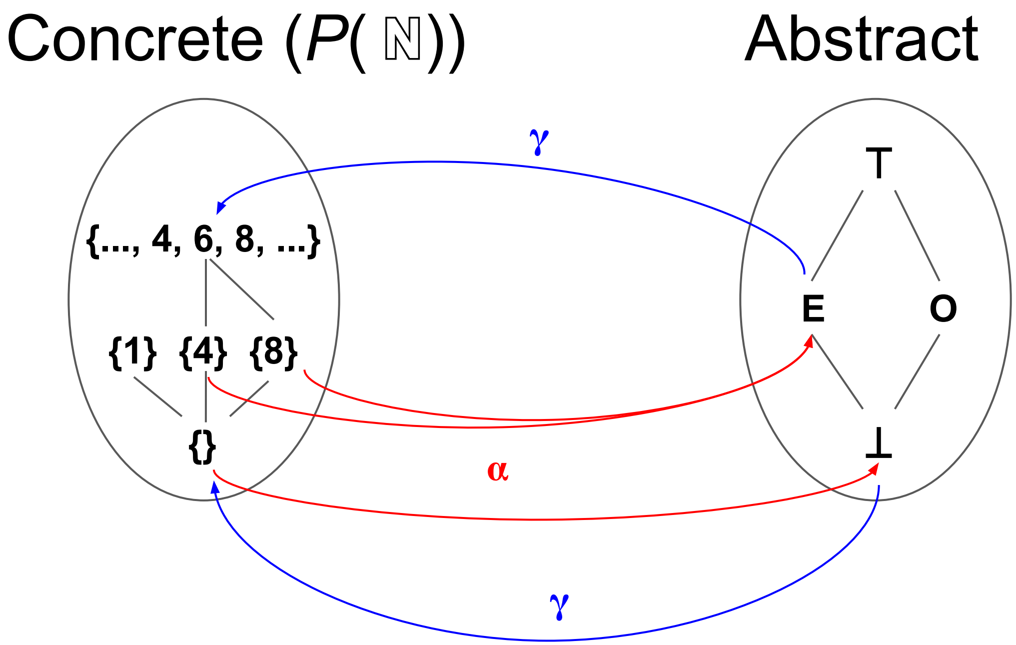
What is the concretization (γ) of $\perp$?

Concretization function

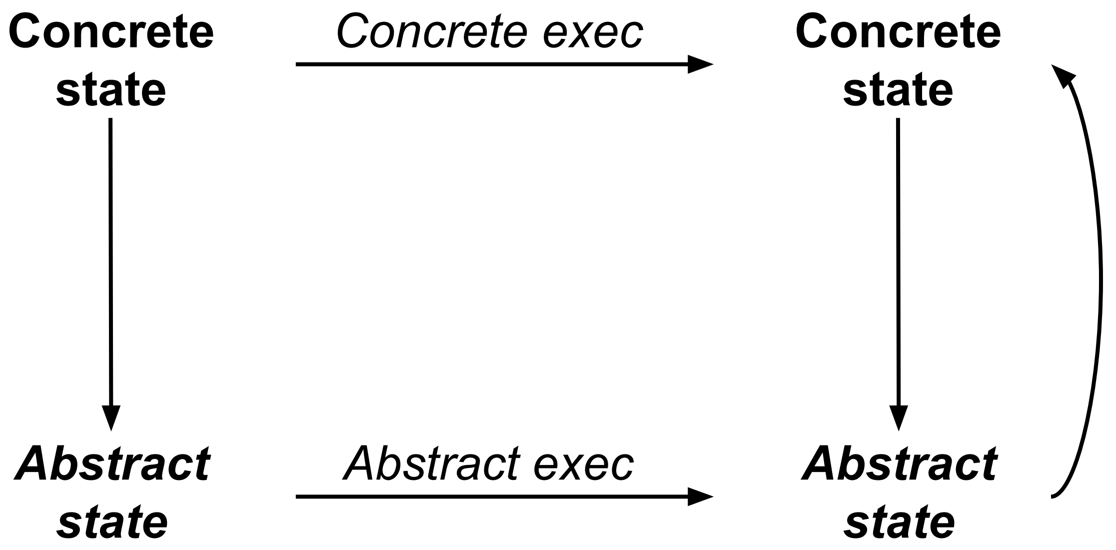


What is the concretization (γ) of **E** ?

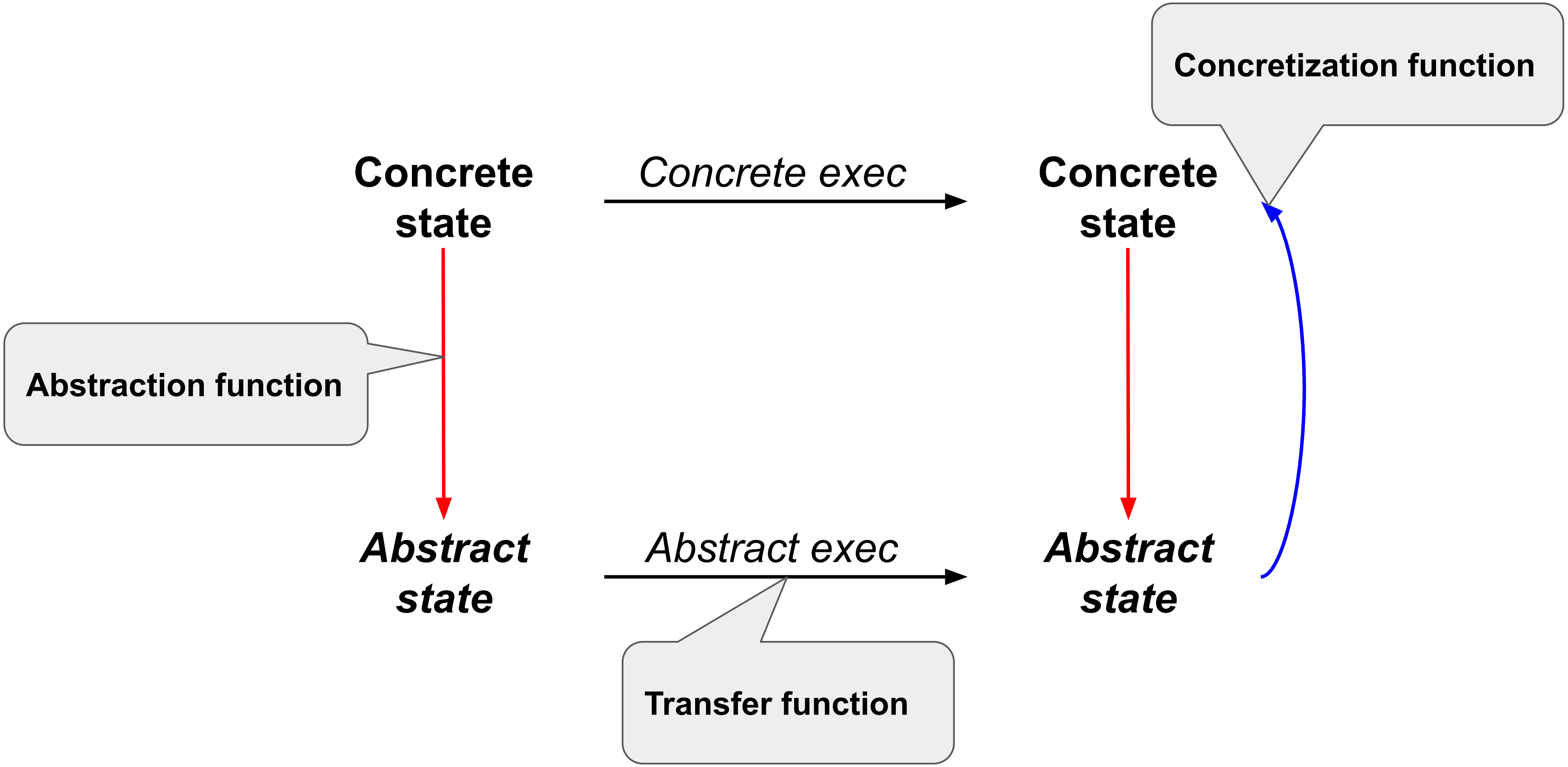
Concretization function



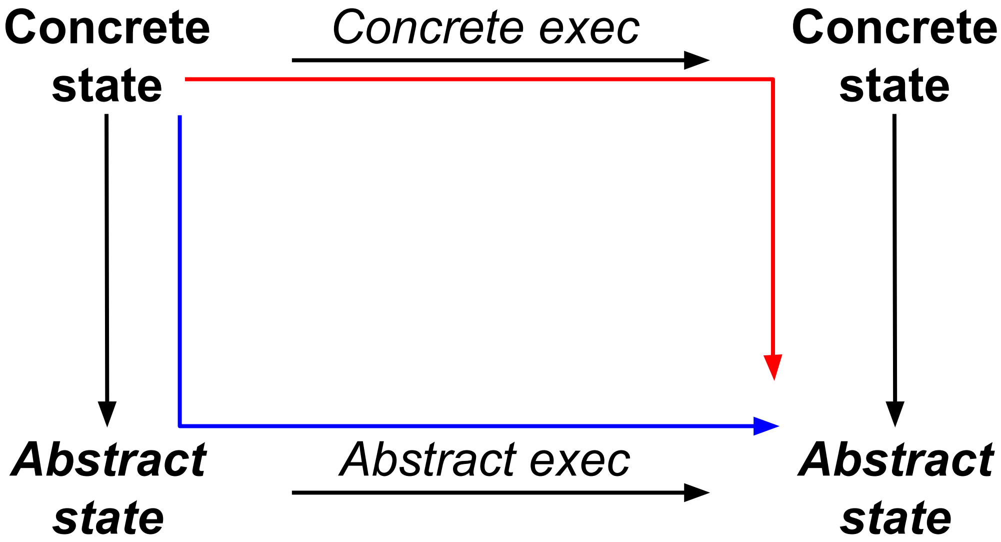
Transfer function



Transfer function

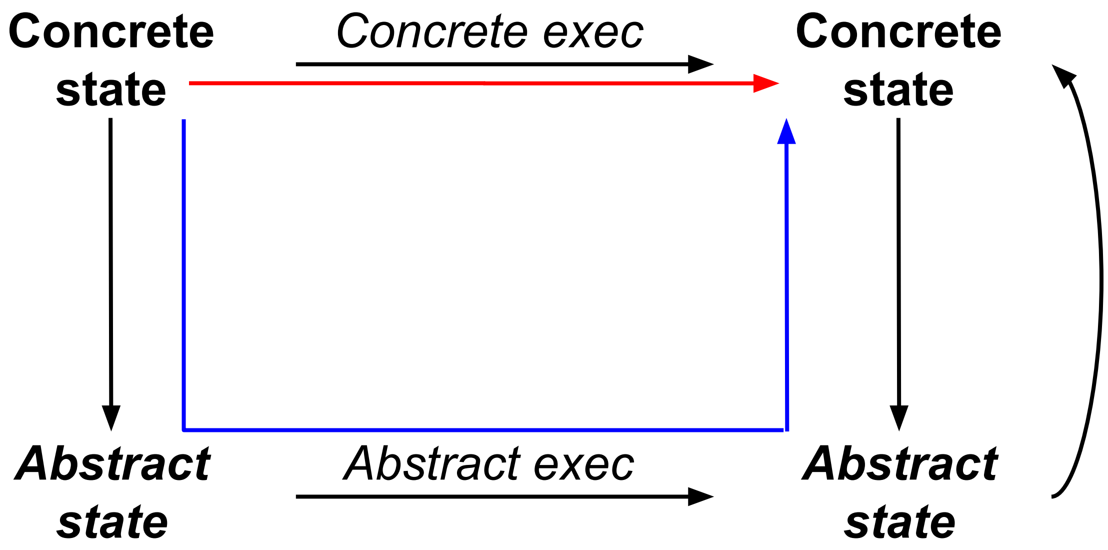


Abstract interpretation: approximation



Do both paths lead to the same abstract state?

Abstract interpretation: approximation



Do both paths lead to the same concrete state?

Set, semilattice, lattice

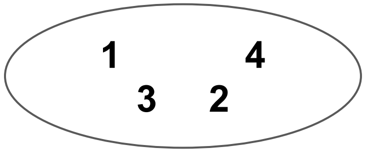
Set, semilattice, lattice

Set

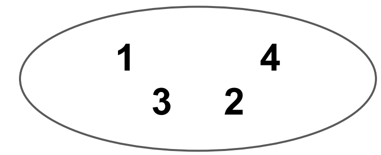
Set, semilattice, lattice

Set

- **unordered** collection of **distinct** objects



Set, semilattice, lattice



Set

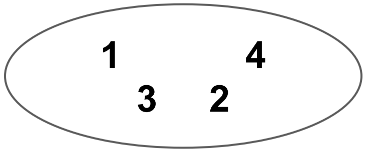
- unordered collection of distinct objects

Partially ordered set

Set, semilattice, lattice

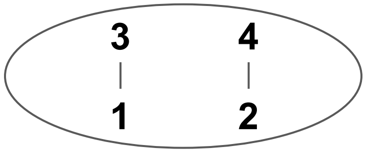
Set

- unordered collection of distinct objects

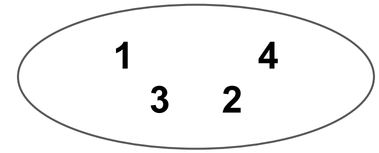


Partially ordered set

- Binary relationship $\leq$:
    - Reflexive: $x \leq x$
    - Anti-symmetric: $x \leq y \land y \leq x \Rightarrow x = y$
    - Transitive: $x \leq y \land y \leq z \Rightarrow x \leq z$

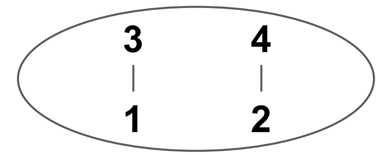


Set, semilattice, lattice



Set

- unordered collection of distinct objects



Partially ordered set

- Binary relationship $\leq$:
  - Reflexive: $x \leq x$
  - Anti-symmetric: $x \leq y \wedge y \leq x \Rightarrow x = y$
  - Transitive: $x \leq y \wedge y \leq z \Rightarrow x \leq z$

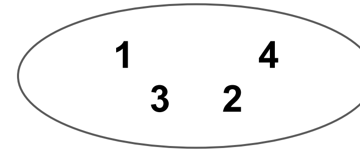
Join semilattice

Meet semilattice

Set, semilattice, lattice

Set

- unordered collection of distinct objects



Partially ordered set

- Binary relationship $\leq$:
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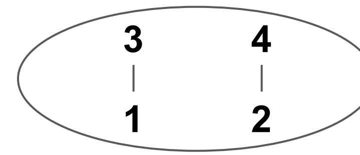
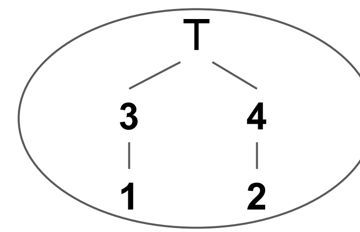
  - Anti-symmetric: $x \leq y \wedge y \leq x \Rightarrow x = y$
  - Transitive: $x \leq y \wedge y \leq z \Rightarrow x \leq z$



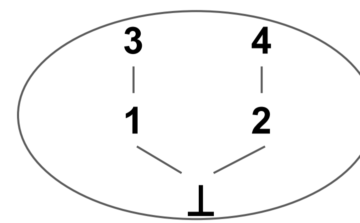
Join semilattice

- Partially ordered set with least upper bound (join)



Meet semilattice

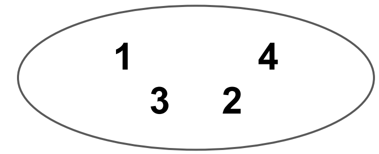
- Partially ordered set with greatest lower bound (meet)



Set, semilattice, lattice

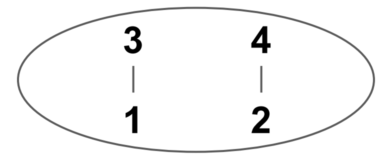
Set

- unordered collection of distinct objects



Partially ordered set

- Binary relationship $\leq$:
  - Reflexive: $x \leq x$
  - Anti-symmetric: $x \leq y \wedge y \leq x \Rightarrow x = y$
  - Transitive: $x \leq y \wedge y \leq z \Rightarrow x \leq z$



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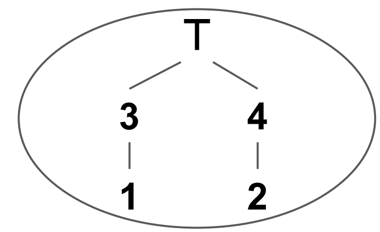
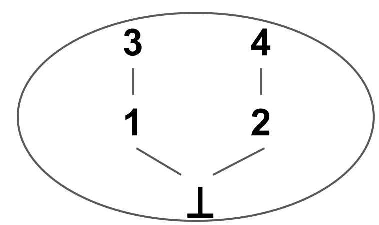


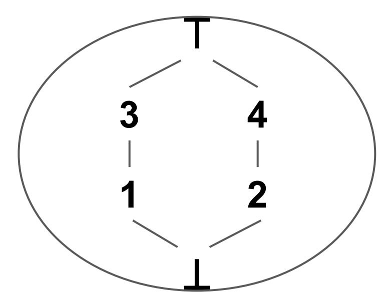
Meet semilattice

- Partially ordered set with greatest lower bound (meet)



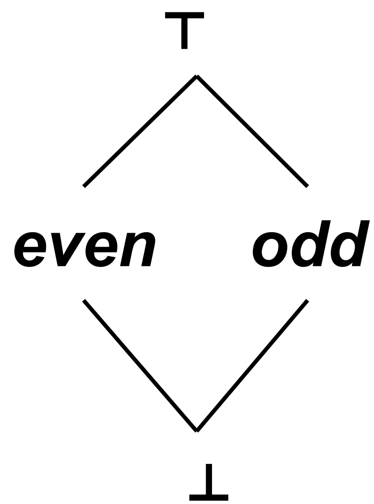
Lattice

- Both a join semilattice and a meet semilattice



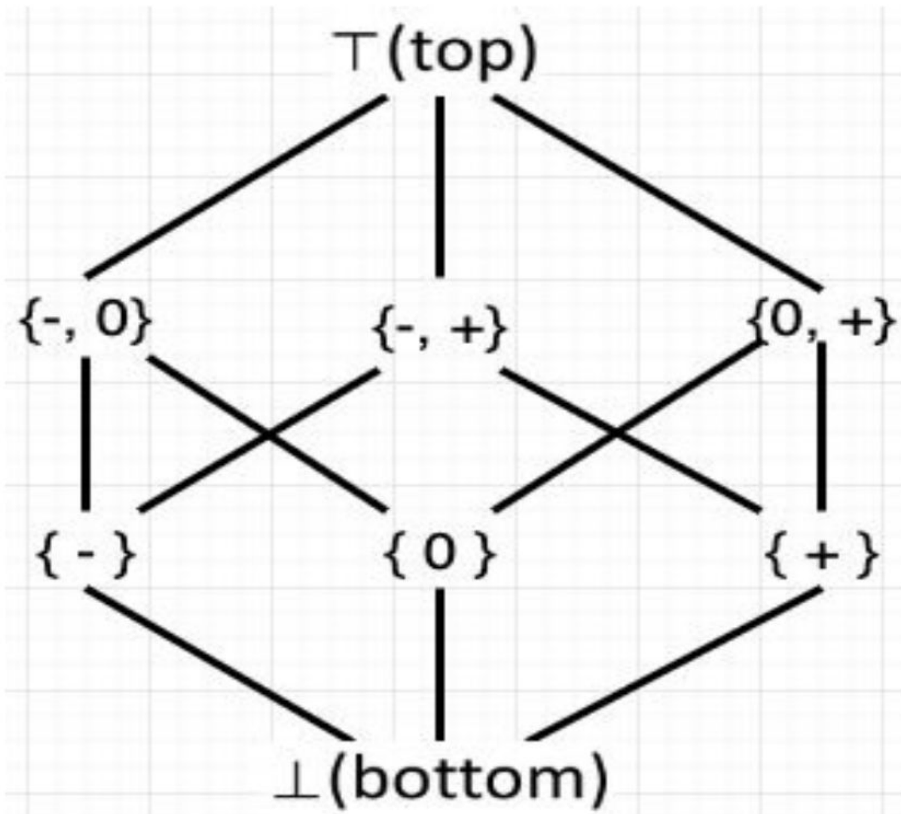
Lattice: example

Abstract domain: *even, odd, unknown* ($\top$ *), {} (* $\perp$*)*



Lattice: example

Abstract domain: *-, 0, +, unknown, {}*



Lattice: example

Goal: approximate the values of x after the loop

```
…
int x = 0;
while (!isDone()) {
  x = x + 1;
}
…
```

What are possible abstract domains and their trade-offs?

Lattice: example

Goal: approximate the values of x after the loop

```
…
int x = 0;
while (!isDone()) {
  x = x + 1;
}
…
```

Possible abstract domains:

- **Powerset of set of integers**
- **Intervals**
- **...**