

CSE 599K

Empirical Research Methods

Winter 2025

Statistical methods: overview

Today

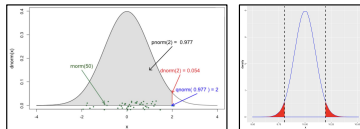
- Uniform vs. stratified sampling
- Statistical vs. practical significance
- Parametric vs non-parametric statistics

3 ways to understand and apply statistics

Math/Proofs

$$\begin{aligned} |f_n(t) - 1| &= \left| \int_{-\infty}^{\infty} e^{itx} dF_n(x) - \int_{-\infty}^{\infty} e^{itx} dF(x) \right| \\ &\leq \int_{-\infty}^{\infty} |e^{itx} - 1| dF_n(x) \\ &= \int_{|x| \leq \epsilon} |e^{itx} - 1| dF_n(x) + \int_{|x| > \epsilon} |e^{itx} - 1| dF_n(x) \\ &\leq \int_{|x| \leq \epsilon} |t| |x| dF_n(x) + 2 \int_{|x| > \epsilon} dF_n(x) \\ &\leq |t| \epsilon P(|X_n| \leq \epsilon) + 2P(|X_n| > \epsilon) \\ &\leq |t| \epsilon + 2P(|X_n| > \epsilon). \end{aligned}$$

Simulations/Visualizations



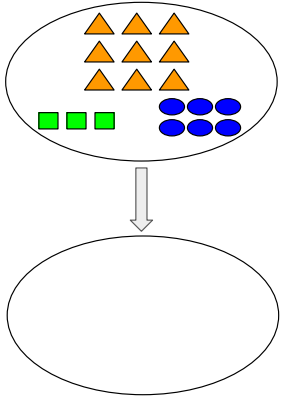
Decision diagrams



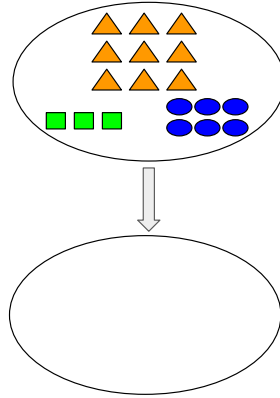
Uniform random vs. stratified random

Sampling: uniform random vs. stratified random

Uniform random



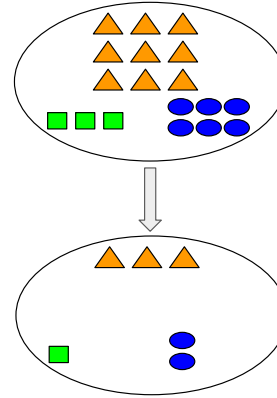
Stratified random



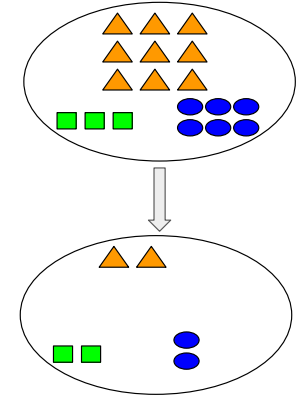
Sample six items: what are the expected outcomes?

Sampling: uniform random vs. stratified random

Uniform random



Stratified random



When would you use which sampling approach?

Statistical vs. practical significance

Statistical significance

Hypothetical study on system performance

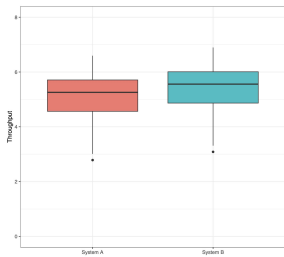
- Compare normalized **throughput** of **two systems**.
- **Statistical test** for the **difference in mean throughput**.

Statistical significance

Hypothetical study on system performance

- Compare normalized throughput of two systems.
- Statistical test for the difference in mean throughput.

Scenario 1: $p = 0.2137$



Scenario 2: $p < 0.05$ (~0.01)

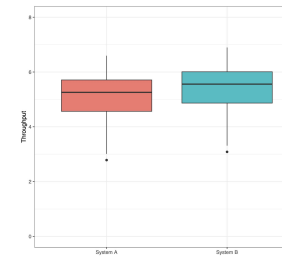
What plot do you expect for Scenario 2?

Statistical significance

Hypothetical study on system performance

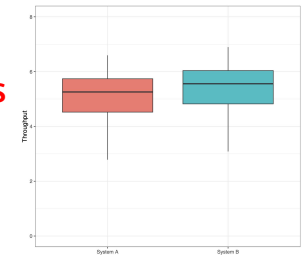
- Compare normalized throughput of two systems.
- Statistical test for the difference in mean throughput.

Scenario 1: $p = 0.2137$



N = 30

Scenario 2: $p < 0.05$ (~0.01)



N = 120

The p value is
conflated
with
sample size!

A little quiz



1. What is the difference between statistical and practical significance?
2. What is the interpretation of the p value?
3. What is an effect size?

Small-group brainstorming

- Explain the answer to a group member.
- Come up with open questions.

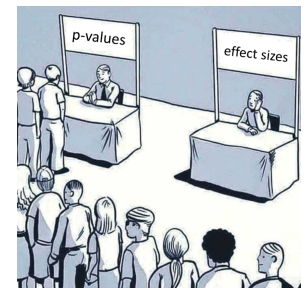
Statistical vs. practical significance

Statistical significance

- Is the difference due to chance?
- *p value*

Practical significance

- Does the difference matter in practice?
- *Effect size*



Effect size measures: examples

Correlation coefficients

- Pearson's r
- Kendall's tau (rank based)
- Spearman's rho (rank based)

“Raw” differences in central tendency

- Difference in means
- Difference in medians

Effect size measures: distinction

Distinction

- Parametric vs. non-parametric
 - Parametric: Pearson's r, Cohen's d
 - Non-parametric: Spearman's rho, A_{12}
- Standardized vs. non-standardized
 - Non-standardized: Difference in means Δ_M
 - Standardized: Δ_M divided by the (pooled) standard deviation
- Variable types
 - Continuous: Cohen's d, A_{12}
 - Ordinal: A_{12} , Cliff's delta, Somers' D
 - Dichotomous: Odds ratio

Interpreting effect sizes

Example (Cohen's d):

- < 0.2: negligible
- ≥ 0.2 : small
- ≥ 0.5 : medium
- ≥ 0.8 : large

Interpreting effect sizes: it's your job!

Example (Cohen's d):

- < 0.2: negligible
- ≥ 0.2 : small
- ≥ 0.5 : medium
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(Standardized) effect sizes are a good starting point, but:

- Is an effect practically significant? Depends on context and domain!
- Raw differences may be easier to interpret (in context).

Generic effect sizes don't provide specific answers!

Contextualizing effect sizes

A statistically significant (large) effect may not be practically relevant:

- System response time: 20ms vs. 10ms
- Analysis runtime: 8h vs. 6h
- Top-5 vs. top-10 ranking
- Magnitude vs. location shift (superiority)

Parametric vs. non-parametric statistics

Parametric vs. non-parametric statistics

Parametric statistics

- Assumptions about the underlying distribution.
Examples for common assumptions:
 - Normal distribution.
 - Equal variance.
- Parametric because of the reliance on distribution parameters.
- Example: Student's t-test, Welch's t-test.

Non-parametric statistics

- Fewer assumptions about the underlying distribution.
- Rank-based -> more robust to outliers.
- Example: Mann Whitney u test (Wilcoxon rank sum test).

Two common statistical tests

Student's/Welch's t test

- Assumes normality
- Hypothesis is related to equality of mean(s).

Mann Whitney u test

- Agnostic to the underlying distribution
- Hypothesis is related to location shift.

A little quiz



1. Why not always use non-parametric statistics (fewer assumptions)?
2. Is the following statement true?
“If a parametric test is not significant, then a non-parametric test cannot be significant either due to less statistical power.”
3. What conclusions can you draw from the Cohen's d vs. A_{12} effect sizes?

My new awesome system

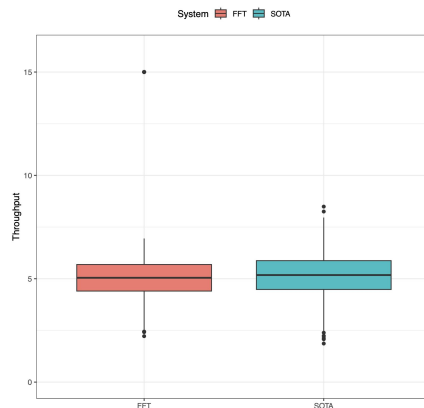
Evaluate system performance

- System: A new system (A) for fast file transfers: **FFT**.
- Goal: Compare the throughput against the state of the art (B): **SOTA**.

Results:

- **Conclusion:** FFT significantly outperforms SOTA:
On average, its throughput of 5.29 files/ms -- a 2.3% increase over SOTA (5.17 files/ms).
- **Statistical significance:** The Mann Whitney U test showed that the difference is significant at the 0.05 significance level ($p=0.0071$).
- **Practical significance:** While a relative increase of 2.3% may seem modest, we argue that this is a big achievement, given how optimized the state of the art is.

My new awesome system



Does this change your perception of the results?
What went wrong?

Statistical analysis: best practices

General advice:

- Be explicit about hypotheses and measures of interest (mean, median, location shift, proportions, etc.).
- Select appropriate statistical tests for a given hypothesis.
- Use data visualization to complement statistical tests.
- Be explicit about the effect size of interest.
- Contextualize effect size (requires domain knowledge).