# CSE 599K

## Empirical Research Methods

Winter 2025

### Statistical methods: overview

# Today

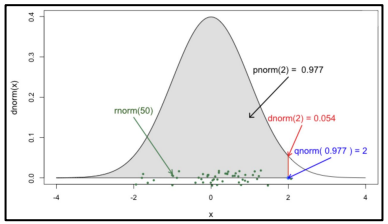
- Uniform vs. stratified sampling
- Statistical vs. practical significance
- Parametric vs non-parametric statistics

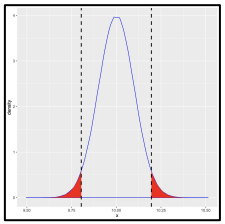
# 3 ways to understand and apply statistics

## Math/Proofs

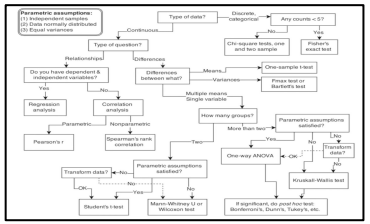
$$
\begin{aligned}
|f_{n_j}(t) - 1| &= \left| \int_{-\infty}^{+\infty} e^{itx} dF_{n_j}(x) - \int_{-\infty}^{+\infty} dF_{n_j}(x) \right| \\
&\leq \int_{-\infty}^{+\infty} \left| e^{itx} - 1 \right| dF_{n_j}(x) \\
&= \int_{|x| \leq \varepsilon} \left| e^{itx} - 1 \right| dF_{n_j}(x) + \int_{|x| > \varepsilon} \left| e^{itx} - 1 \right| dF_{n_j}(x) \\
&\leq \int_{|x| \leq \varepsilon} |tx| dF_{n_j}(x) + 2 \int_{|x| > \varepsilon} dF_{n_j}(x) \\
&\leq |t|\varepsilon P(|X_{n_j}| \leq \varepsilon) + 2P(|X_{n_j}| > \varepsilon) \\
&\leq |t|\varepsilon + 2P(|X_{n_j}| > \varepsilon).
\end{aligned}
$$

## Simulations/Visualizations



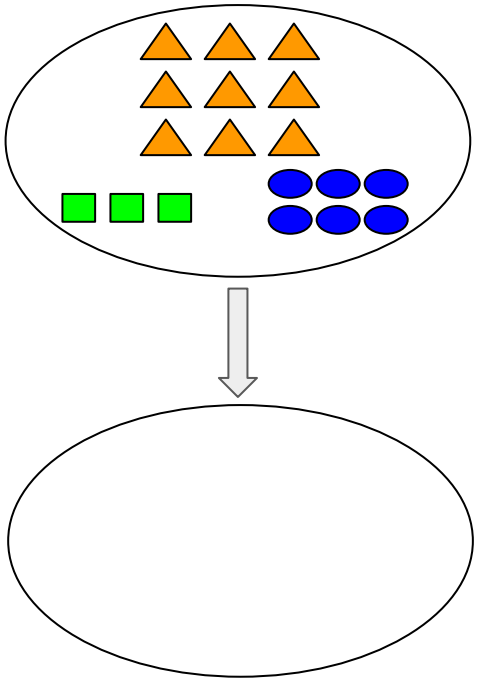
## Decision diagrams



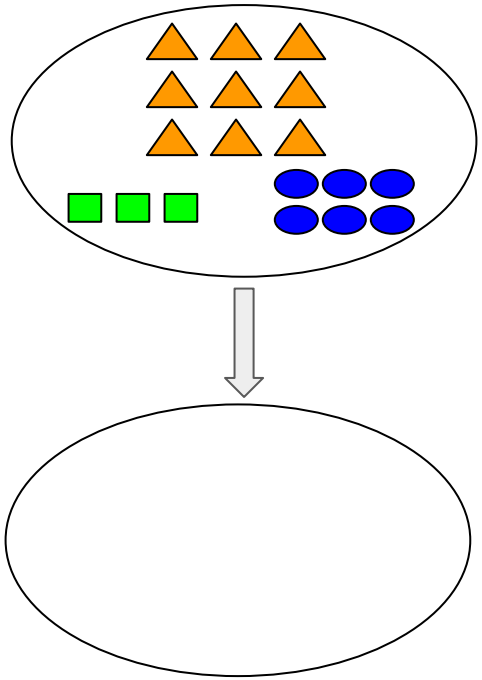
# Uniform random vs. stratified random

# Sampling: uniform random vs. stratified random

**Uniform random**



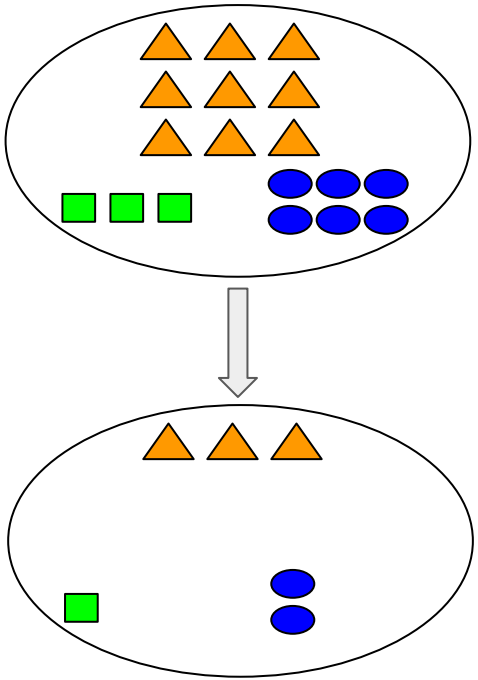
**Stratified random**



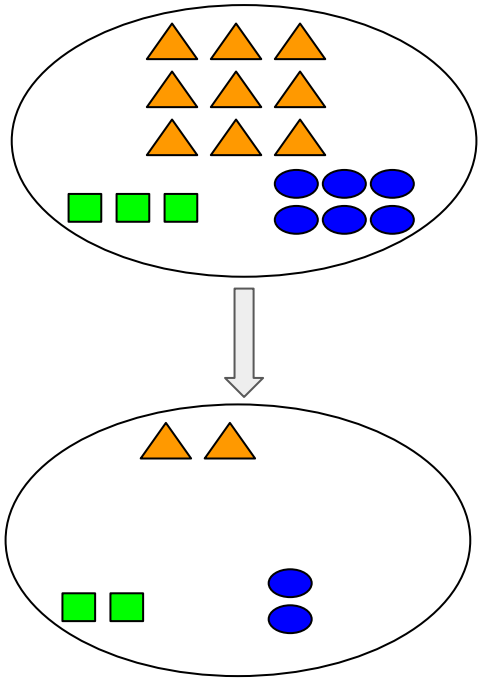
Sample six items: what are the expected outcomes?

# Sampling: uniform random vs. stratified random

**Uniform random**



**Stratified random**



When would you use which sampling approach?

# Statistical vs. practical significance

# Statistical significance

## Hypothetical study on system performance

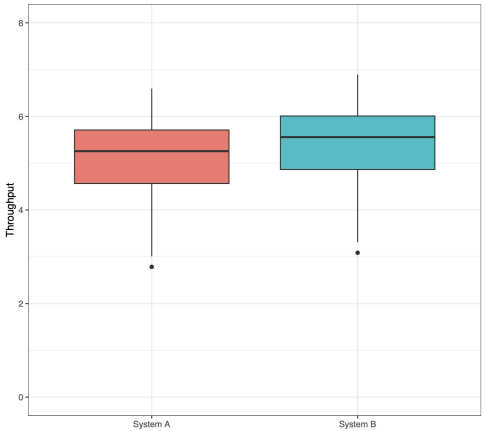
- Compare normalized **throughput** of **two systems**.
- **Statistical test** for the **difference in mean throughput**.

# Statistical significance

## Hypothetical study on system performance

- Compare normalized throughput of two systems.
- Statistical test for the difference in mean throughput.

**Scenario 1**: p = 0.2137




**Scenario 2**: p < 0.05 (~0.01)

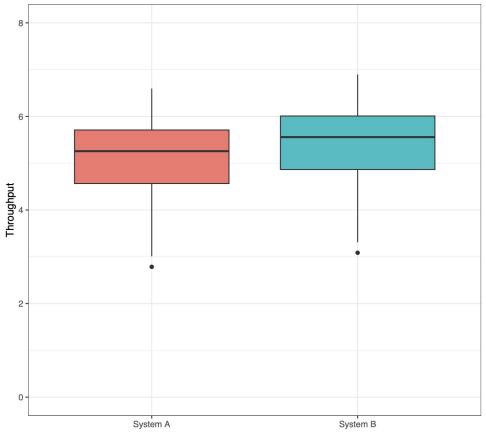
What plot do you expect for Scenario 2?

# Statistical significance

## Hypothetical study on system performance

- Compare normalized throughput of two systems.
- Statistical test for the difference in mean throughput.

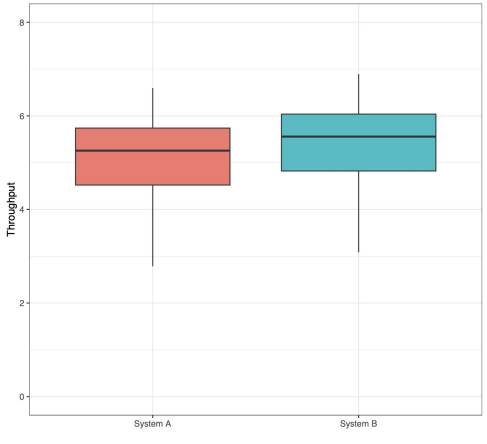
**Scenario 1**: p = 0.2137




N = 30

The p value is conflated with **sample size!**

**Scenario 2**: p < 0.05 (~0.01)




N = 120

# A little quiz

1. What is the difference between statistical and practical significance?
2. What is the interpretation of the p value?
3. What is an effect size?

Small-group brainstorming

- Explain the answer to a group member.
- Come up with open questions.

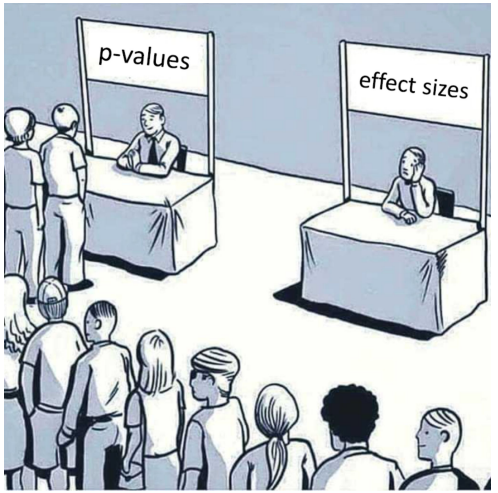
# Statistical vs. practical significance

**Statistical significance**

- Is the difference due to chance?
- *p value*

**Practical significance**

- Does the difference matter in practice?
- *Effect size*



# Effect size measures: examples

## Correlation coefficients

- Pearson's r
- Kendall's tau (rank based)
- Spearman's rho (rank based)

## "Raw" differences in central tendency

- Difference in means
- Difference in medians

# Effect size measures: distinction

## Distinction

- Parametric vs. non-parametric
  - Parametric: Pearson's r, Cohen's d
  - Non-parametric: Spearman's rho, $A_{12}$
- Standardized vs. non-standardized
  - Non-standardized: Difference in means $\Delta_M$
  - Standardized: $\Delta_M$ divided by the (pooled) standard deviation
- Variable types
  - Continuous: Cohen's d, $A_{12}$
  - Ordinal: $A_{12,}$ Cliff's delta, Somers' D
  - Dichotomous: Odds ratio

# Interpreting effect sizes

**Example (Cohen's d):**

- < 0.2: negligible
- >= 0.2: small
- >= 0.5: medium
- >= 0.8: large

# Interpreting effect sizes: it's your job!

**Example (Cohen's d):**

- < 0.2: negligible
- >= 0.2: small
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**(Standardized) effect sizes are a good starting point, but:**

- Is an effect practically significant? Depends on context and domain!
- Raw differences may be easier to interpret (in context).

Generic effect sizes don't provide specific answers!

# Contextualizing effect sizes

**A statistically significant (large) effect may not be practically relevant:**

- System response time: 20ms vs. 10ms
- Analysis runtime: 8h vs. 6h
- Top-5 vs. top-10 ranking
- Magnitude vs. location shift (superiority)

# Parametric vs. non-parametric statistics

# Parametric vs. non-parametric statistics

## Parametric statistics

- Assumptions about the underlying distribution.

  Examples for common assumptions:
  - Normal distribution.
  - Equal variance.
- Parametric because of the reliance on distribution parameters.
- Example: Student’s t-test, Welch’s t-test.

## Non-parametric statistics

- Fewer assumptions about the underlying distribution.
- Rank-based -> more robust to outliers.
- Example: Mann Whitney u test (Wilcoxon rank sum test).

# Two common statistical tests

## Student's/Welch's t test

- Assumes normality
- Hypothesis is related to equality of mean(s).

## Mann Whitney u test

- Agnostic to the underlying distribution
- Hypothesis is related to location shift.

# A little quiz

1. Why not always use non-parametric statistics (fewer assumptions)?
2. Is the following statement true?
   "If a parametric test is not significant, then a non-parametric test cannot be significant either due to less statistical power."
3. What conclusions can you draw from the Cohen's d vs. $A_{12}$ effect sizes?

# My new awesome system

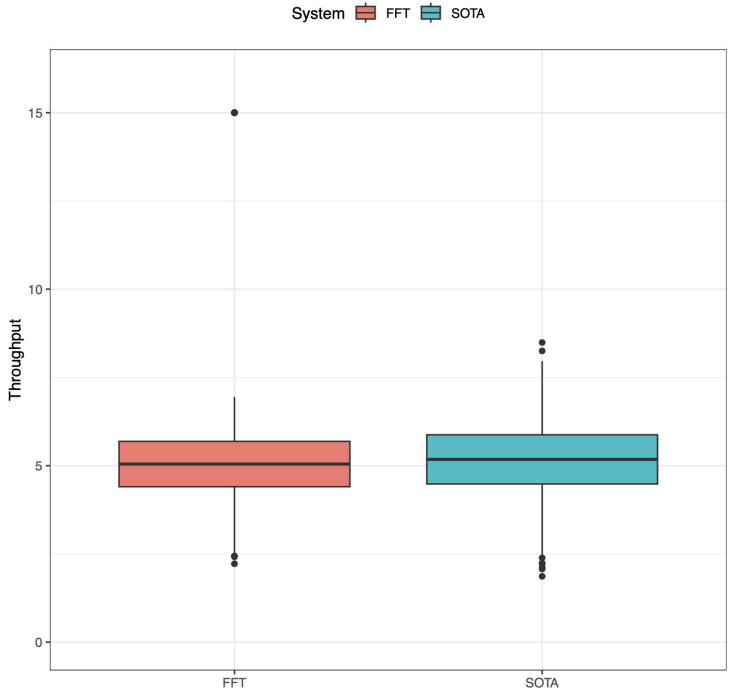
## Evaluate system performance

- System: A new system (**A**) for fast file transfers: **FFT**.
- Goal: Compare the throughput against the state of the art (**B**): **SOTA**.

## Results:

- ***Conclusion:*** FFT significantly outperforms SOTA:
  On average, its throughput of 5.29 files/ms -- a 2.3% increase over SOTA (5.17 files/ms).
- ***Statistical significance:*** The Mann Whitney U test showed that the difference is significant at the 0.05 significance level (p=0.0071).
- ***Practical significance***: While a relative increase of 2.3% may seem modest, we argue that this is a big achievement, given how optimized the state of the art is.

# My new awesome system




Does this change your perception of the results?
What went wrong?

# Statistical analysis: best practices

## General advice:

- Be explicit about hypotheses and measures of interest (mean, median, location shift, proportions, etc.).
- Select appropriate statistical tests for a given hypothesis.
- Use data visualization to complement statistical tests.
- Be explicit about the effect size of interest.
- Contextualize effect size (requires domain knowledge).