

# CSE P 504


Advanced topics in Software Systems

Fall 2022

## Invariants and partial test oracles

November 07, 2022

## Course overview: the big picture

- **10/03:** Course introduction HW 1
  - **10/10:** Best practices and version control In-class exercise
  - **10/17:** Coverage-based testing In-class exercise
  - **10/24:** Mutation-based testing In-class exercise
  - **10/31:** Delta debugging In-class exercise
  - **11/07:** Invariants and partial oracles In-class exercise
  - **11/14:** Statistical fault localization In-class exercise
  - **11/21:** Static analysis Happy Thanksgiving
  - **11/28:** Abstract interpretation HW 2
  - **12/05:** Formal methods In-class exercise
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## Reasoning about programs

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### Use cases

- Testing: increase confidence in correctness
- Verification: prove facts to be true, e.g.:
  - x is never null
  - y is always greater than 0
  - a happens before b
- Debugging: understand why code is incorrect

## Reasoning about programs

### Use cases

- Testing: increase confidence in correctness
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  - a happens before b
- Debugging: understand why code is incorrect

### Approaches

- Testing
- Abstract interpretation
- Theorem proving
- Delta debugging
- Slicing
- ...

## Forward vs. backward reasoning

### Forward reasoning

- Knowing a fact that is true before execution.
- Reasoning about **what must be true after execution**.
- Given a precondition, what postcondition(s) are true?

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### Backward reasoning

- Knowing a fact that is true after execution.
- Reasoning about **what must be true before execution**.
- Given a postcondition, what precondition(s) must hold?

What are the pros and cons for each approach?

## Forward vs. backward reasoning

### Forward reasoning

- More intuitive for most people
- Helps understand what will happen (simulates the code)
- Introduces facts that may be irrelevant to the goal
- Set of current facts may get large
- Takes longer to realize that the task is hopeless

### Backward reasoning

- Usually more helpful
- Helps understand what should happen
- Given a specific goal, indicates how to achieve it
- Given an error, gives a test case that exposes it

## Pre/Post-conditions and Invariants

### Terminology

#### Pre-condition (to a function)

- A condition that must be true when entering (the function)
- May include expectations about the arguments

#### Post-condition (to a function)

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- Must be true at the beginning and end of the loop body

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Pre-conditions define execution validity. Post-conditions and loop invariants define expected properties of a correct implementation, given a valid execution.

## Pre-conditions and post-conditions



```
1 double avgAbs(double[] nums) {
2   int n = nums.length;
3   double sum = 0;
4
5   int i = 0;
6   while (i != n) {
7     if(nums[i]>0) {
8       sum = sum + nums[i];
9     } else {
10      sum = sum - nums[i];
11    }
12    i = i + 1;
13  }
14
15  return sum / n;
16 }
```

Entry point

What are pre-conditions  
and post-conditions of  
this method (at the entry  
and exit points)?

Exit point

## Pre-conditions and post-conditions

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### Pre-conditions

- `nums` is not null
- `nums.length > 0`

### Post-conditions

- `nums` has not changed
- `n > 0`
- `sum >= 0`
- return value `>= 0`
- ...

## (Loop) invariants



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```

Does this loop terminate?  
What are pre-conditions,  
post-conditions,  
and loop invariants?

## (Loop) invariants

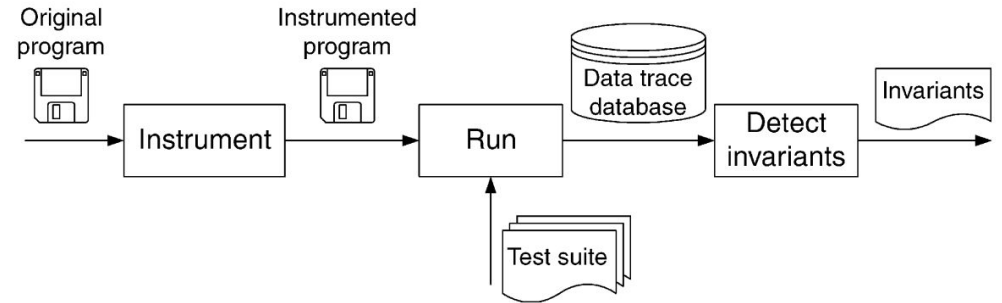
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Explicitly stating invariants  
is hard -- reasoning about  
inferred variants might be  
easier.

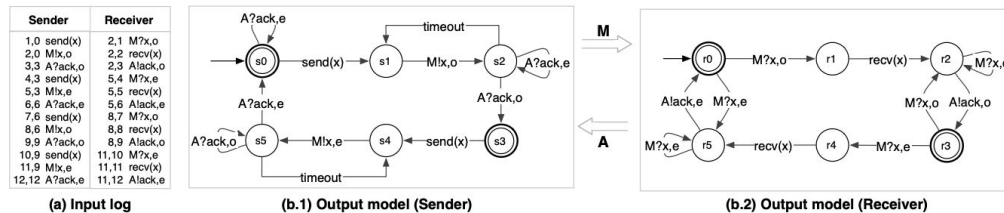
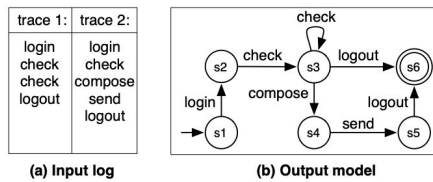
# Daikon live example

<https://plse.cs.washington.edu/daikon/download/doc/daikon/Example-usage.html#Detecting-invariants-in-Java-programs>

# Daikon: general workflow



# Log-based model inference



**Partial test oracles,  
Property-based testing  
Metamorphic testing\***

Beschastnikh et al., Synoptic: Studying Logged Behavior with Inferred Models; Inferring Models of Concurrent Systems from Logs of their Behavior with CSight

\*Chen et al. coined the term metamorphic testing in 1998, but the key idea was first described by Ammann and Knight as data diversity in 1988.

## Partial test oracles

### Partial test oracle

- Necessary (but not sufficient) conditions
- Example:  $\text{abs}(x) \geq 0$

## Property-based testing

### Partial test oracle

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### Property-based testing

- Check property (necessary condition) that must hold for any input, which requires knowledge about the system
- Commonly used with random input generation

How is property-based testing different from testing with input-output pairs and how is it different from fuzzing?

## Property-based testing

### Partial test oracle

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### Property-based testing

- Check property (necessary condition) that must hold for any input, which requires knowledge about the system
  - Commonly used with random input generation
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- Contrast: testing with input-output pairs usually checks for sufficient conditions for a (small) subset of all possible inputs
  - Contrast: fuzzing is usually a black-box approach that checks for a simple property (“should not crash”)

## Data diversity and metamorphic testing

### Simple case: related inputs with identical outcomes

- Expected output for a given input is unknown
- Two related inputs must result in the same output
- Example:  $\text{abs}(x) == \text{abs}(-x)$

## Data diversity and metamorphic testing

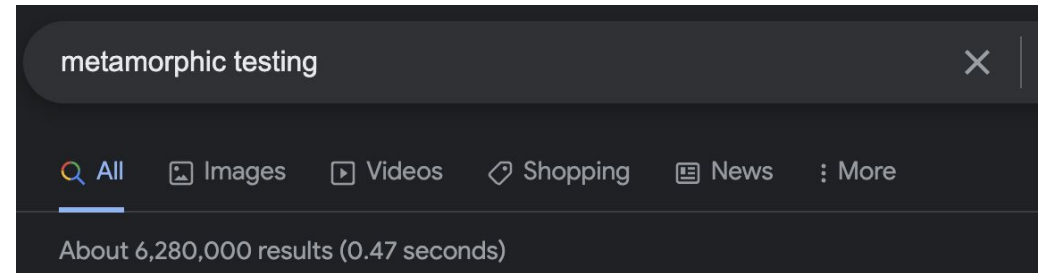
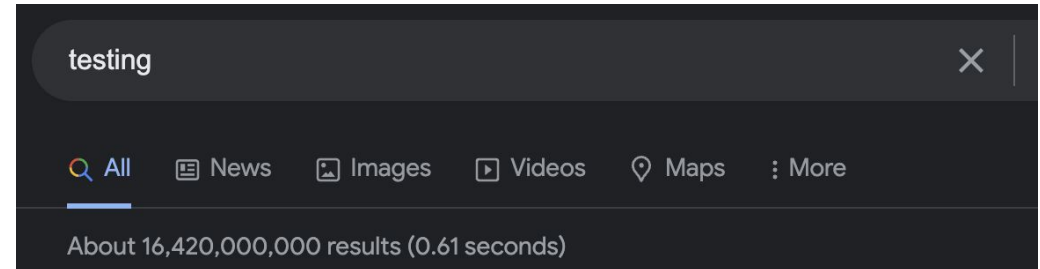
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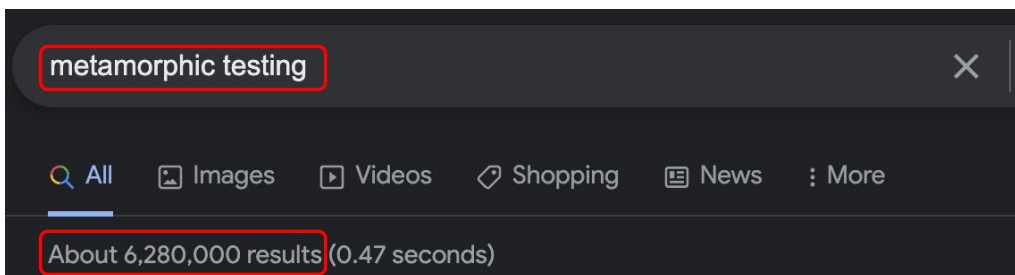
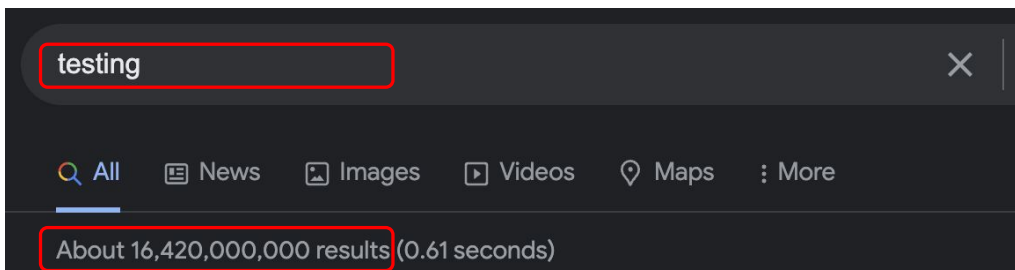
### Generalization: related inputs and related outputs

- Input  $i_1$  yields (unknown)  $o_1$  (initial input)
- $R_i: i_1 \implies i_2$  (follow-up input)
- $R_o: o_1 \implies o_2$  (necessary condition)

## Metamorphic testing: a first example



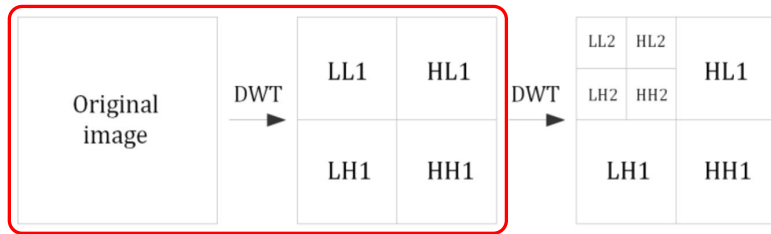
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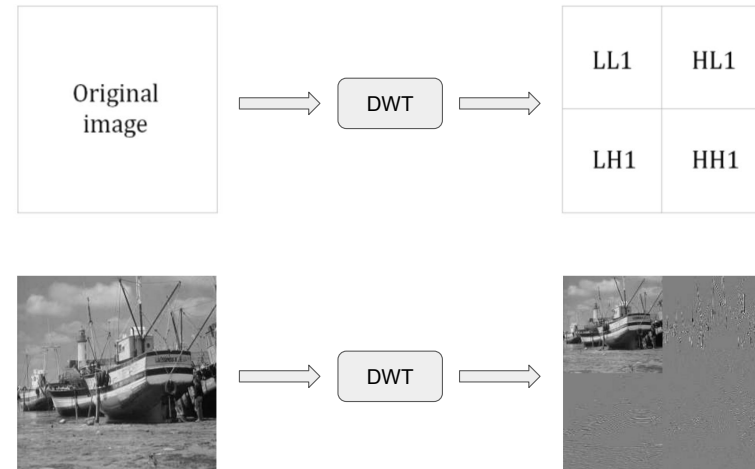
## Discrete wavelet transformation



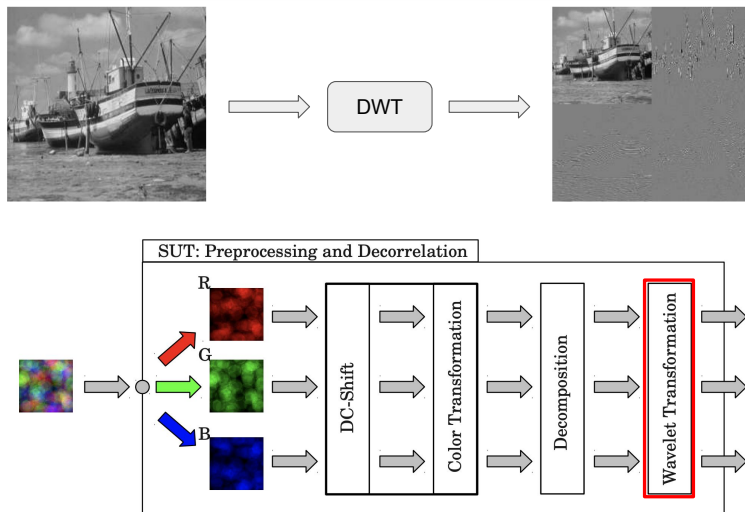
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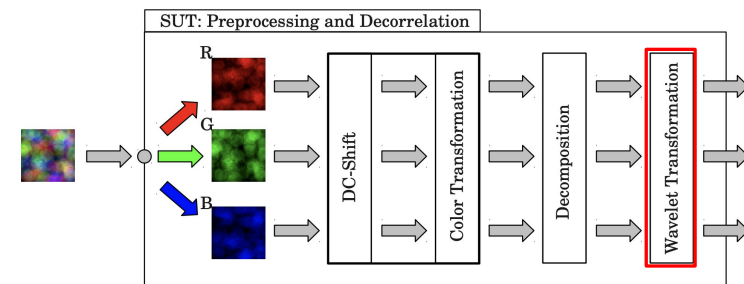
## Discrete wavelet transformation



## A concrete SUT: jpeg2000 encoder



## Metamorphic testing: three requirements

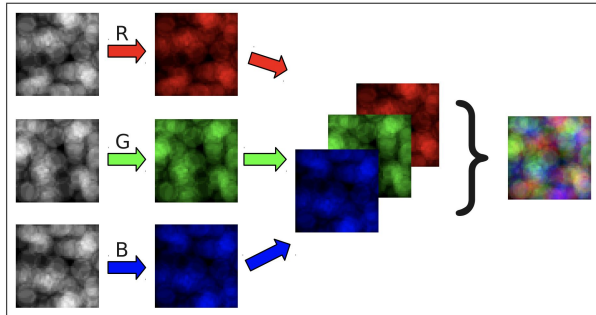


### MT requires

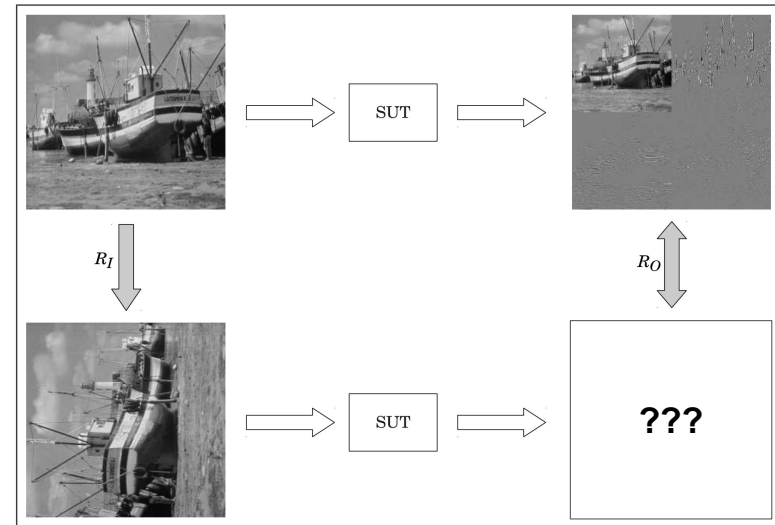
1. A set of initial inputs (or a generator)
2. A relation  $R_i$ : generates follow-up inputs
3. A relation  $R_o$ : necessary correctness condition



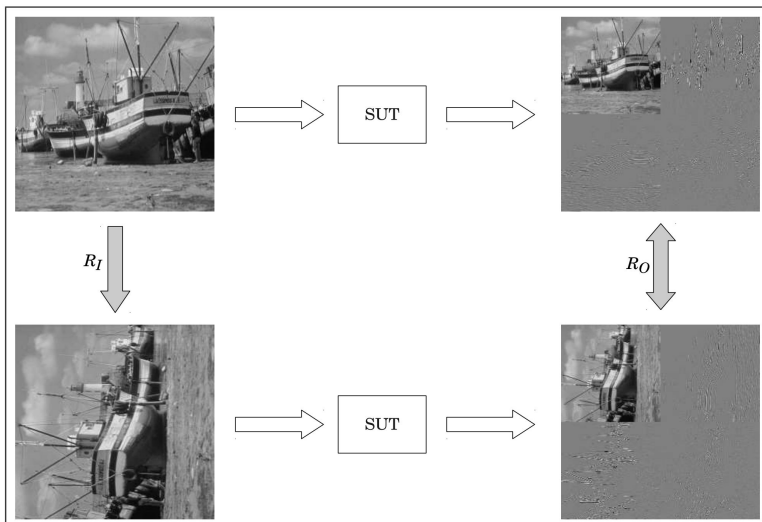
## Metamorphic testing: Input generation



## Metamorphic testing: relations $R_i$ and $R_o$

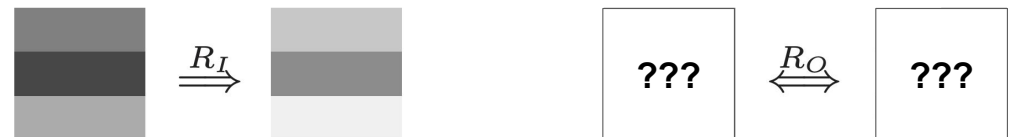


## Metamorphic testing: relations $R_i$ and $R_o$



## Metamorphic testing: Relations

1.  $R_i$ : Add a constant offset to all color values  
 $R_o$ : ???



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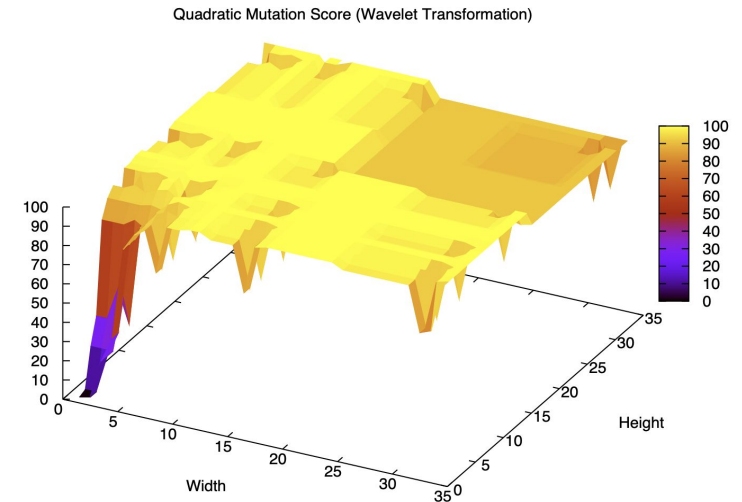
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Commutative

Time-invariant

It turns out that MR compositions are effective

## Metamorphic testing: effectiveness



## Putting it all together

1. (Random) input generation
2. Metamorphic testing: follow-up inputs and partial oracles
3. Delta debugging: Minimize bug-exposing inputs
4. Mutation analysis: assess the effectiveness of relations

Examples:

- GraphicsFuzz
- Testing ML-based systems