

# CSE P 504


Advanced topics in Software Systems

Fall 2022

**Invariants and partial test oracles**

November 07, 2022

# Course overview: the big picture

- 
- **10/03:** Course introduction HW 1
  - **10/10:** Best practices and version control In-class exercise
  - **10/17:** Coverage-based testing In-class exercise
  - **10/24:** Mutation-based testing In-class exercise
  - **10/31:** Delta debugging In-class exercise
  - **11/07:** Invariants and partial oracles In-class exercise
  - **11/14:** Statistical fault localization In-class exercise
  - **11/21:** Static analysis Happy Thanksgiving
  - **11/28:** Abstract interpretation HW 2
  - **12/05:** Formal methods In-class exercise

# Reasoning about programs

# Reasoning about programs

## Use cases

- Testing: increase confidence in correctness
- Verification: prove facts to be true, e.g.:
  - x is never null
  - y is always greater than 0
  - a happens before b
- Debugging: understand why code is incorrect

# Reasoning about programs

## Use cases

- Testing: increase confidence in correctness
- Verification: prove facts to be true, e.g.:
  - x is never null
  - y is always greater than 0
  - a happens before b
- Debugging: understand why code is incorrect

## Approaches

- Testing
- Abstract interpretation
- Theorem proving
- Delta debugging
- Slicing
- ...

# Forward vs. backward reasoning

## Forward reasoning

- Knowing a fact that is true before execution.
- Reasoning about **what must be true after execution.**
- Given a precondition, what postcondition(s) are true?

# Forward vs. backward reasoning

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## Backward reasoning

- Knowing a fact that is true after execution.
- Reasoning about **what must be true before execution.**
- Given a postcondition, what precondition(s) must hold?

What are the pros and cons for each approach?

# Forward vs. backward reasoning

## **Forward reasoning**

- More intuitive for most people
- Helps understand what will happen (simulates the code)
- Introduces facts that may be irrelevant to the goal
- Set of current facts may get large
- Takes longer to realize that the task is hopeless

## **Backward reasoning**

- Usually more helpful
- Helps understand what should happen
- Given a specific goal, indicates how to achieve it
- Given an error, gives a test case that exposes it



# **Pre/Post-conditions and Invariants**

# Terminology

## **Pre-condition** (to a function)

- A condition that must be true when entering (the function)
- May include expectations about the arguments

## **Post-condition** (to a function)

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- A condition that must be true for every loop iteration
- Must be true at the beginning and end of the loop body

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## **Loop invariant**

- A condition that must be true for every loop iteration
- Must be true at the beginning and end of the loop body

Pre-conditions define execution validity. Post-conditions and loop invariants define expected properties of a correct implementation, given a valid execution.

# Pre-conditions and post-conditions



```
1 double avgAbs(double[] nums) {  
2   int n = nums.length;  
3   double sum = 0;  
4  
5   int i = 0;  
6   while (i != n) {  
7     if(nums[i]>0) {  
8       sum = sum + nums[i];  
9     } else {  
10      sum = sum - nums[i];  
11    }  
12    i = i + 1;  
13  }  
14  
15  return sum / n;  
16 }
```

Entry point

Exit point

What are pre-conditions  
and post-conditions of  
this method (at the entry  
and exit points)?

# Pre-conditions and post-conditions

```
1 double avgAbs(double[] nums) {
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12    i = i + 1;
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14
15  return sum / n;
16 }
```

## Pre-conditions

- `nums` is not null
- `nums.length > 0`

## Post-conditions

- `nums` has not changed
- `n > 0`
- `sum >= 0`
- `return value >= 0`
- ...

# (Loop) invariants



```
1 double avgAbs(double[] nums) {
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```

Does this loop terminate?  
What are pre-conditions,  
post-conditions,  
and loop invariants?

# (Loop) invariants

```
1 double avgAbs(double[] nums) {
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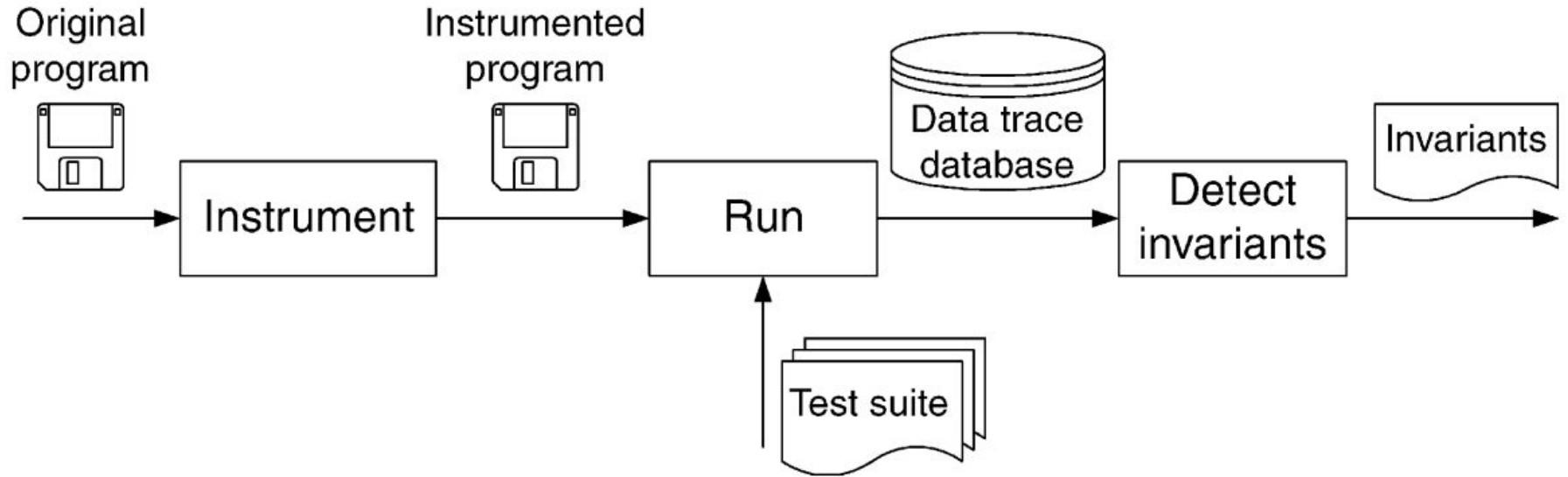
Explicitly stating invariants  
is hard -- reasoning about  
inferred variants might be  
easier.



# Daikon live example

<https://plse.cs.washington.edu/daikon/download/doc/daikon/Example-usage.html#Detecting-invariants-in-Java-programs>

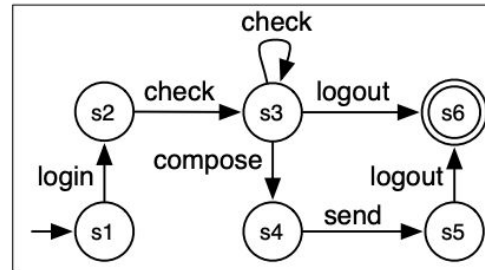
# Daikon: general workflow



# Log-based model inference

trace 1:	trace 2:
login	login
check	check
check	compose
logout	send
	logout

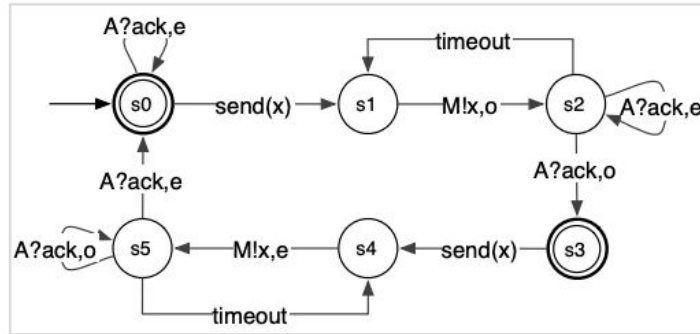
(a) Input log



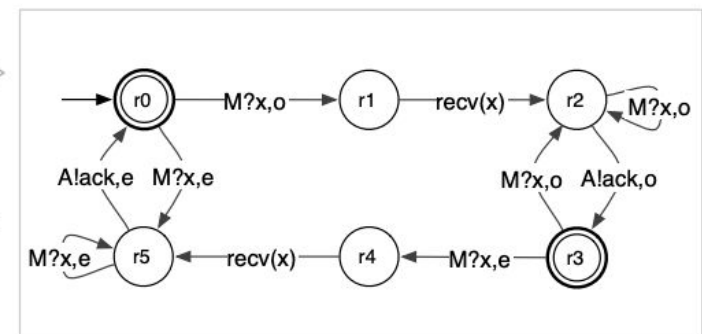
(b) Output model

Sender	Receiver
1,0 send(x)	2,1 M?x,o
2,0 M!x,o	2,2 recv(x)
3,3 A?ack,o	2,3 A!ack,o
4,3 send(x)	5,4 M?x,e
5,3 M!x,e	5,5 recv(x)
6,6 A?ack,e	5,6 A!ack,e
7,6 send(x)	8,7 M?x,o
8,6 M!x,o	8,8 recv(x)
9,9 A?ack,o	8,9 A!ack,o
10,9 send(x)	11,10 M?x,e
11,9 M!x,e	11,11 recv(x)
12,12 A?ack,e	11,12 A!ack,e

(a) Input log



(b.1) Output model (Sender)



(b.2) Output model (Receiver)

**Partial test oracles,  
Property-based testing  
Metamorphic testing\***

# Partial test oracles

## **Partial test oracle**

- Necessary (but not sufficient) conditions
- Example:  $\text{abs}(x) \geq 0$

# Property-based testing

## Partial test oracle

- Necessary (but not sufficient) conditions
- Example:  $\text{abs}(x) \geq 0$

## Property-based testing

- Check property (necessary condition) that must hold for any input, which requires knowledge about the system
- Commonly used with random input generation

How is property-based testing different from testing with input-output pairs and how is it different from fuzzing?

# Property-based testing

## Partial test oracle

- Necessary (but not sufficient) conditions
- Example:  $\text{abs}(x) \geq 0$

## Property-based testing

- Check property (necessary condition) that must hold for any input, which requires knowledge about the system
  - Commonly used with random input generation
- 
- Contrast: testing with input-output pairs usually checks for sufficient conditions for a (small) subset of all possible inputs
  - Contrast: fuzzing is usually a black-box approach that checks for a simple property (“should not crash”)

# Data diversity and metamorphic testing

## **Simple case: related inputs with identical outcomes**

- Expected output for a given input is unknown
- Two related inputs must result in the same output
- Example:  $\text{abs}(x) == \text{abs}(-x)$



# Data diversity and metamorphic testing

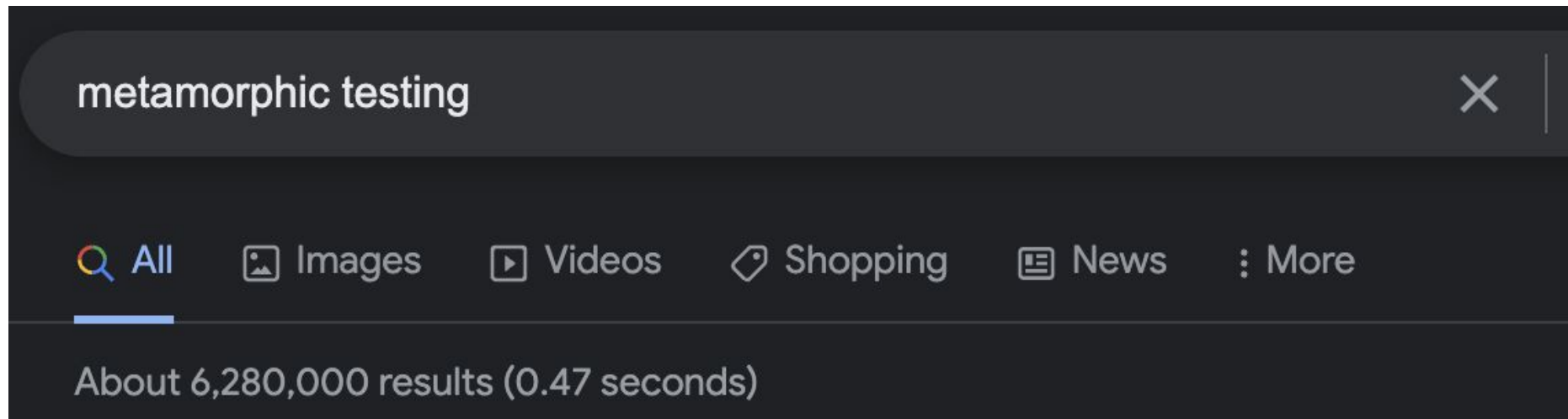
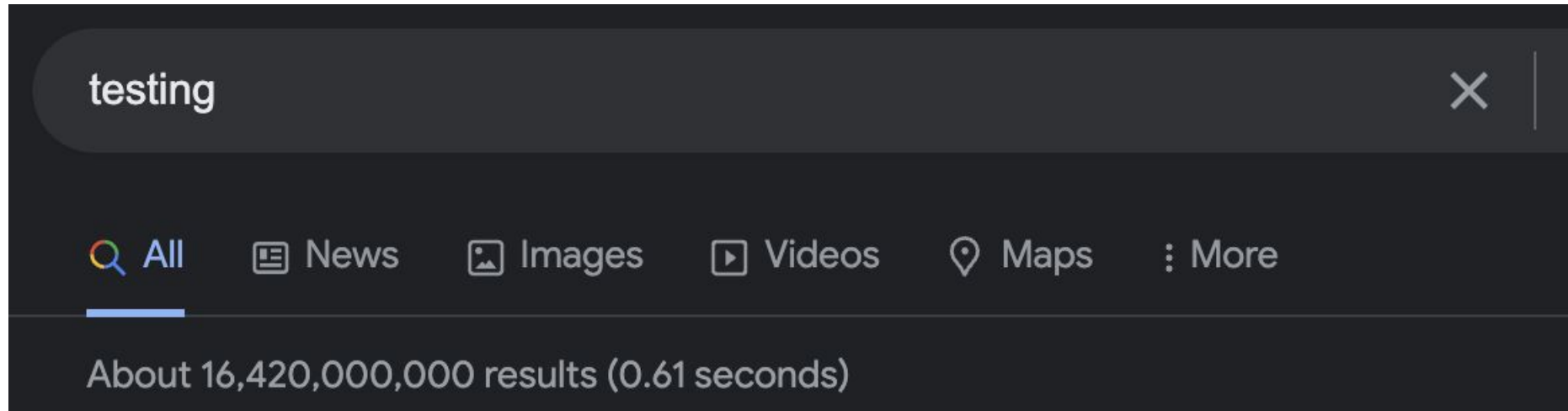
## Simple case: related inputs with identical outcomes

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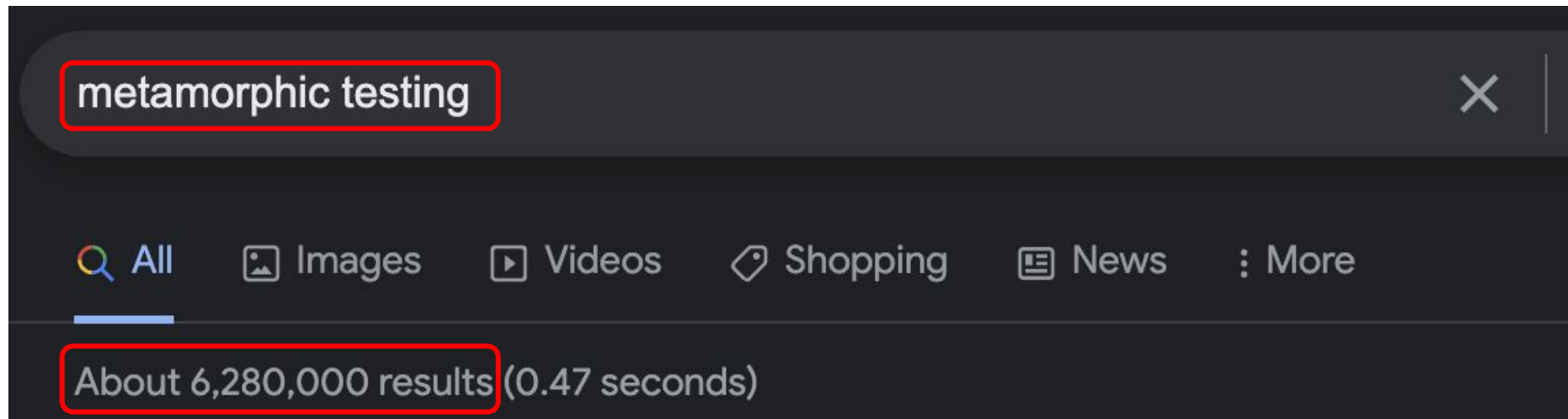
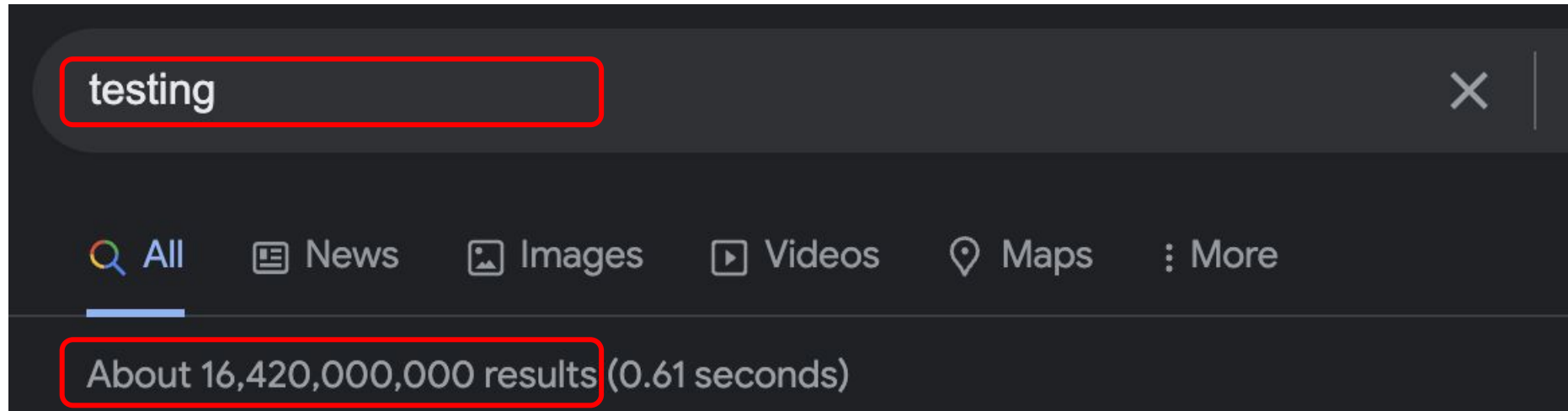
## Generalization: related inputs and related outputs

- Input  $i_1$  yields (unknown)  $o_1$  (initial input)
- $R_i: i_1 \Longrightarrow i_2$  (follow-up input)
- $R_o: o_1 \Longrightarrow o_2$  (necessary condition)

# Metamorphic testing: a first example



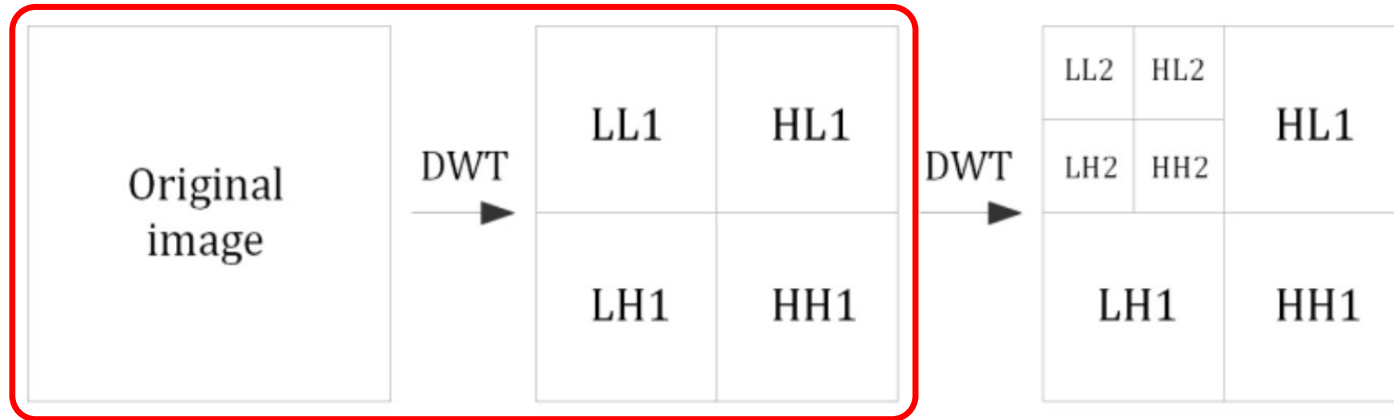
# Metamorphic testing: a first example



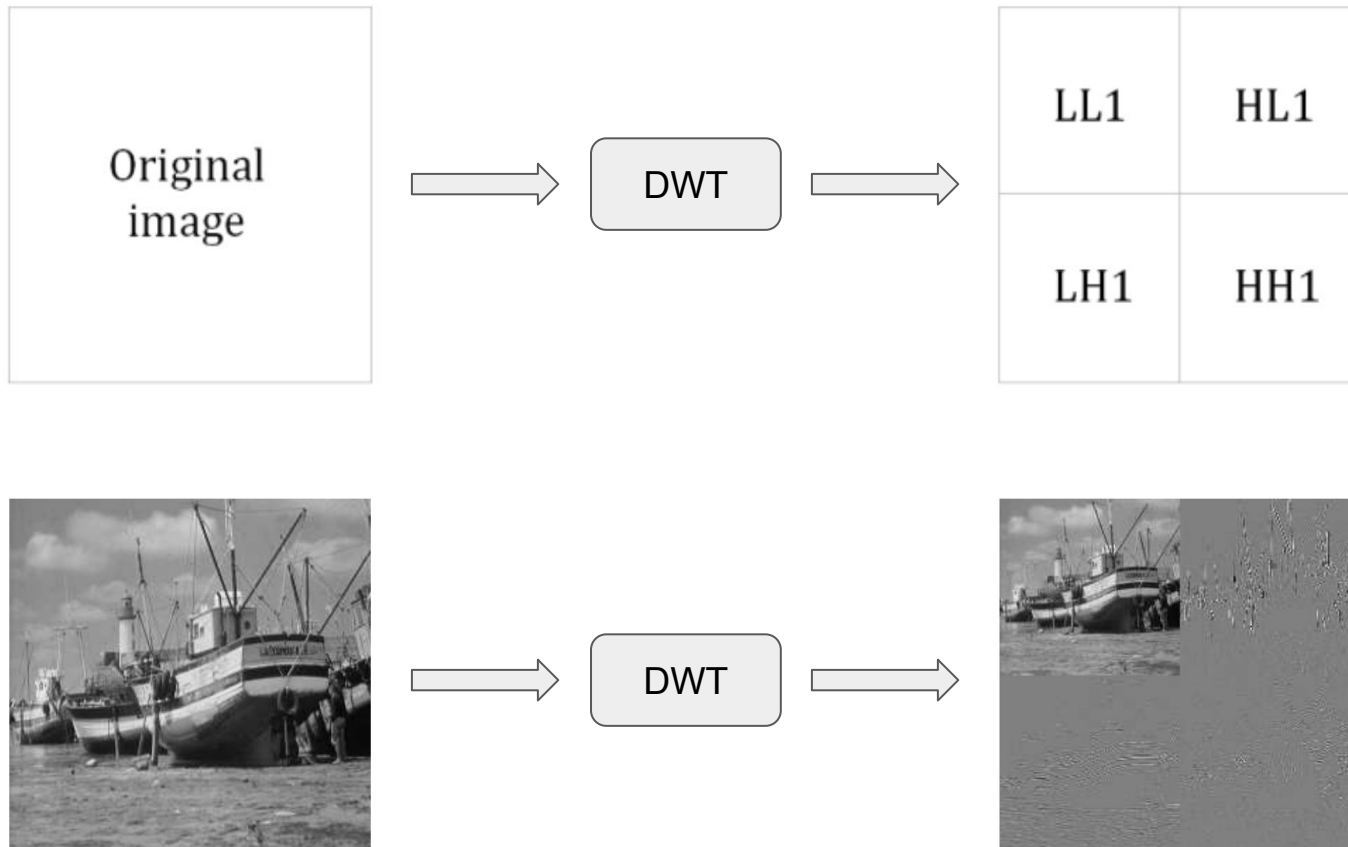
# Discrete wavelet transformation



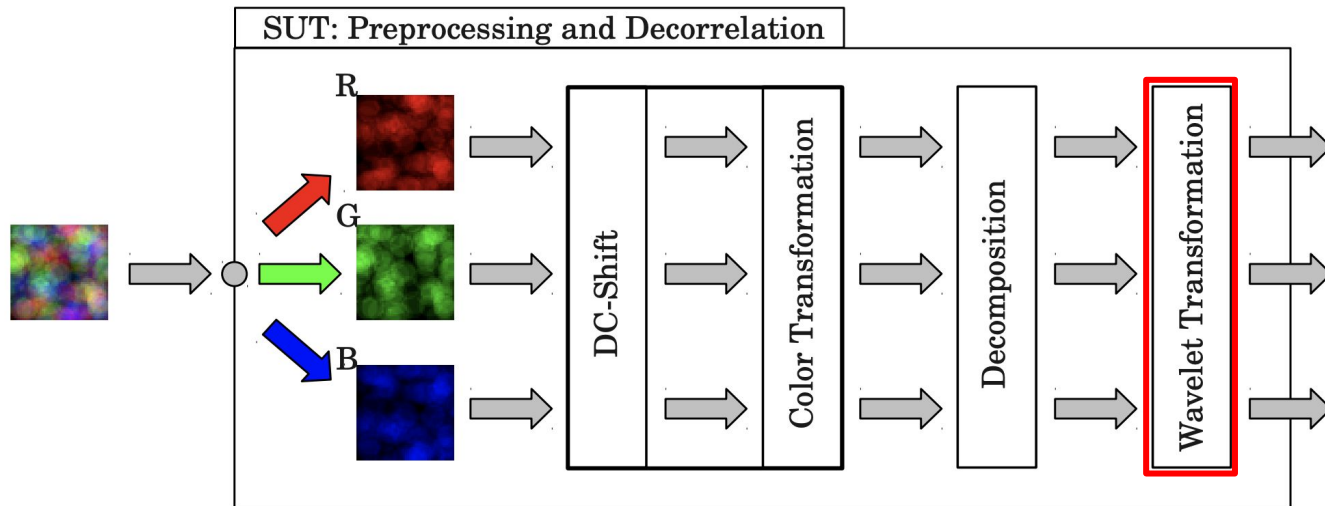
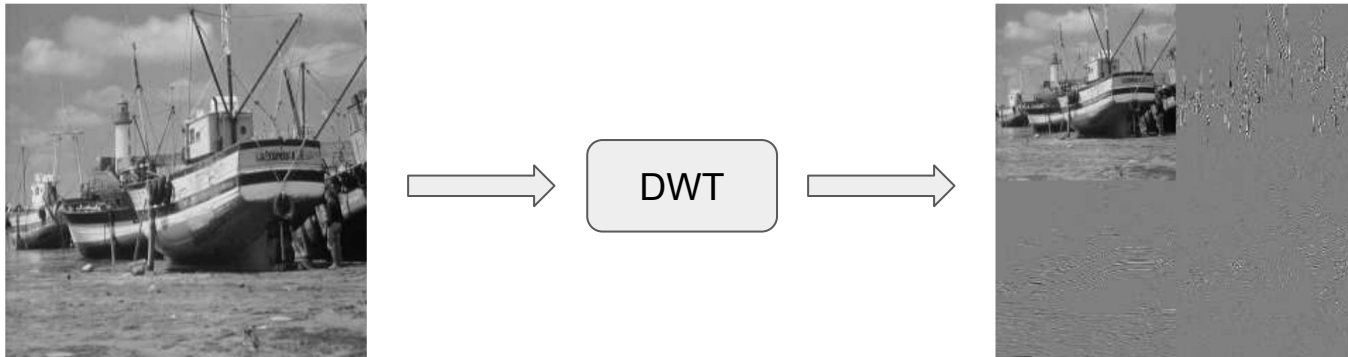
# Discrete wavelet transformation



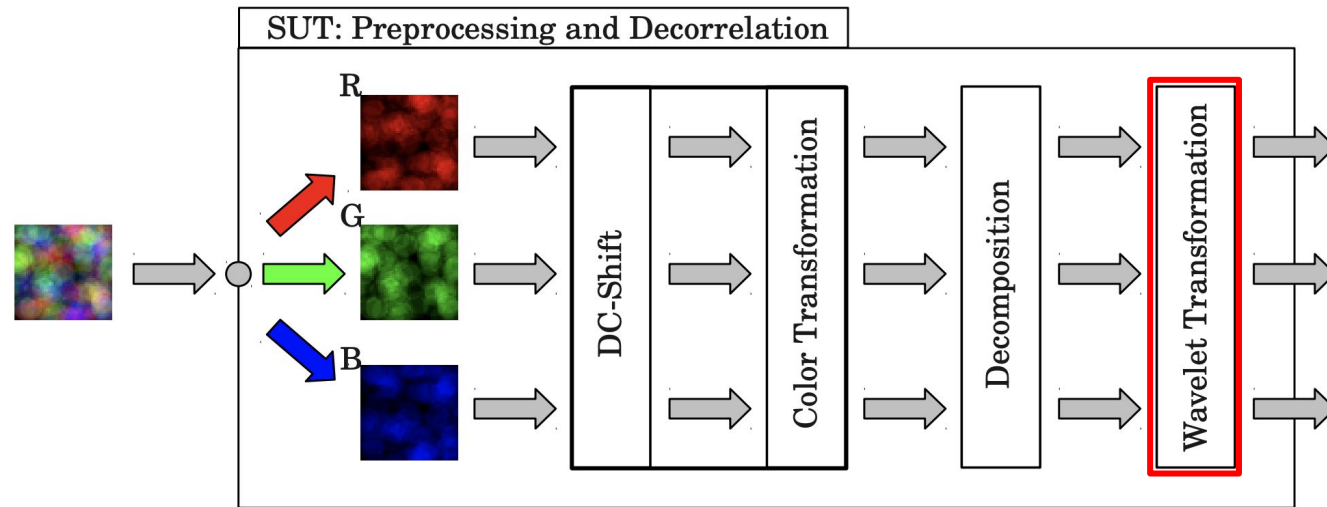
# Discrete wavelet transformation



# A concrete SUT: jpeg2000 encoder



# Metamorphic testing: three requirements

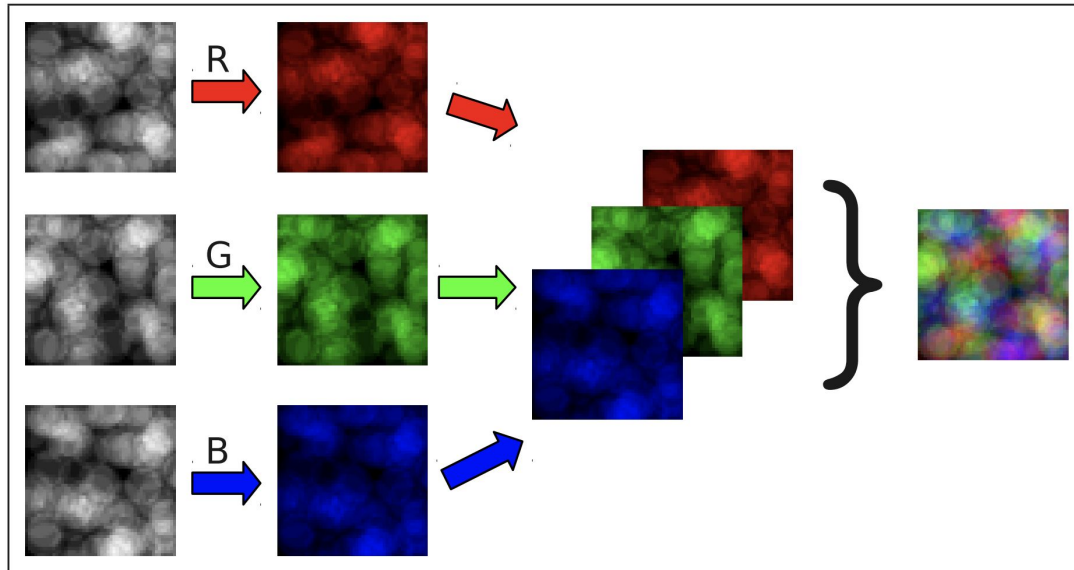


## MT requires

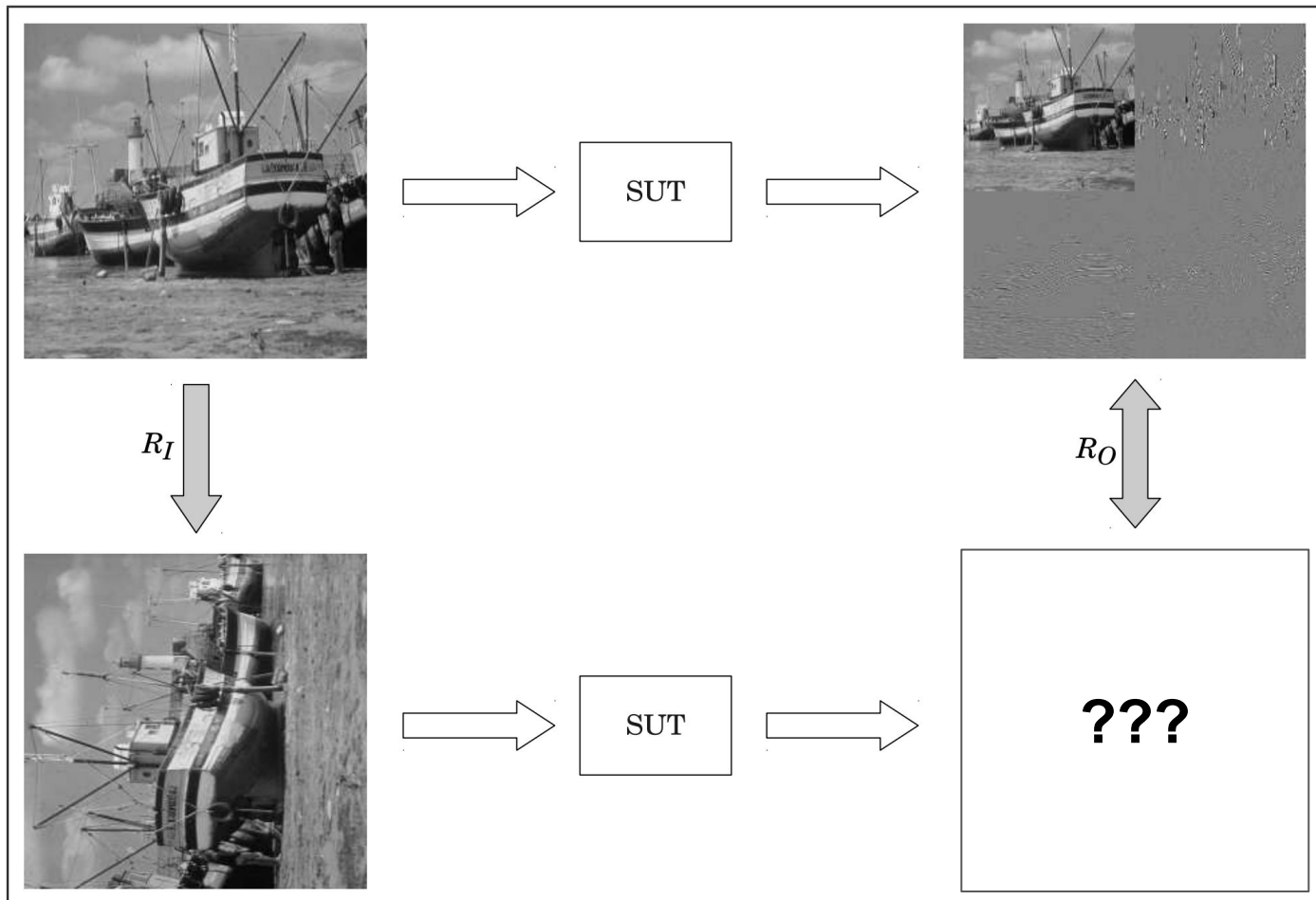
1. A set of initial inputs (or a generator)
2. A relation  $R_i$ : generates follow-up inputs
3. A relation  $R_o$ : necessary correctness condition



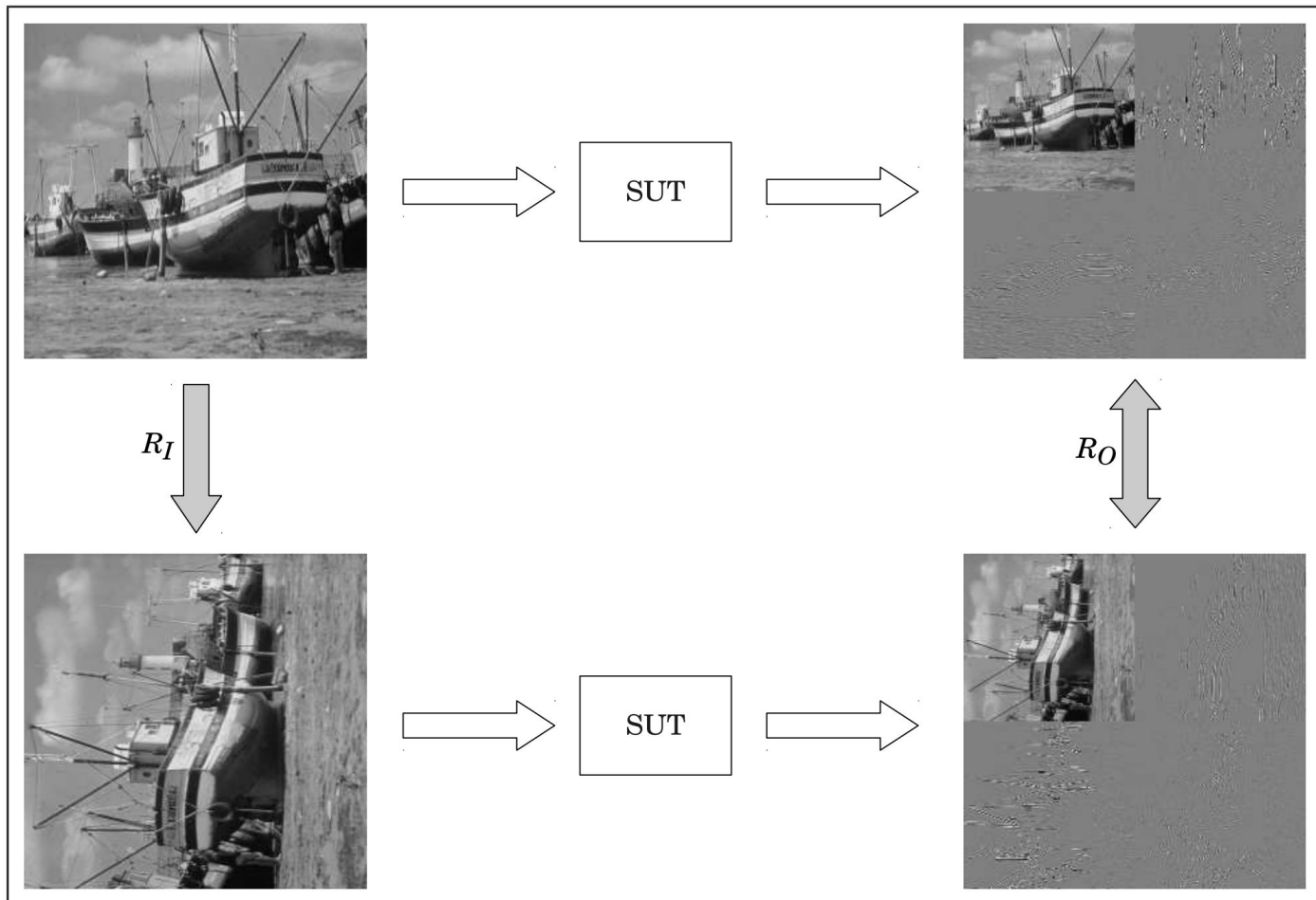
# Metamorphic testing: Input generation



# Metamorphic testing: relations $R_i$ and $R_o$

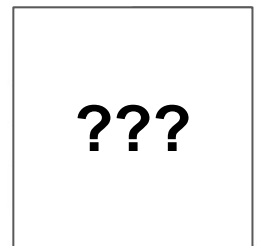
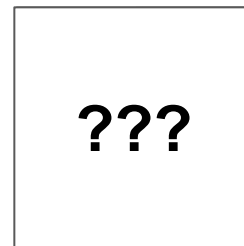
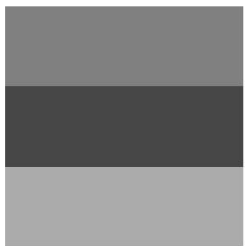


# Metamorphic testing: relations $R_i$ and $R_o$



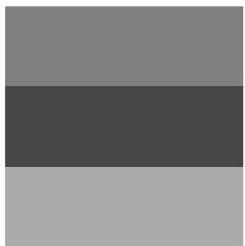
# Metamorphic testing: Relations

1.  $R_i$ : Add a constant offset to all color values  
 $R_o$ : ???

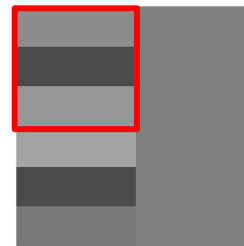


# Metamorphic testing: Relations

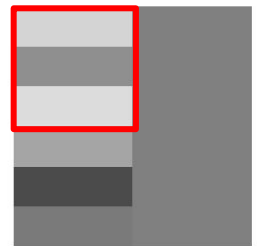
1.  $R_i$ : Add a constant offset to all color values  
 $R_o$ : Only the DC component must change



$\Rightarrow R_I$

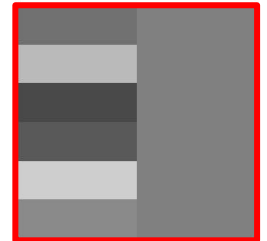
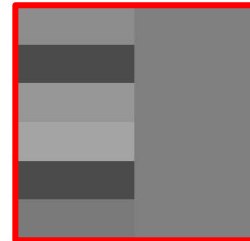


$\Leftrightarrow R_O$



# Metamorphic testing: Relations

1.  $R_i$ : Add a constant offset to all color values  
 $R_o$ : Only the DC component must change
2.  $R_i$ : Invert the color values  
 $R_o$ : The color values of the output must be inverted



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3.  $R_i$ : Transpose the input image  
 $R_o$ : The output components must be transposed



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3.  $R_i$ : Transpose the input image  
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4.  $R_i$ : Enlarge the input image (“zero-padding”)  
 $R_o$ : The output components must be shifted





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Commutative

4.  $R_i$ : Enlarge the input image ("zero-padding")

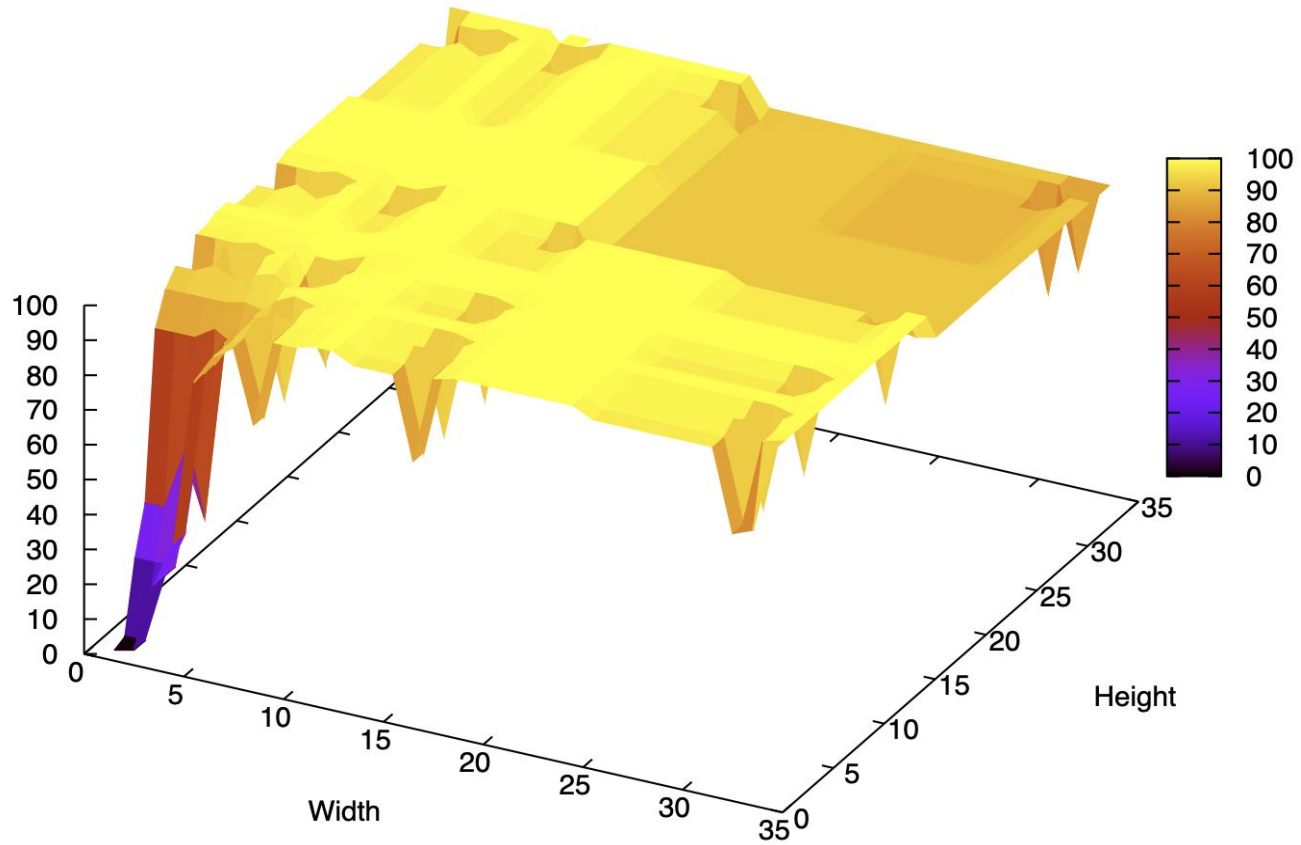
$R_o$ : The output components must be shifted

Time-invariant

**It turns out that MR compositions are effective**

# Metamorphic testing: effectiveness

Quadratic Mutation Score (Wavelet Transformation)



# Putting it all together

1. (Random) input generation
2. Metamorphic testing: follow-up inputs and partial oracles
3. Delta debugging: Minimize bug-exposing inputs
4. Mutation analysis: assess the effectiveness of relations

## Examples:

- GraphicsFuzz
- Testing ML-based systems