

# KO codes

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# Outline

- Motivation

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- Motivation
- Learning codes

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- Learning codes
- **KO codes:** novel neural codes
  - KO codes, *ICML 2021*

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- Motivation
- Learning codes
- KO codes: novel neural codes
  - KO codes, *ICML 2021*
- Future directions

# Age of Information



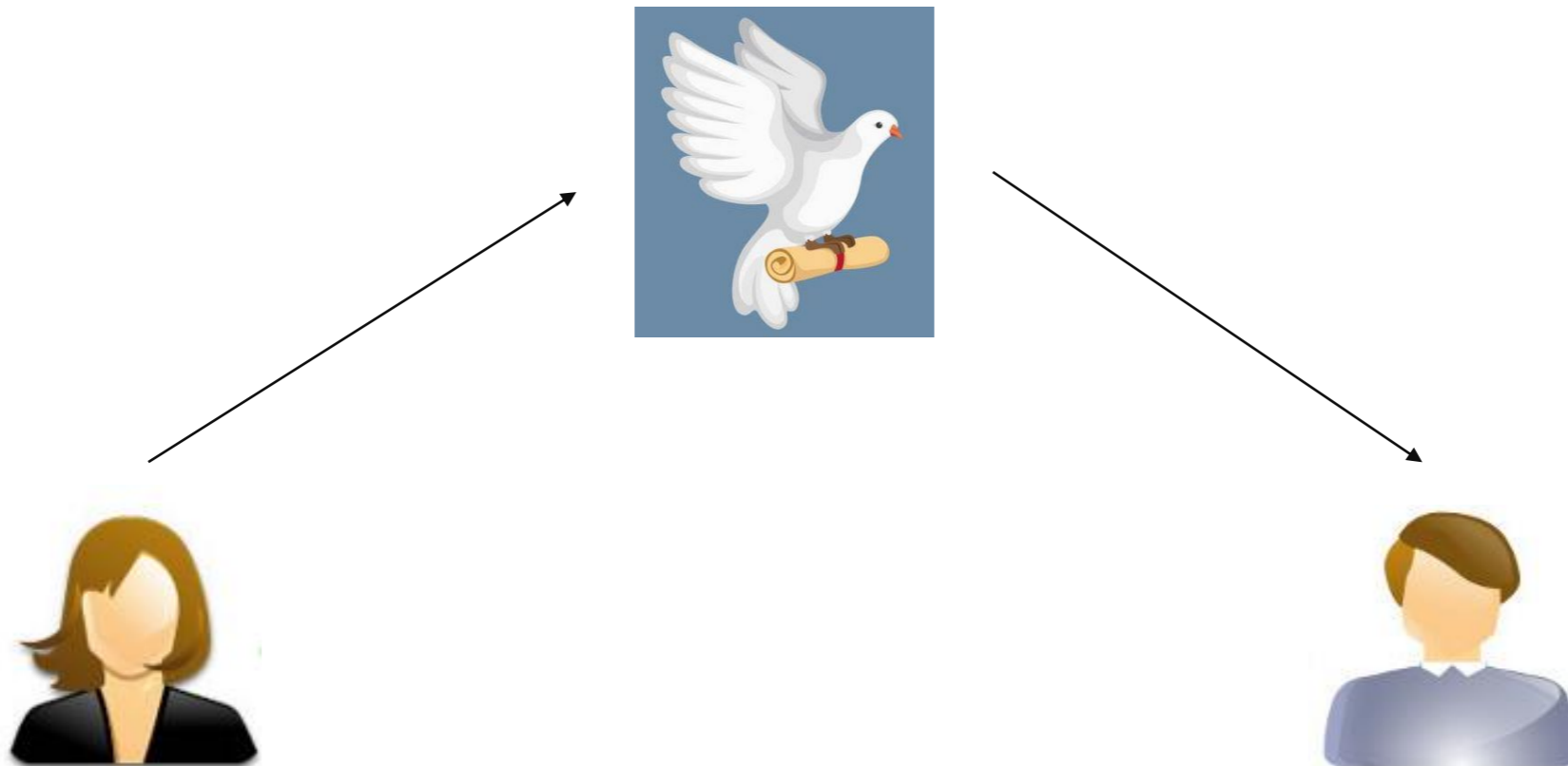
**Once upon a time...**







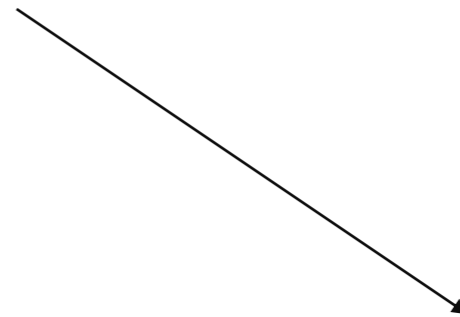
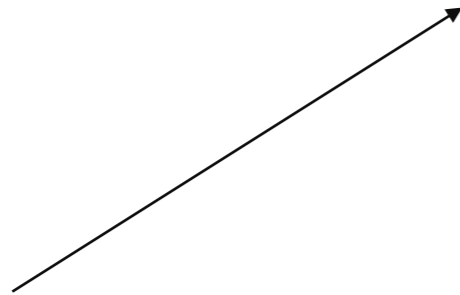
**3000 BC**



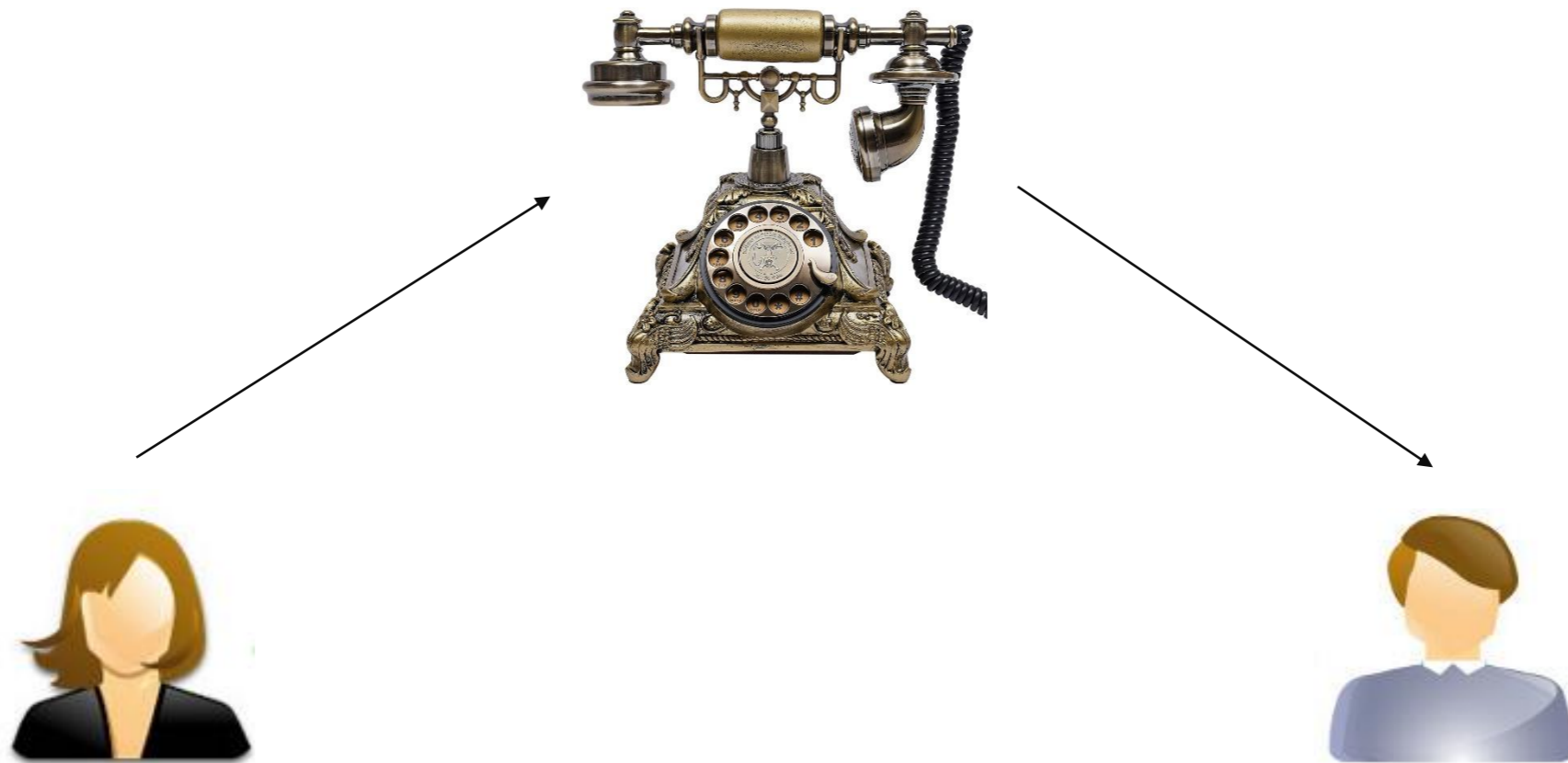
**500 BC**



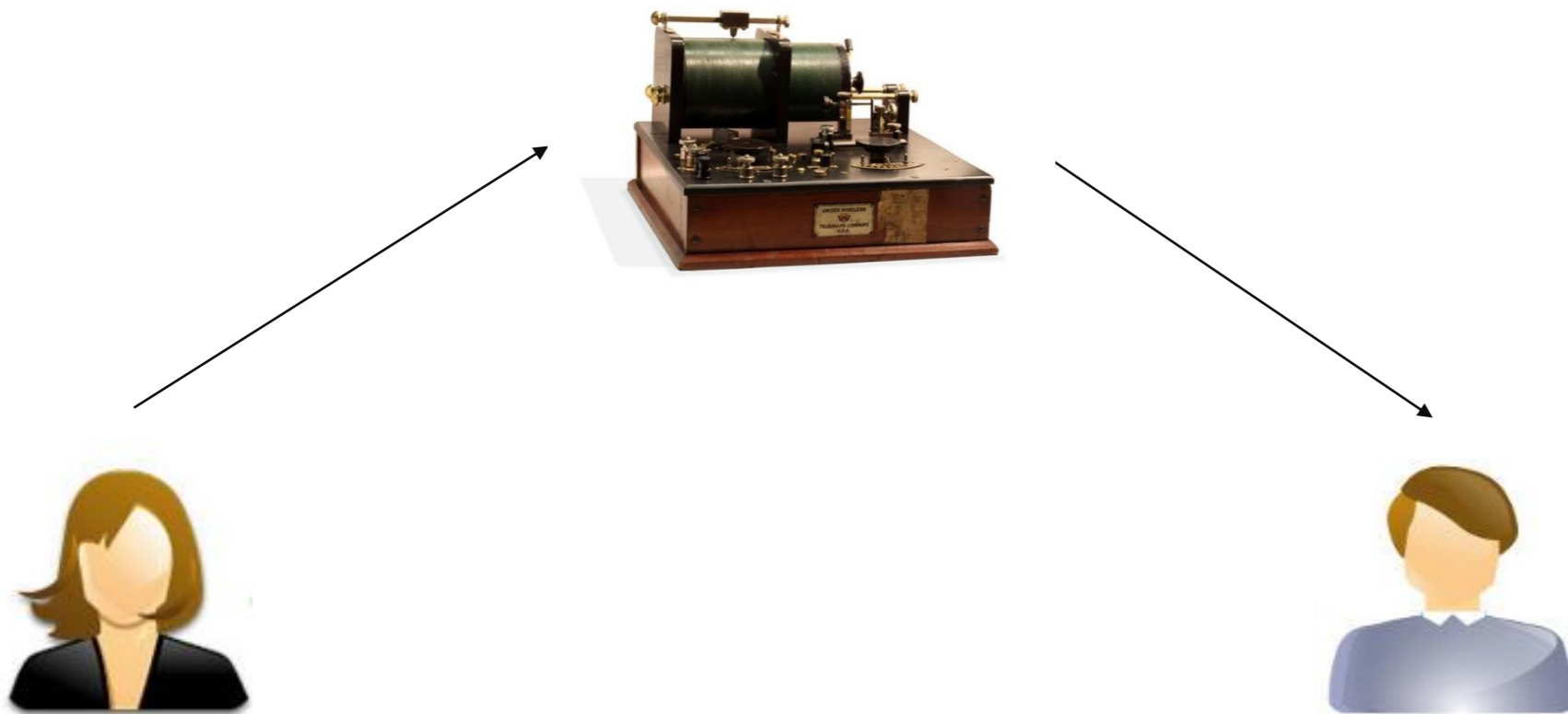
# Early 19<sup>th</sup> century



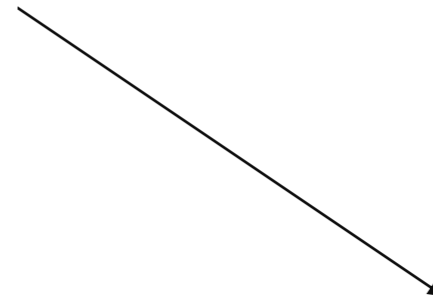
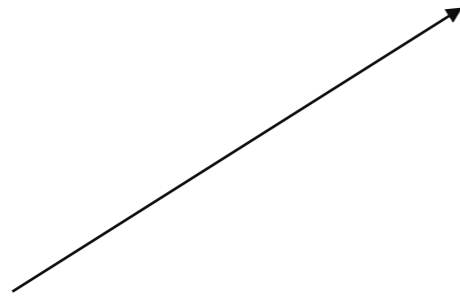
# Late 19<sup>th</sup> century



# Early 20<sup>th</sup> century



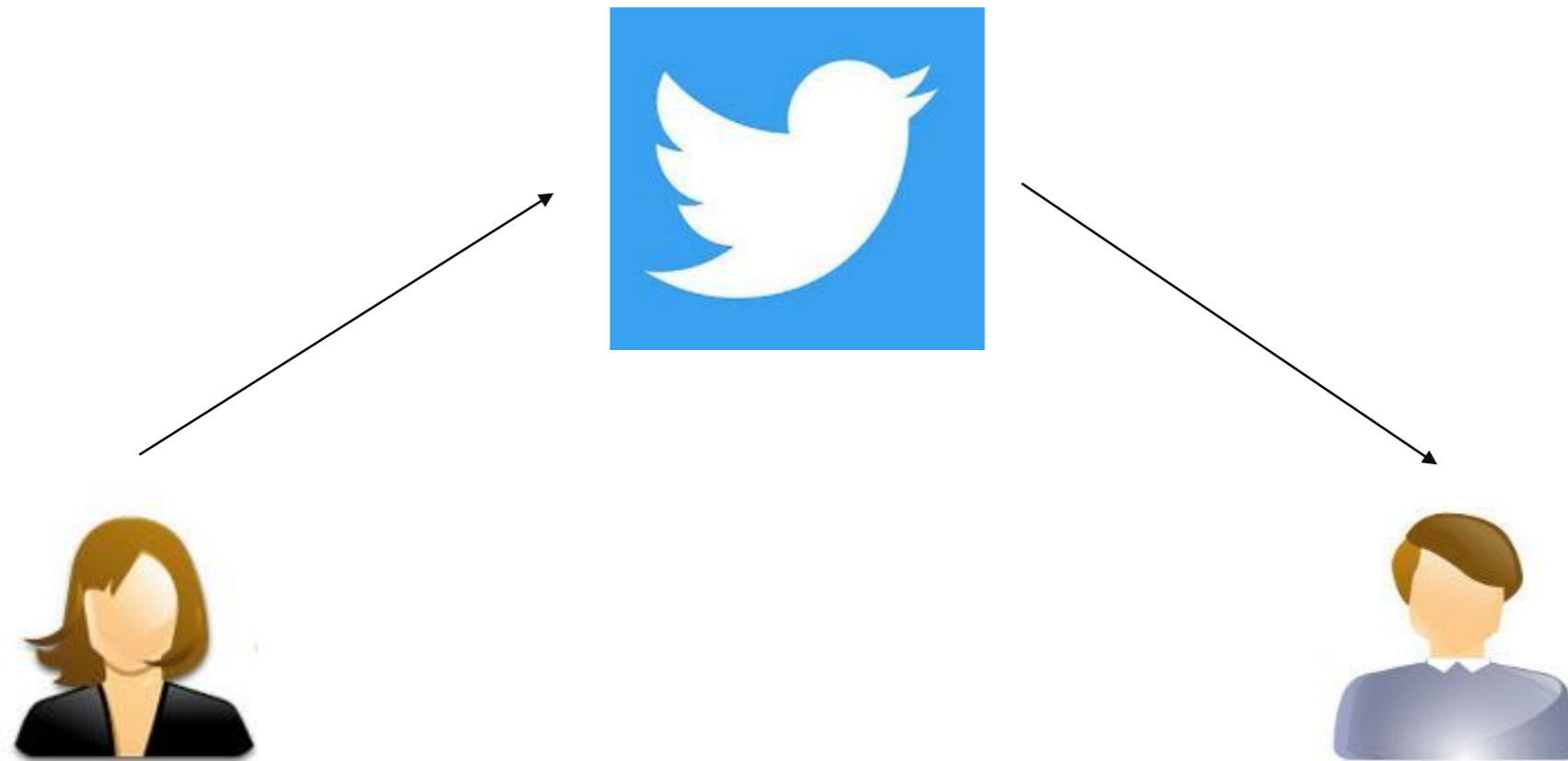
1974



# 21<sup>st</sup> century

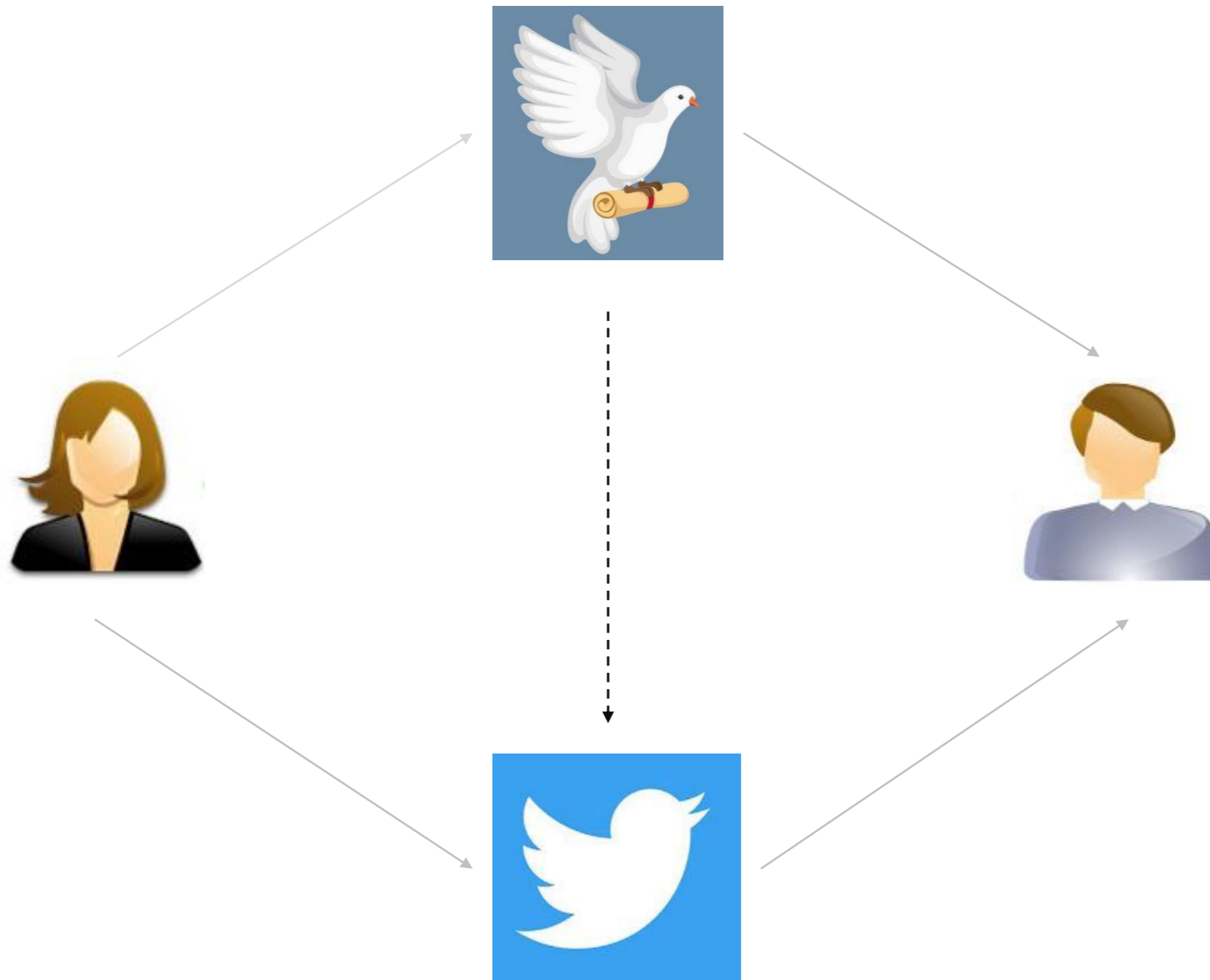


# 21<sup>st</sup> century



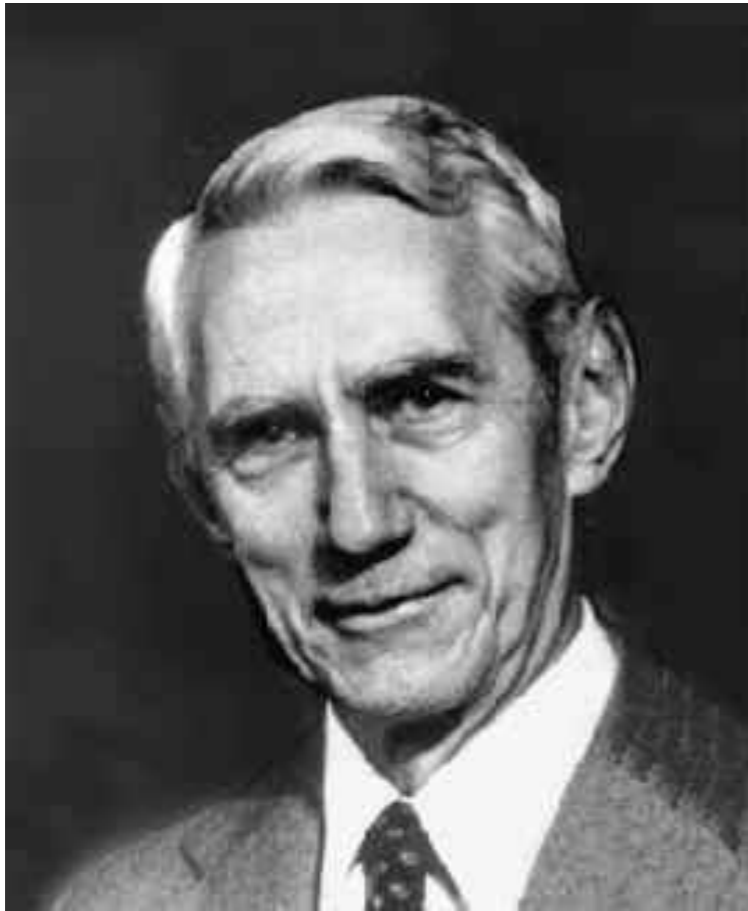


# A communication odyssey

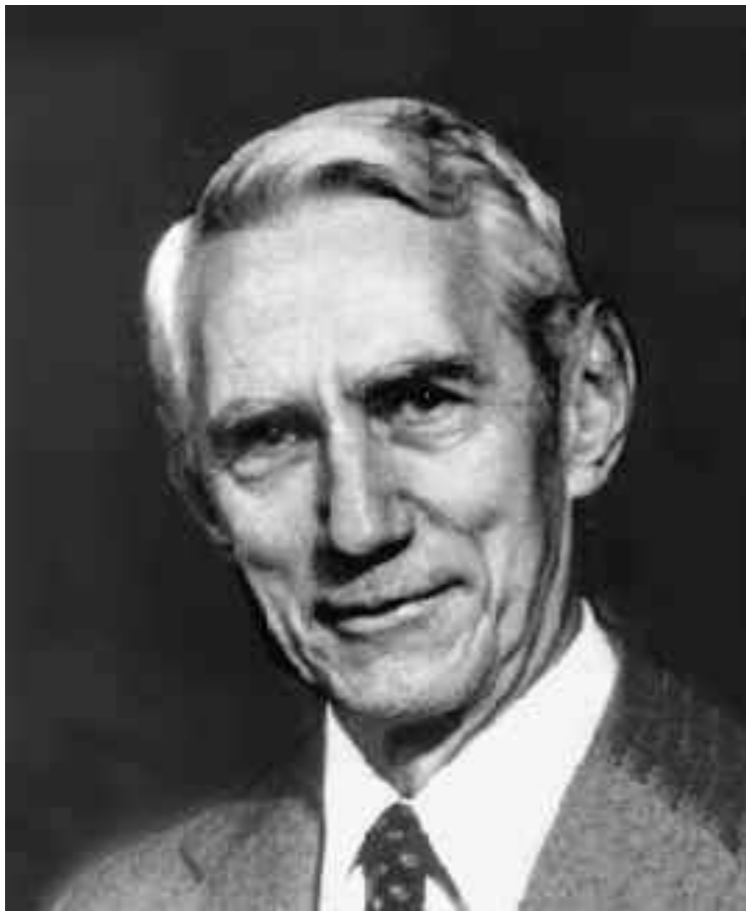


**1948: Then there was light**

# 1948: Then there was light



# 1948: Then there was light



## The Bell System Technical Journal

Vol. XXVII

July, 1948

No. 3

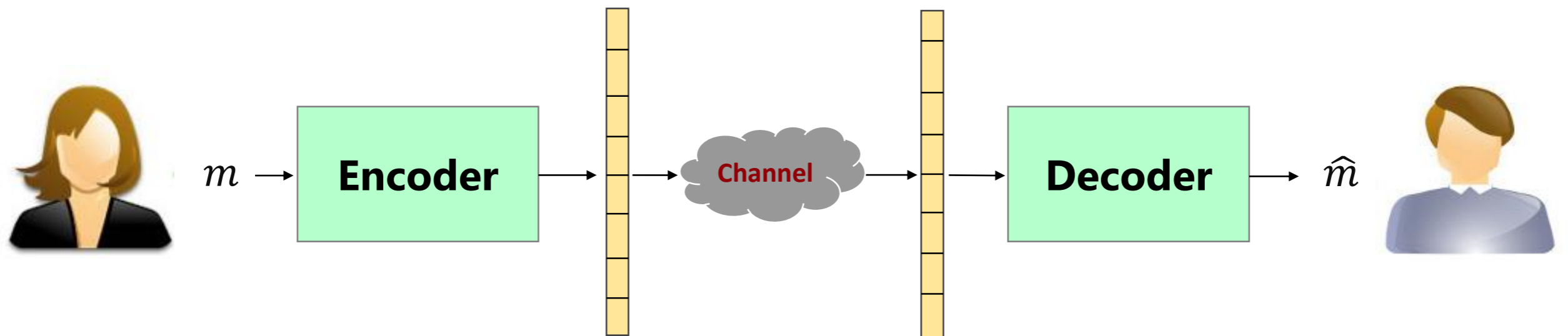
### A Mathematical Theory of Communication

By C. E. SHANNON

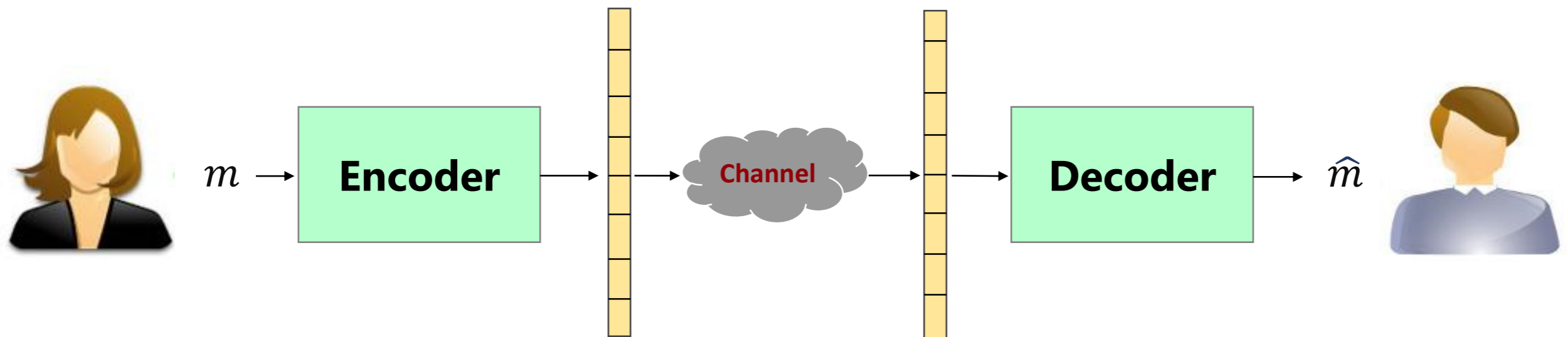
#### INTRODUCTION

THE recent development of various methods of modulation such as PCM and PPM which exchange bandwidth for signal-to-noise ratio has intensified the interest in a general theory of communication. A basis for such a theory is contained in the important papers of Nyquist<sup>1</sup> and Hartley<sup>2</sup> on this subject. In the present paper we will extend the theory to include a number of new factors, in particular the effect of noise in the channel, and the savings possible due to the statistical structure of the original message and due to the nature of the final destination of the information.

# Codes: a mathematical lens



# Codes: a mathematical lens



Code = (Encoder, Decoder)

# Communication codes

- **AWGN channel**
  - Precise performance metrics

# Communication codes

- **AWGN channel**
  - Precise performance metrics
- **Challenge:** Space of (encoders, decoders) very large
  - Rate =  $\frac{1}{2}$ ,  $k = 100$ :  $2^{100}$  codewords in **200** dimensional space



# Communication codes

- **AWGN channel**
  - Precise performance metrics
- **Challenge:** Space of (encoders, decoders) very large
  - Rate =  $\frac{1}{2}$ ,  $k = 100$ :  $2^{100}$  codewords in 200 dimensional space
- Information theory, Coding theory, Comm. theory

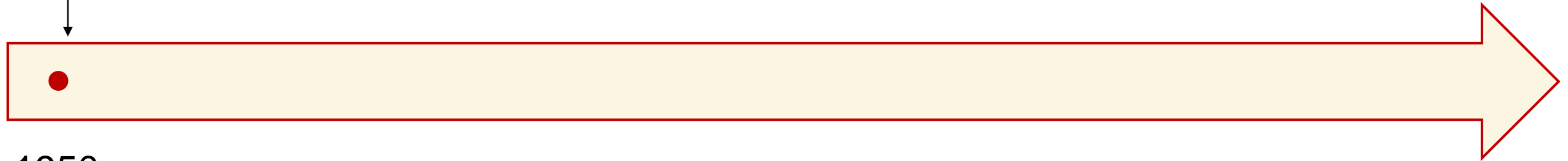
**Landmark codes: AWGN**

# Landmark codes: AWGN

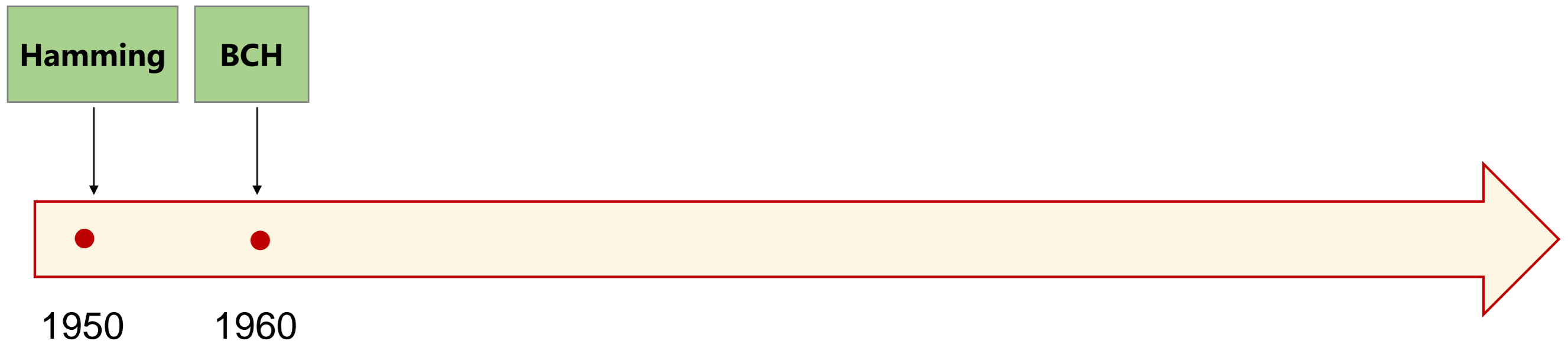
Hamming



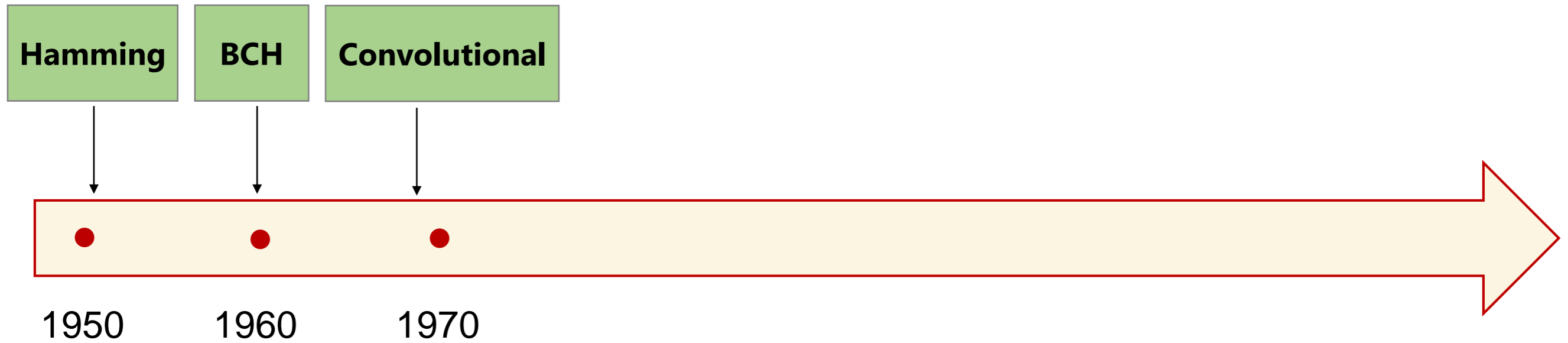
1950



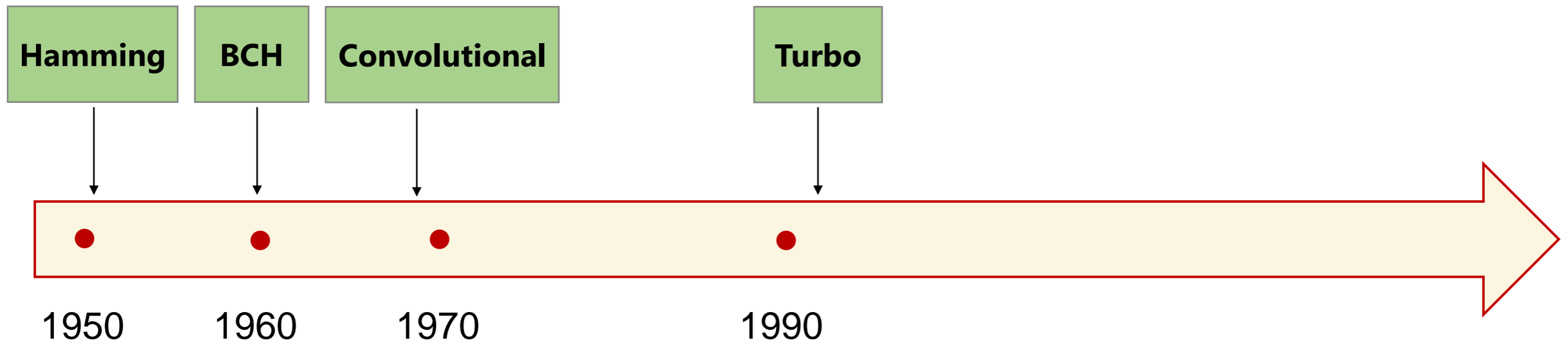
# Landmark codes: AWGN



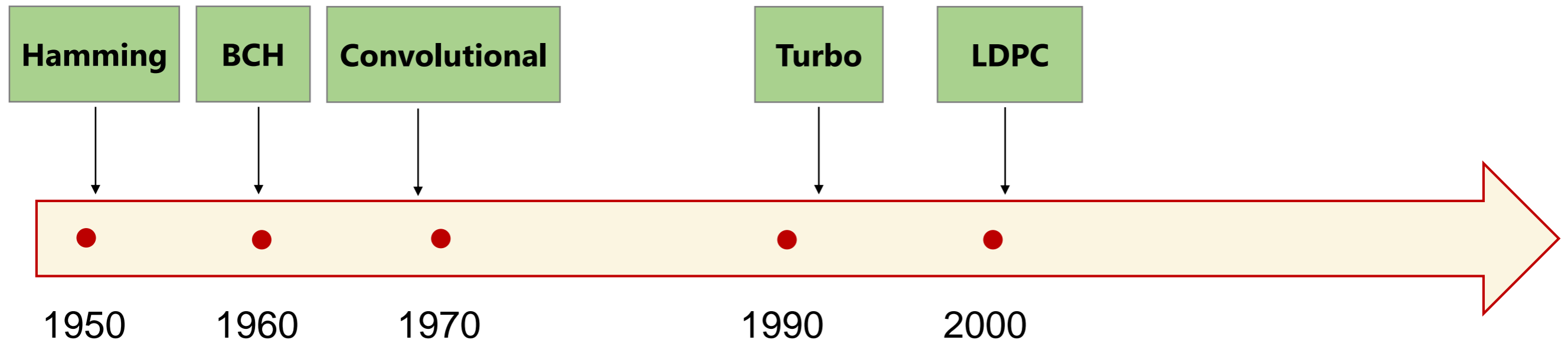
# Landmark codes: AWGN



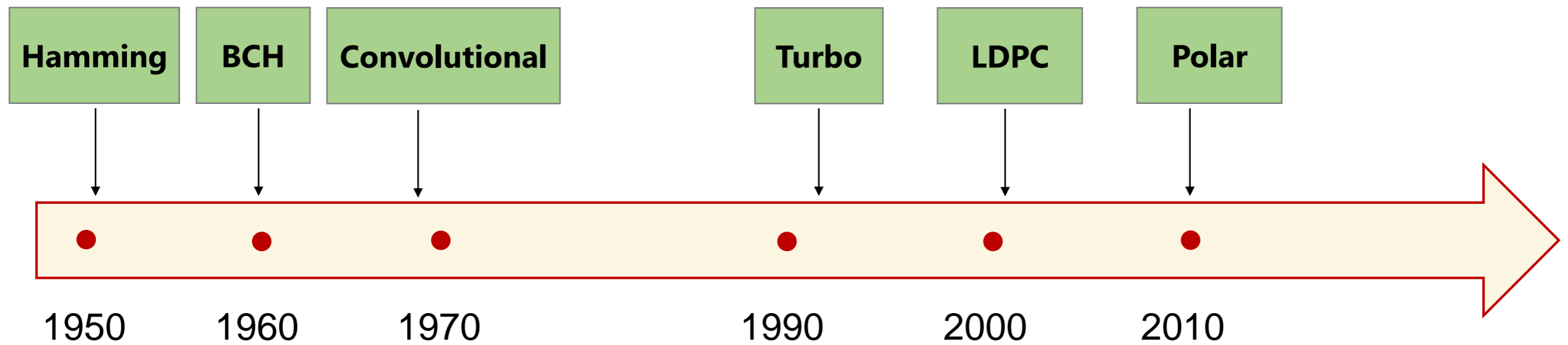
# Landmark codes: AWGN



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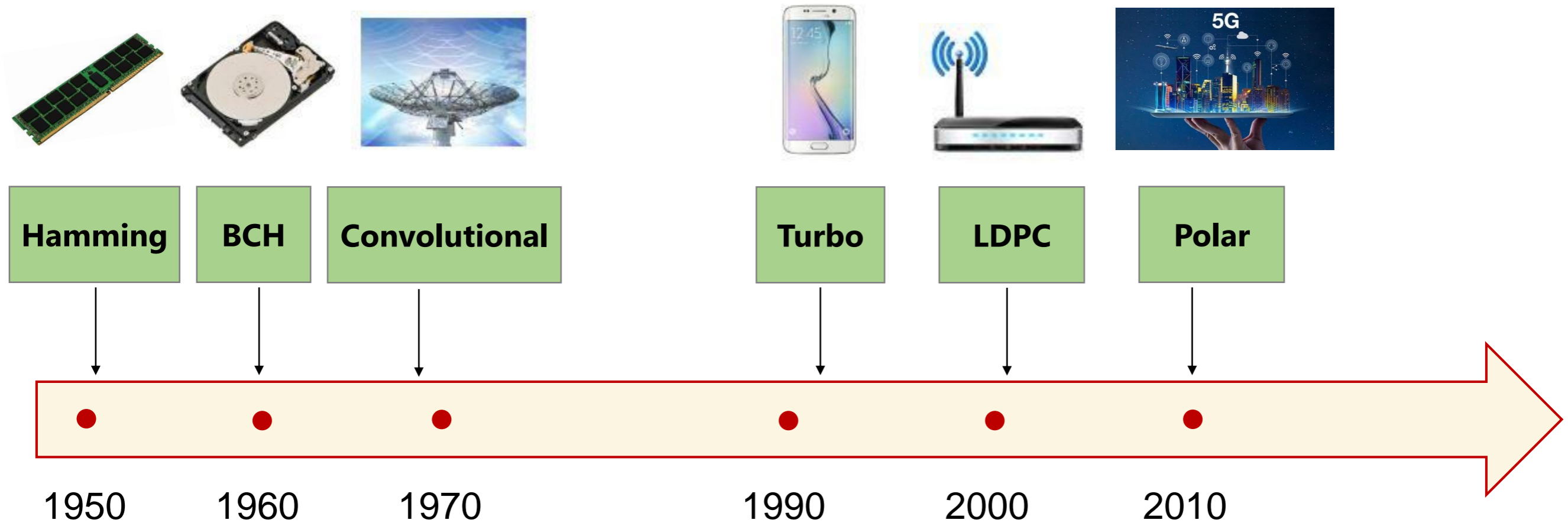


# Landmark codes: AWGN





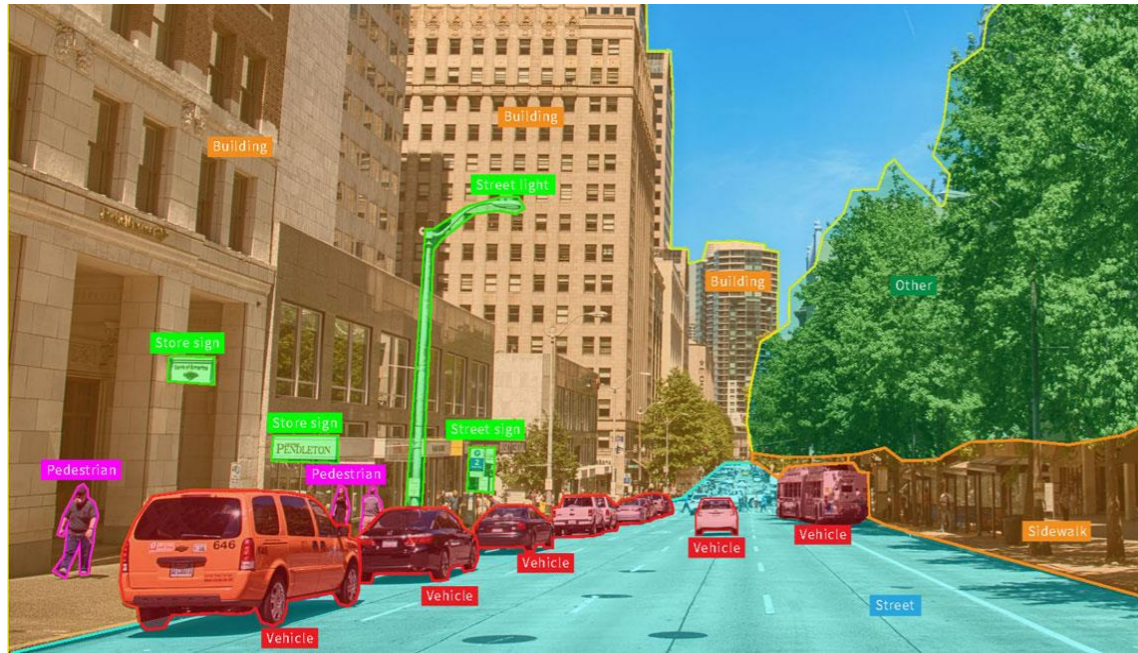
# Huge practical impact



# Discovery of codes

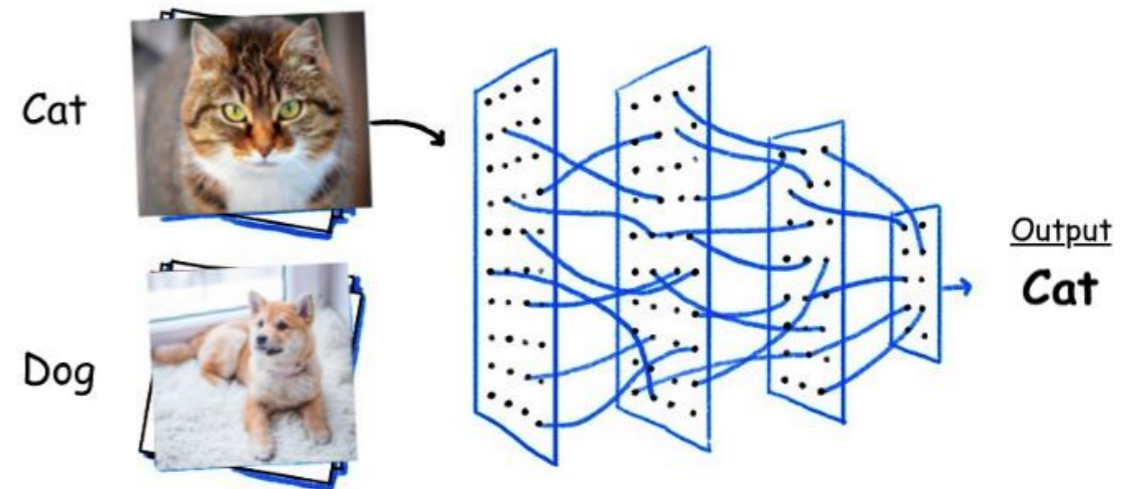
- Human eureka moments
  - Sporadic
- **Goal:** Automate the discovery

# Deep learning (DL)

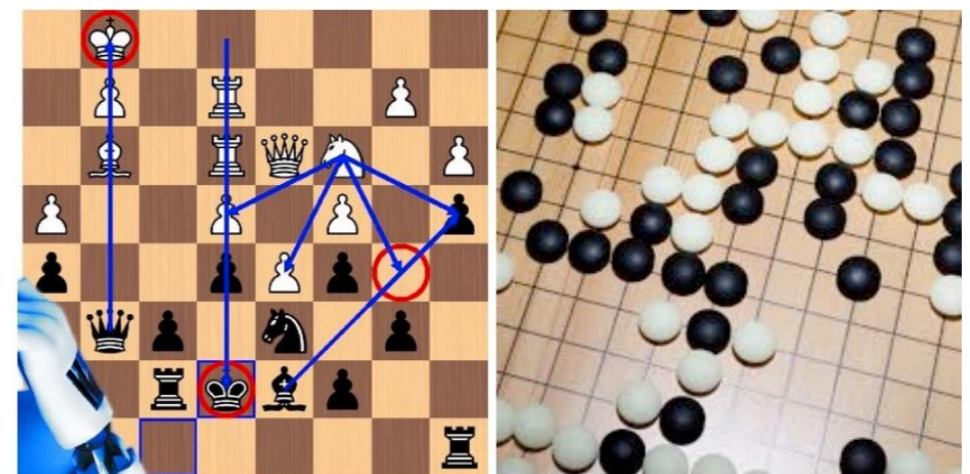


# DL success story

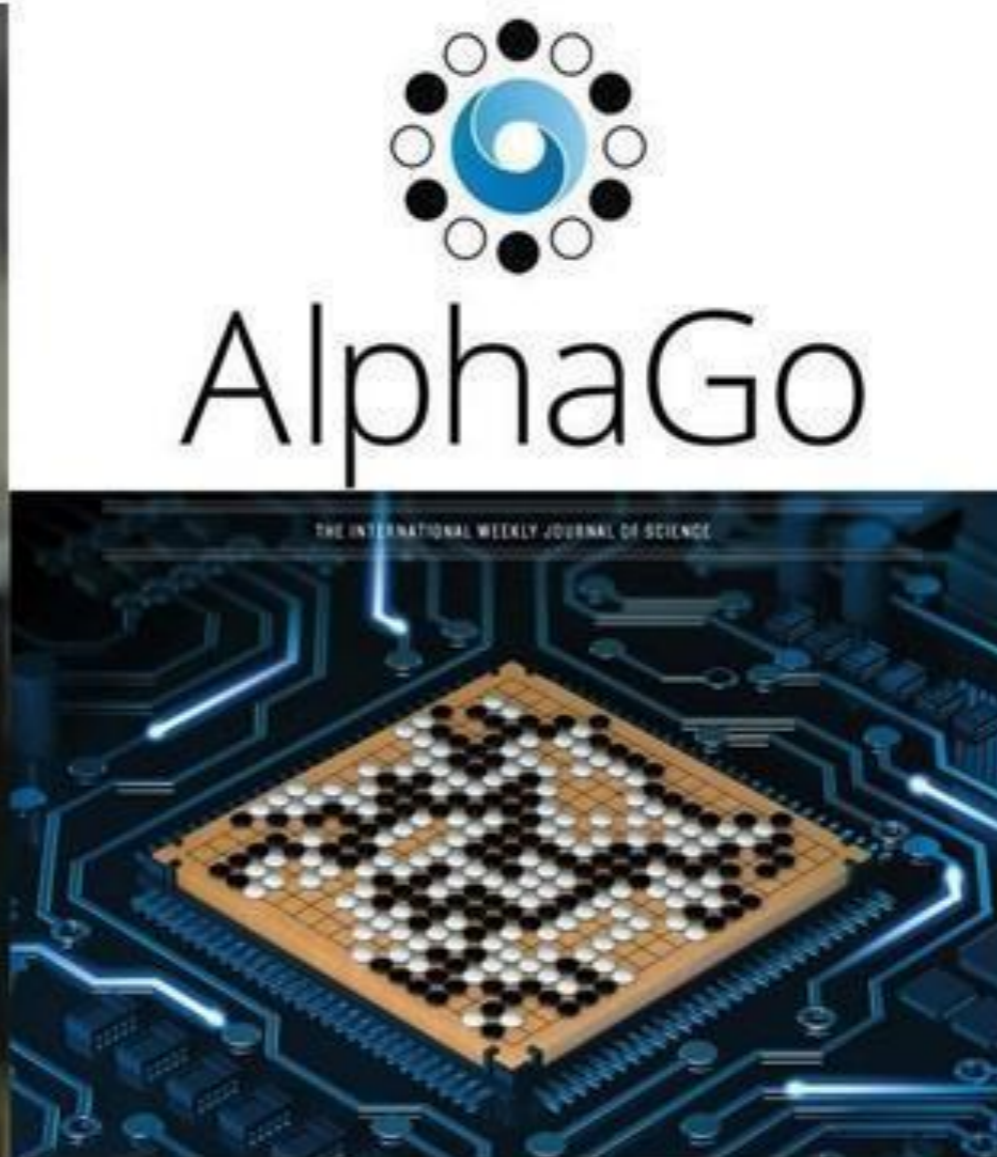
- Model deficiency
  - No analytical model
  - **AlexNet**



- Algorithm deficiency
  - Clear model
  - Space of algorithms large
  - **AlphaGo**



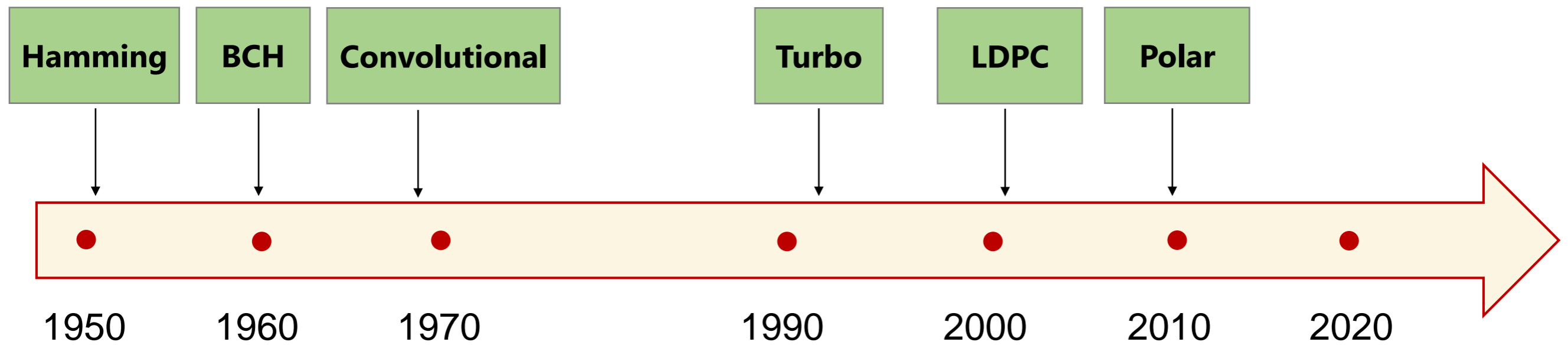
# Breakthroughs of DL



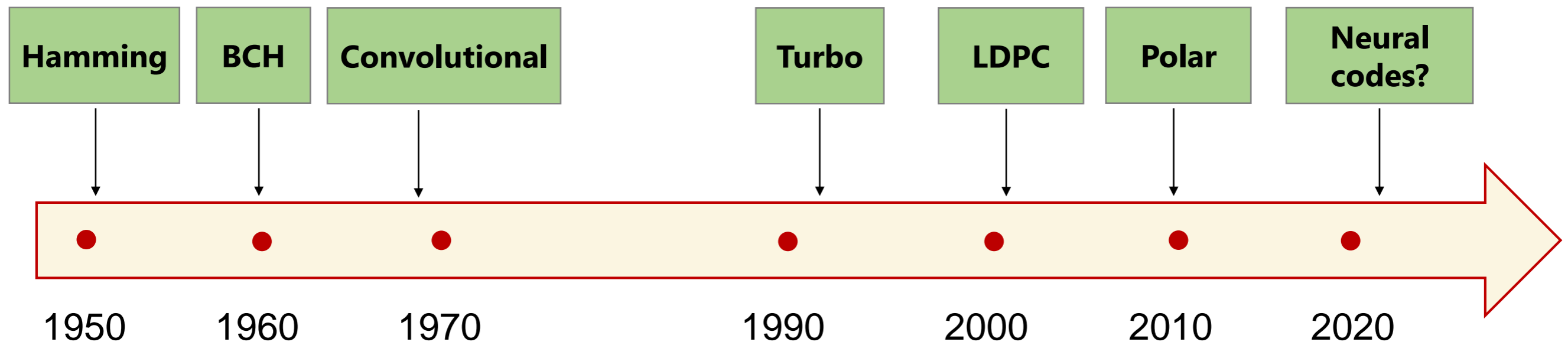
# Main goal

**Can we automate the search for codes via DL?**

# Main goal



# Main goal

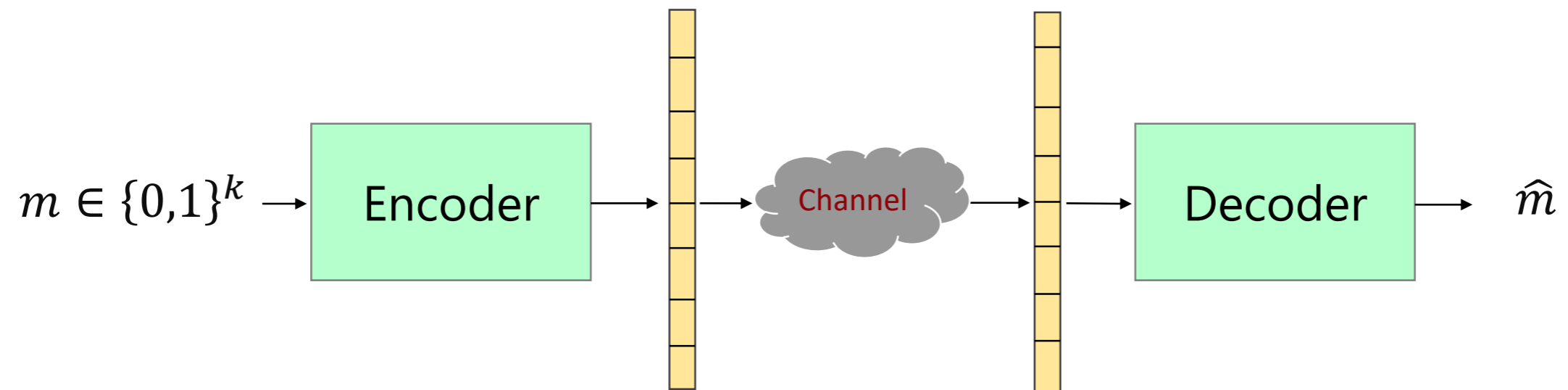




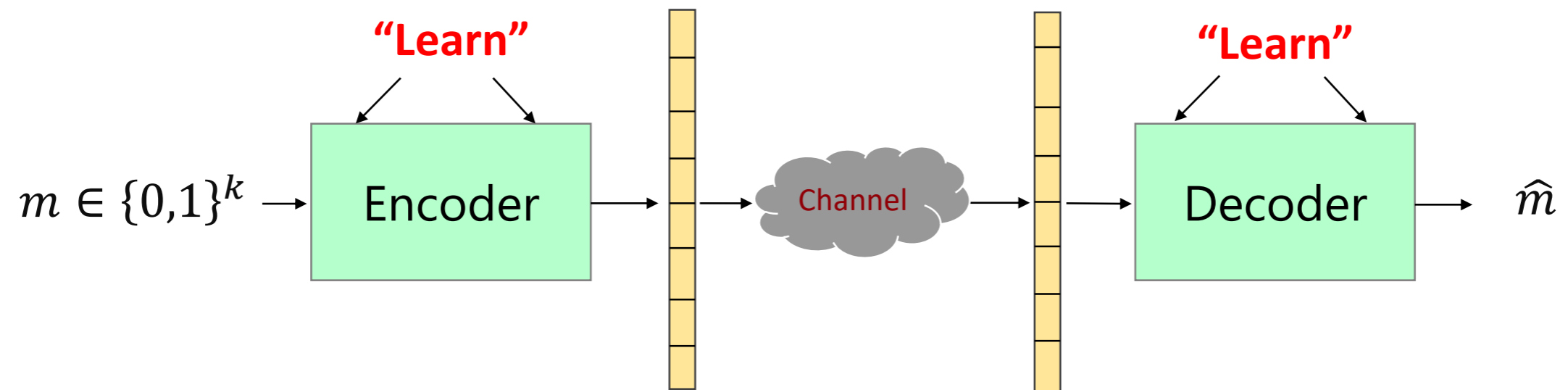
# Agenda

- New (deep learning) tools for classical problems
  - New state-of-the-art codes
  - Inherent practical value
  
- Insight into deep learning methods
  - Communication framework as a lens

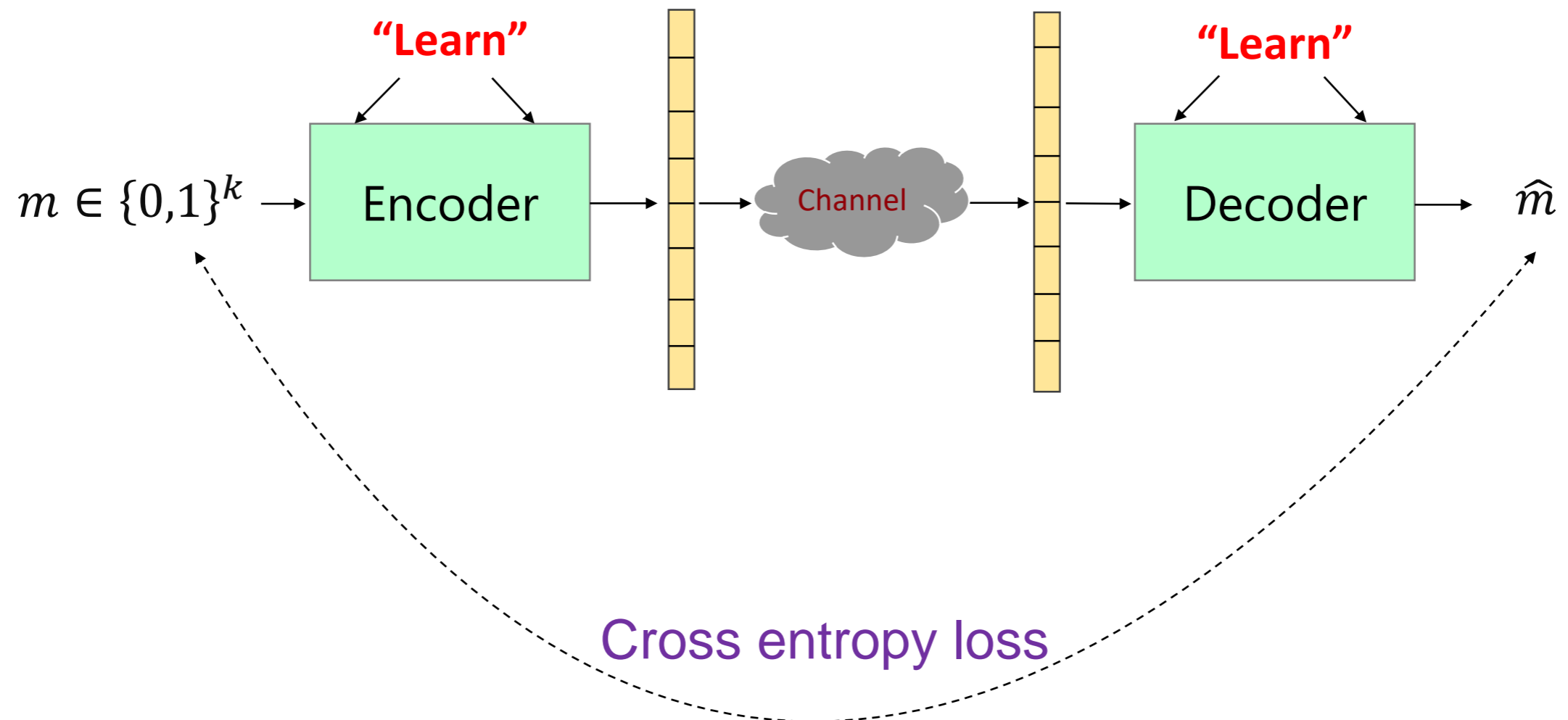
# Learning a new code



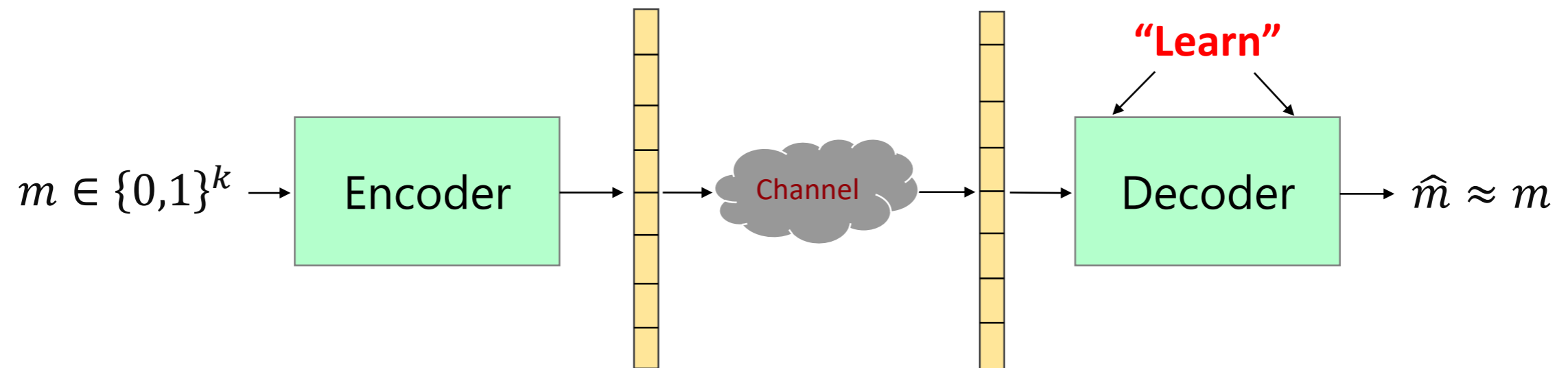
# Learning a new code



# Learning a new code



# Learning to decode



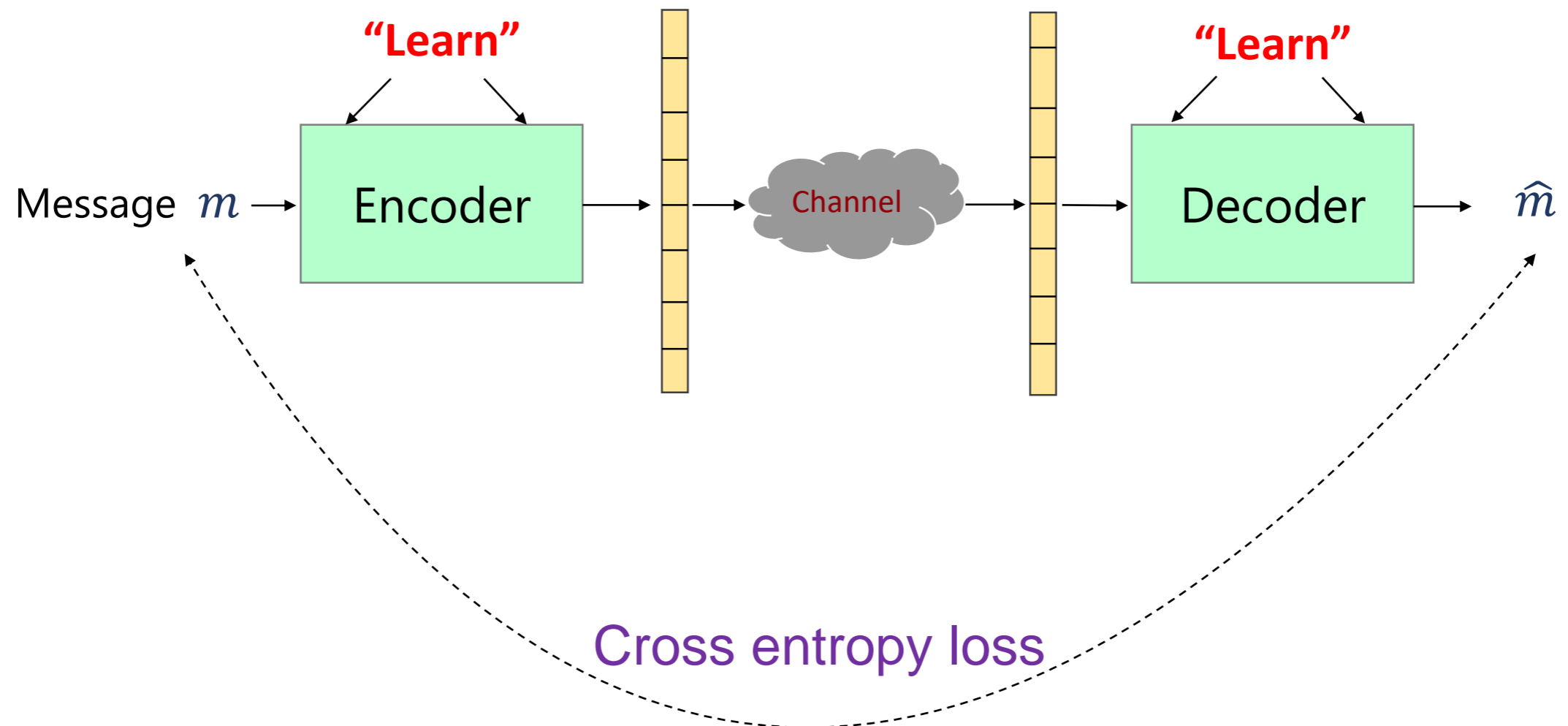
# Vast literature

- **Supervised learning**
  - Nachmani et al., 2016
  - Gruber et al. 2017
  - Cammerer et al., 2017
  - Nachmani et al., 2018
  - Kim et al., 2018a;b
  - Vasic et al., 2018
  - Teng et al., 2019
  - Nachmani & Wolf, 2019
  - Buchberger et al., 2020
  - Chen & Ye, 2021
  
- **Reinforcement learning**
  - Carpi et al., 2019
  - Habib et al., 2020
  - Doan et al., 2020

# Learning to decode: summary

- Fix the encoding
- DL decoders **learn** state-of-the-art decoders
  - Convolutional codes: Viterbi, BCJR, dynamic programming
  - Turbo codes: BCJR
  - RM & Polar codes: Successive Cancellation
- Clever architectural choices
  - Recurrent neural networks  $\longleftrightarrow$  dynamic programming

# Learning a new code





# Code structure

# Code structure

- Linear and binary: Classical codes

# Code structure

- Linear and binary: Classical codes
- Non-linear and real valued: **Neural networks (NNs)**
  - Fully connected NNs worse than repetition codes (Jiang et. al '19)
  - **Still need a structure**

# Imparting structure

- Capitalize on state-of-the-art codes

# Imparting structure

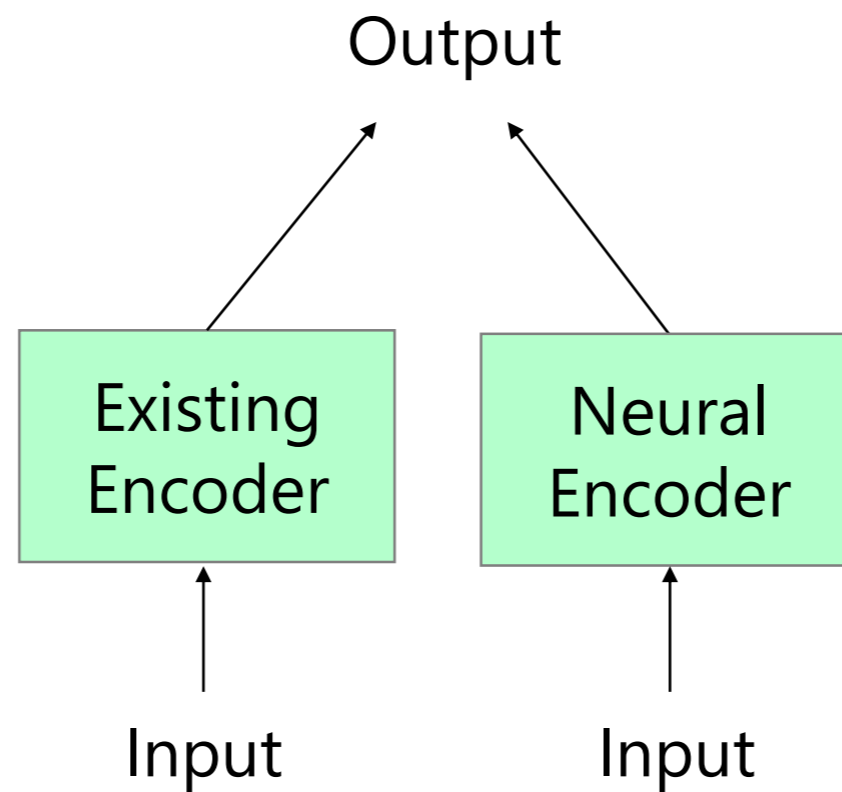
- Capitalize on state-of-the-art codes
- **How?**
- **What class of codes?**

# Imparting structure

- Capitalize on state-of-the-art codes
- **How?**

# Neural Augmentation

- (Welling '20)



# Imparting structure

- Capitalize on state-of-the-art codes
- How?
  - Neural Augmentation
- **What class of codes?**



# Taxonomy of codes

## **Sequential codes**

Convolutional and  
Turbo codes.

## **Graphical codes**

LDPC codes.

## **Algebraic codes**

Reed-Solomon, BCH,  
Reed-Muller and Polar codes.

# Taxonomy of codes

## Sequential codes

Convolutional and Turbo codes.

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# Algebraic codes

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# Reed-Muller and Polar codes

## Sequential codes

Convolutional and  
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# Reed-Muller (RM) codes

- **Classical**

- Muller, 1954
- Efficient decoder by Reed, 1954

- **Recent Interest**

- Polar codes
- Achieve capacity (very recent!)

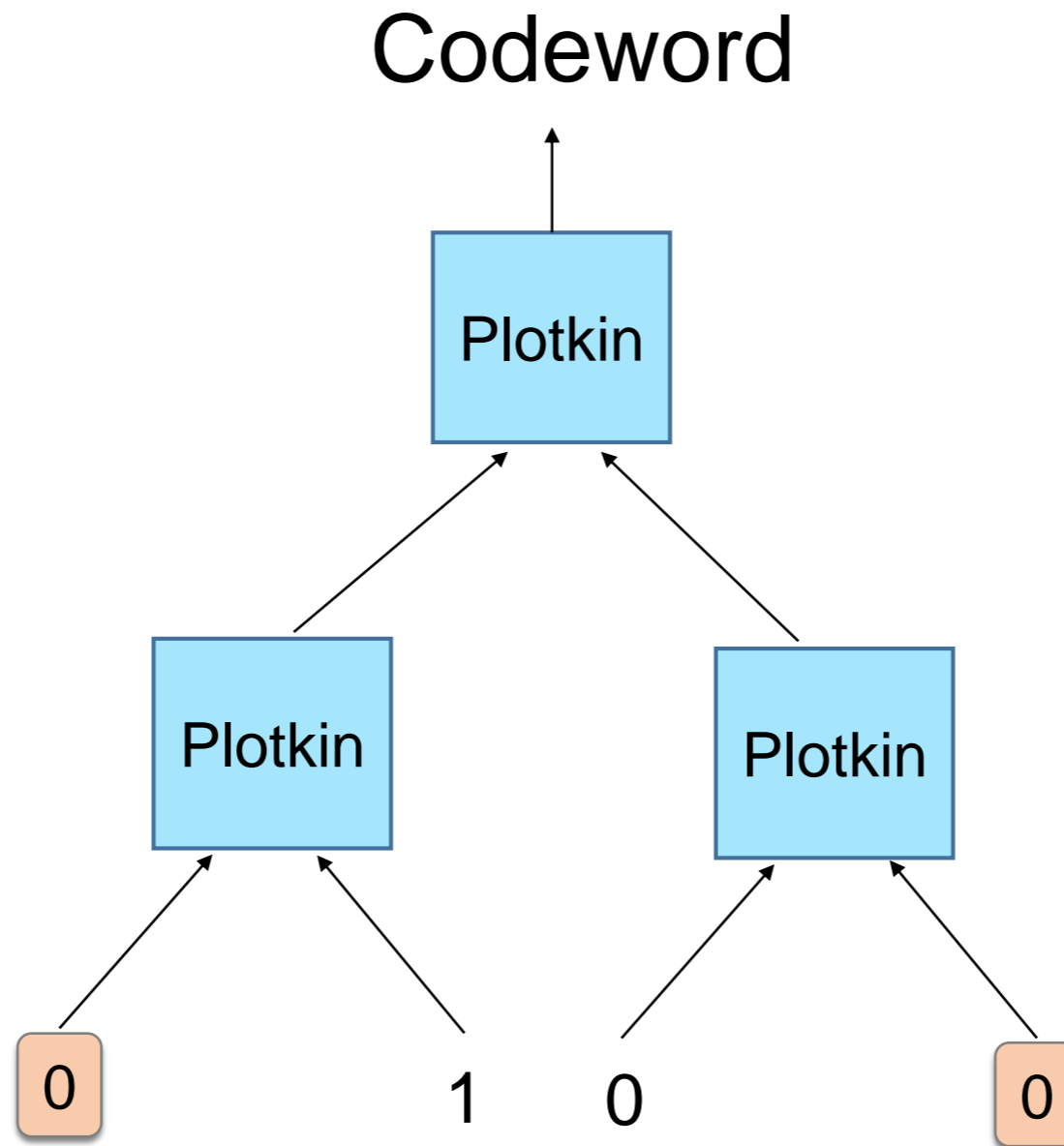
# Polar codes

- Arikan, 2009
- **First codes proven to achieve capacity**
- Recent interest: 5G

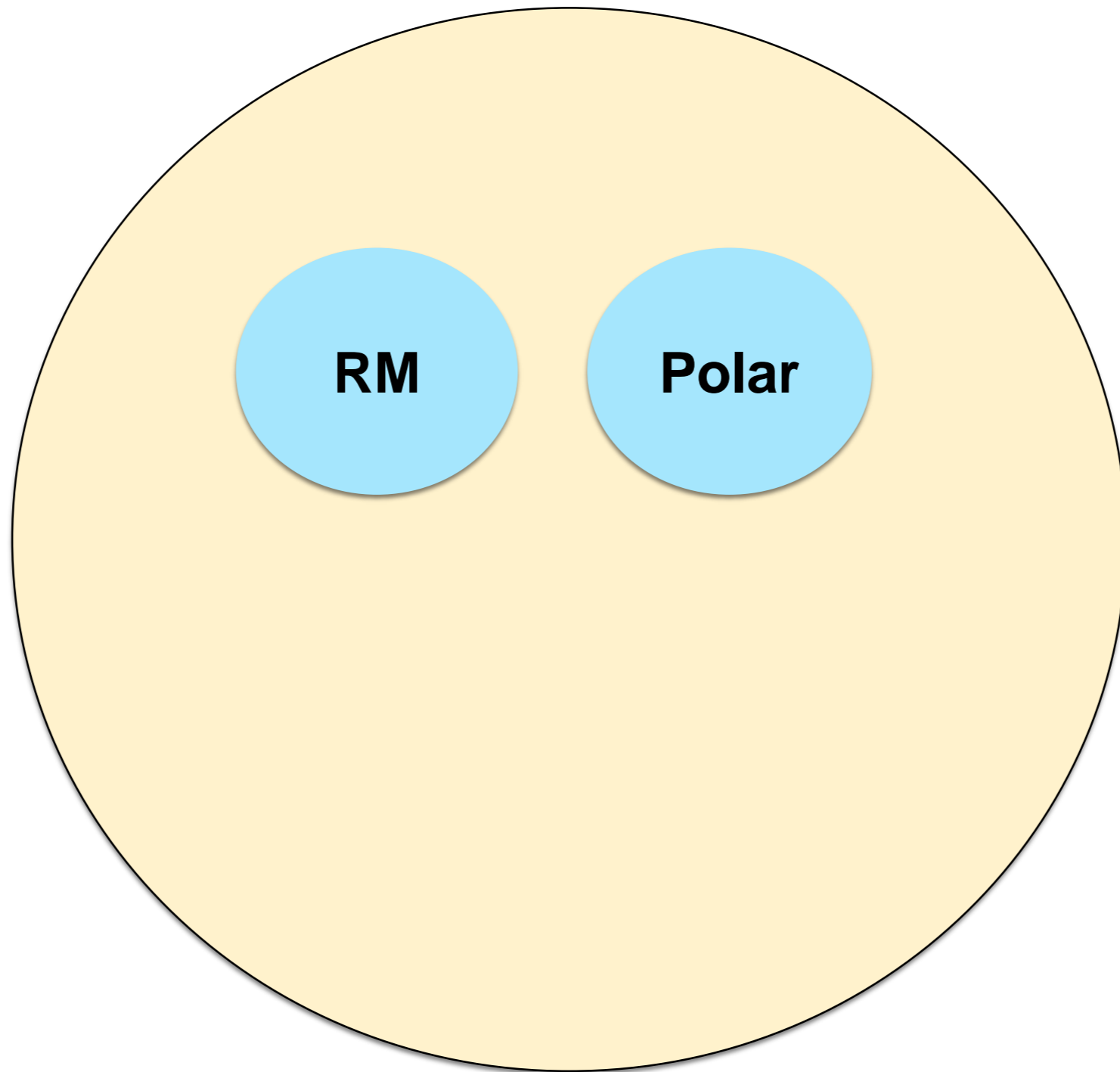
# RM and Polar

- Common structure
- **Kronecker Operation on the Plotkin transform**

# Structure: Kronecker Operation (KO)



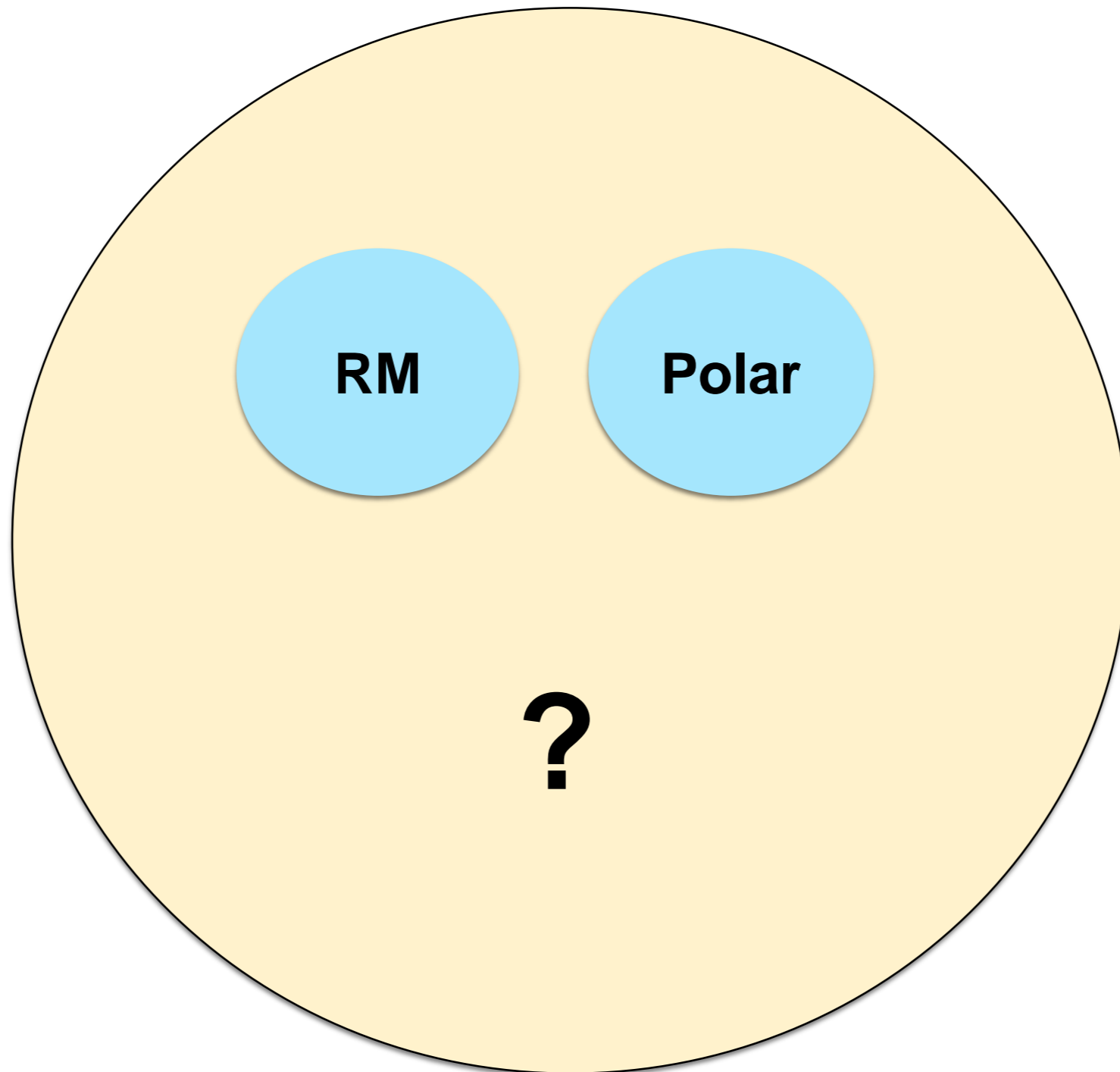




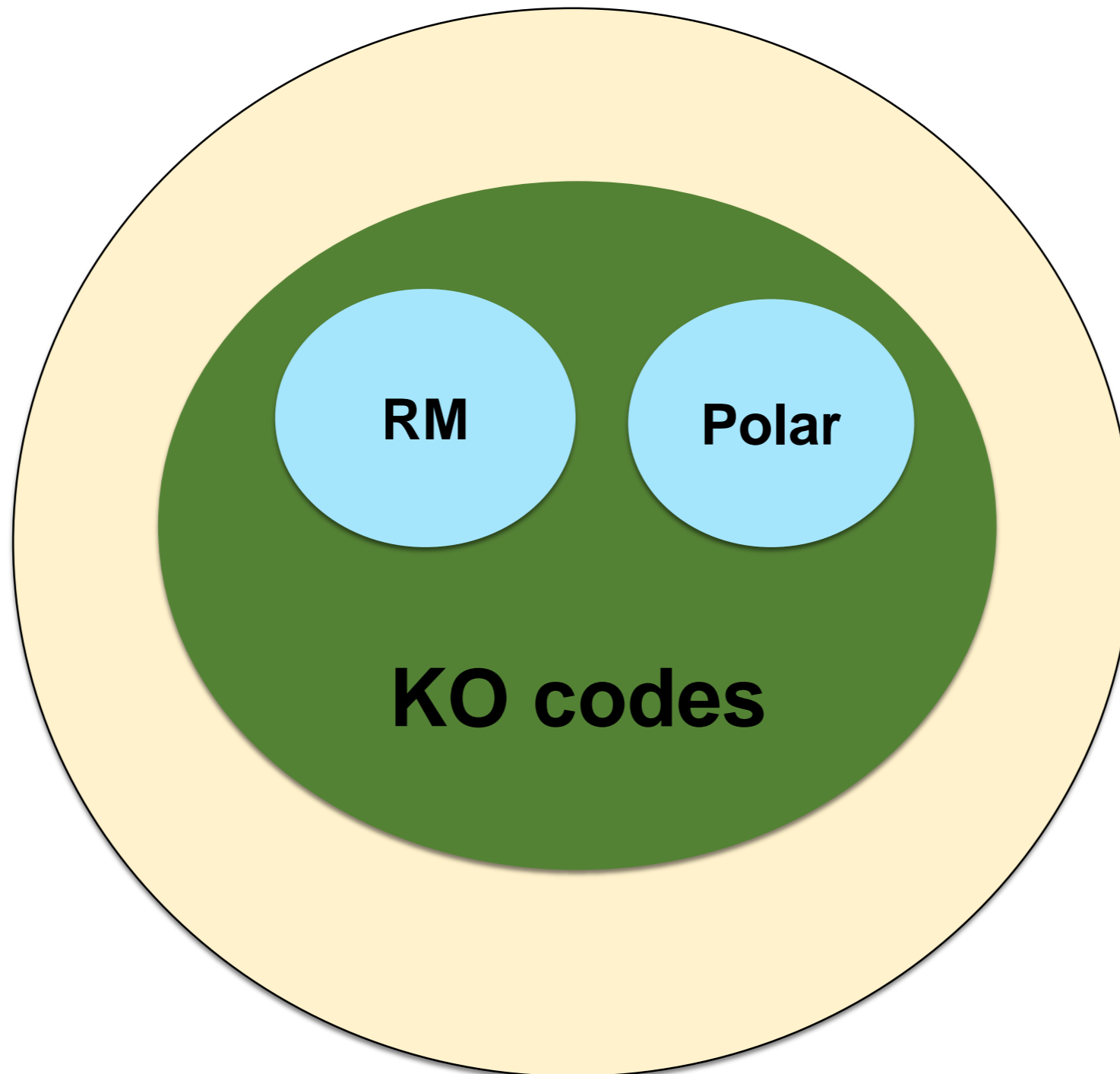
**RM**

**Polar**

# More codes?



# KO codes



# KO codes

**A. V. Makkuva\***, X. Liu\*, M. V. Jamali, H. Mahdavifar, S. Oh, and P. Viswanath, “KO codes: inventing nonlinear encoding and decoding for reliable wireless communication via deep-learning,” *in Proceedings of the 38th International Conference on Machine Learning (ICML), 2021.*

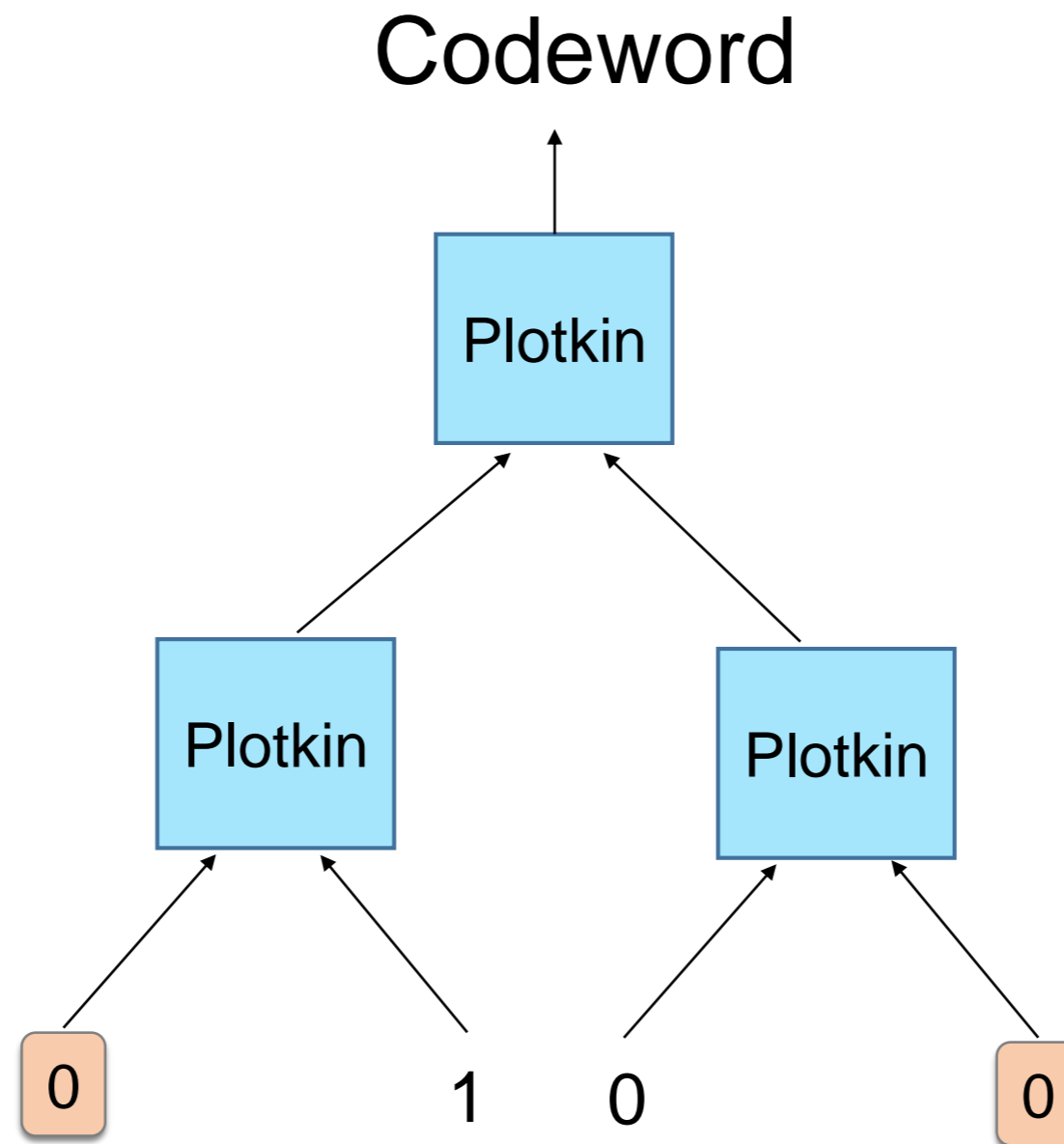
# KO codes: An overview

- Novel family of neural codes
- Outperform both RM and Polar in certain regimes
- Fascinating properties

# KO codes: An overview

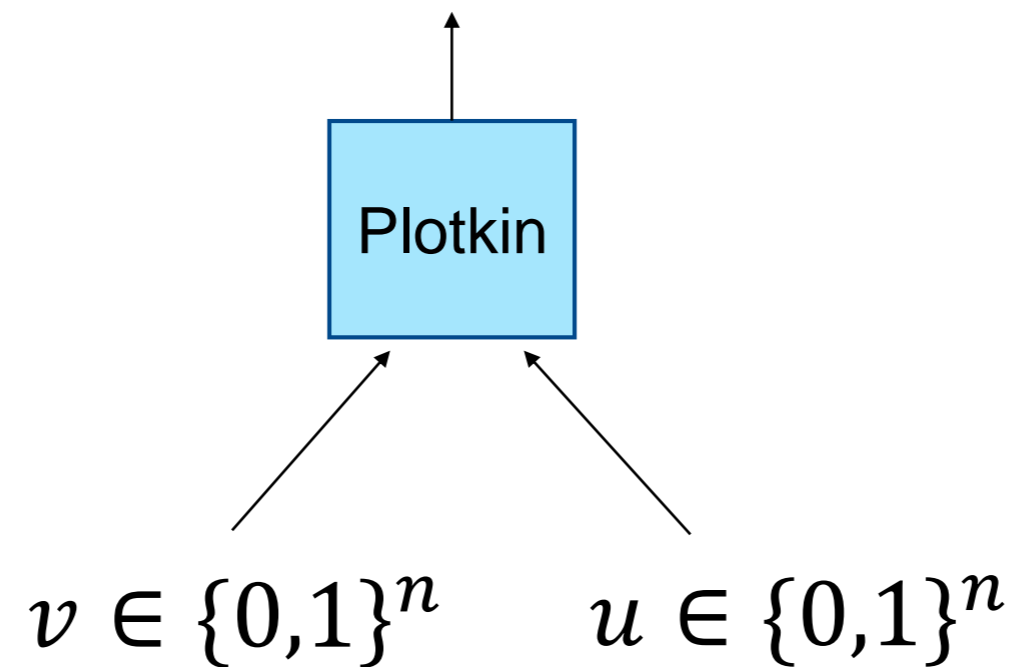
- **Novel family of neural codes**
- Outperform both RM and Polar in certain regimes
- Fascinating properties

# Encoding: RM and Polar



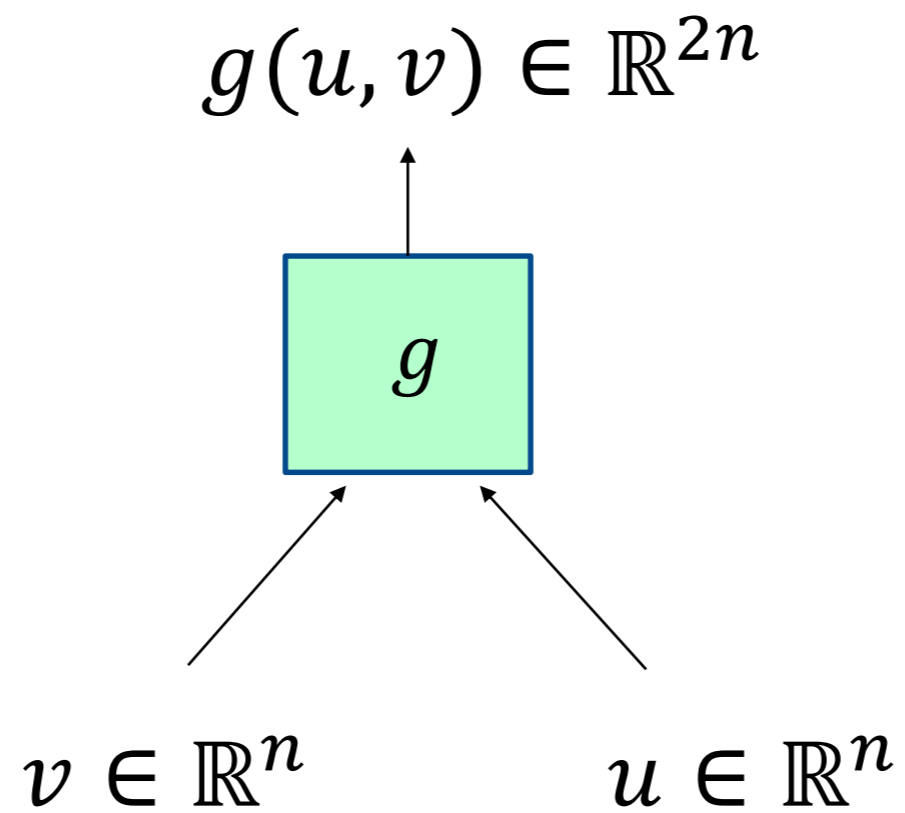
# Plotkin mapping

$$(u, u \oplus v) \in \{0,1\}^{2n}$$

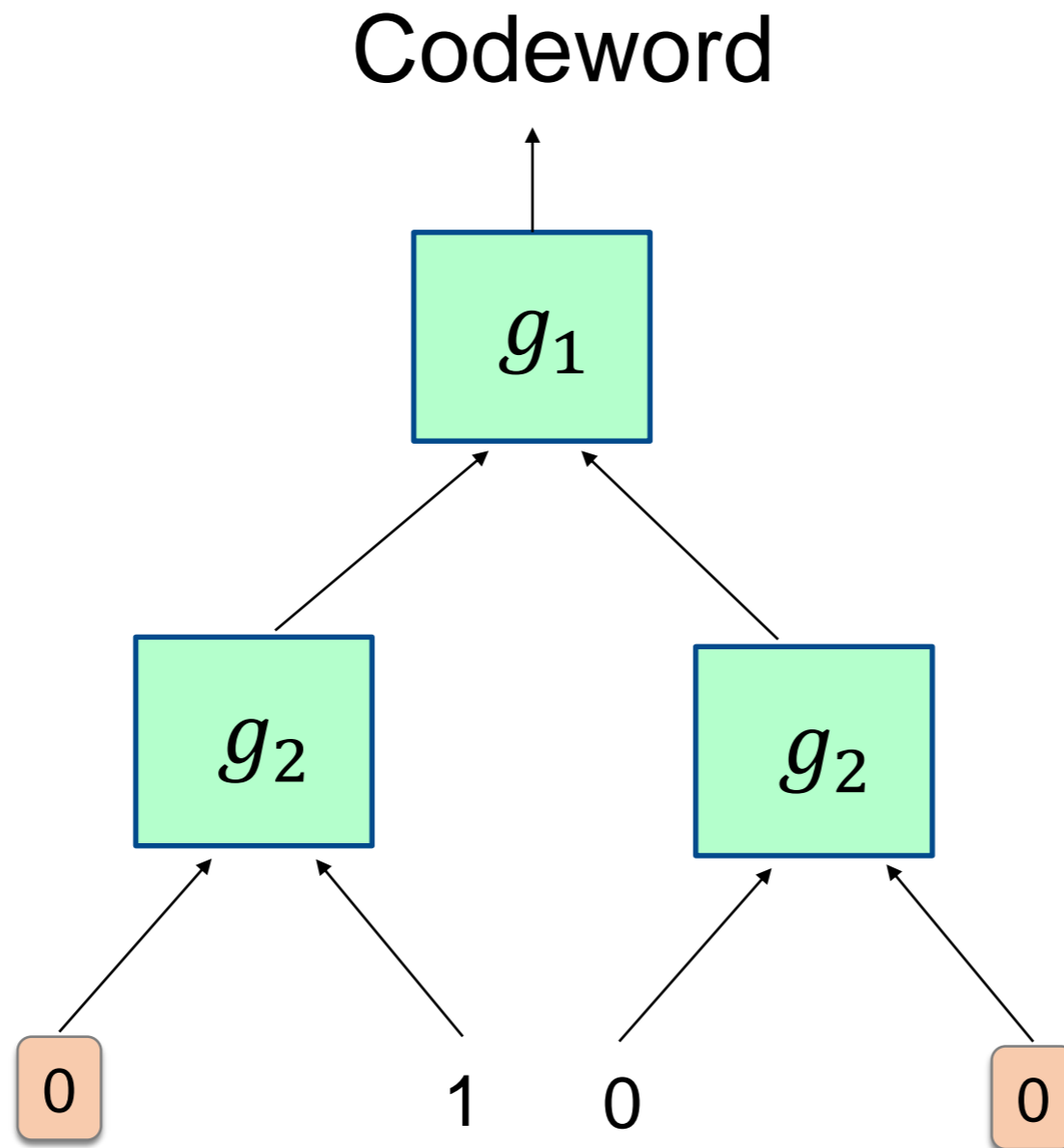




# KO neural network



# KO encoder

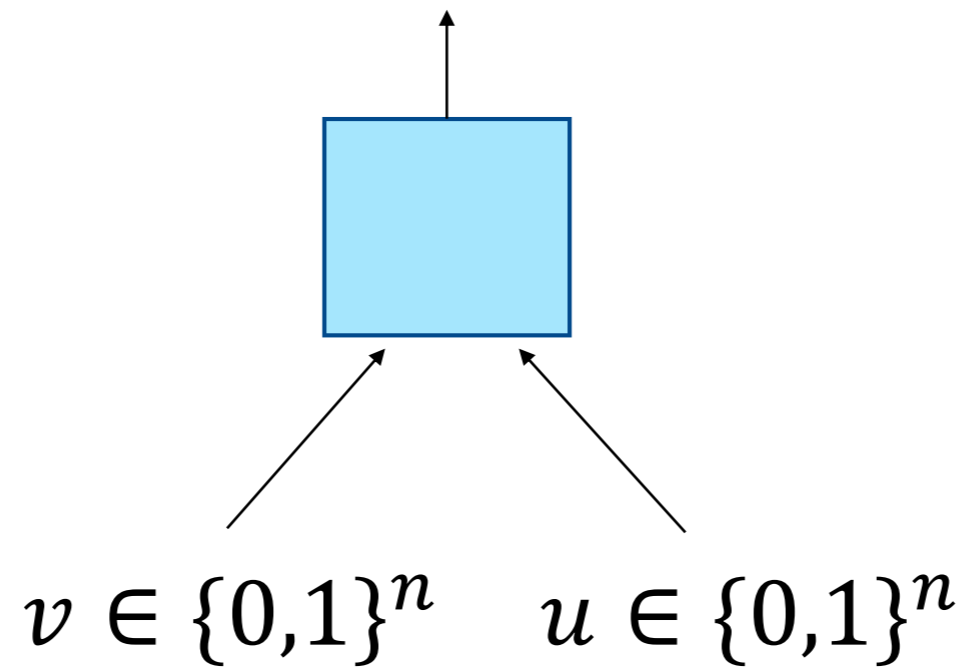


# Decoder

- Matching decoder for KO encoder?
- **Dumer's decoder / Successive Cancellation (SC)**

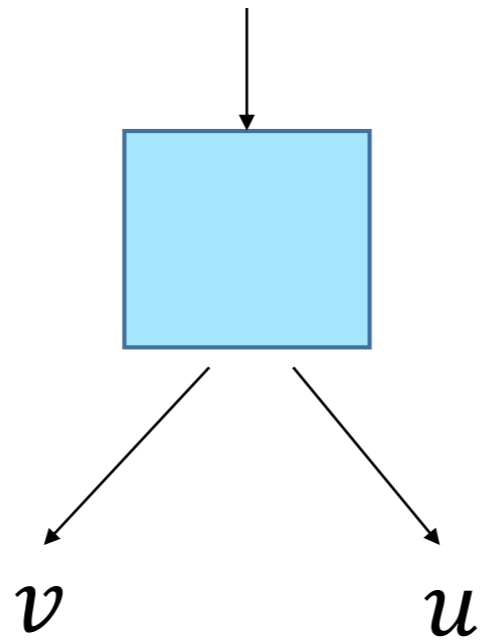
# Plotkin revisited

$$(u, u \oplus v) \in \{0,1\}^{2n}$$



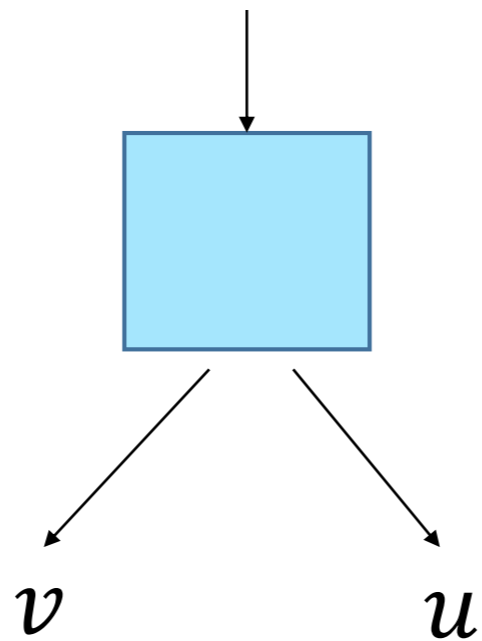
# Decoding

$(\text{LLR}_u, \text{LLR}_{u \oplus v})$



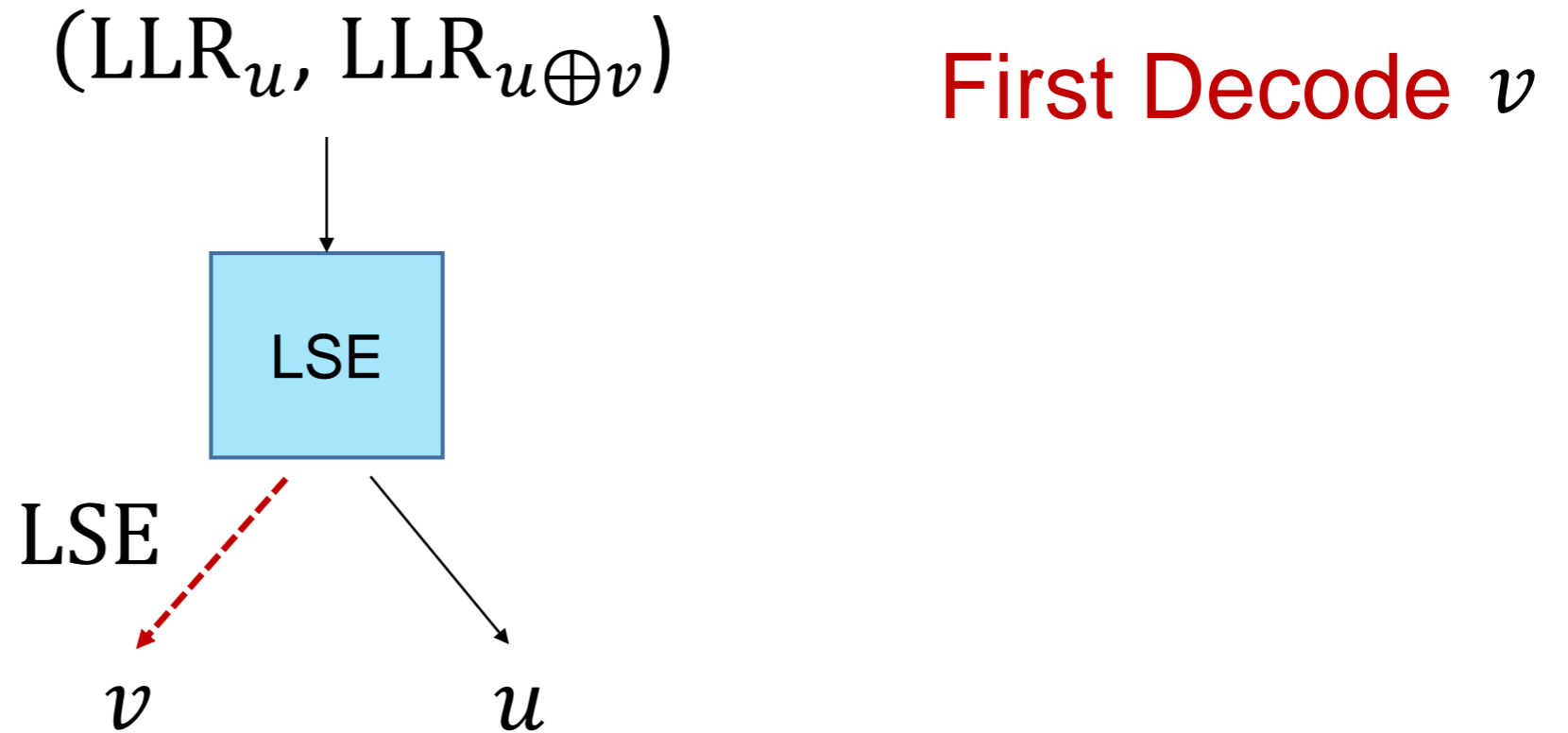
# SC decoder

$(\text{LLR}_u, \text{LLR}_{u \oplus v})$

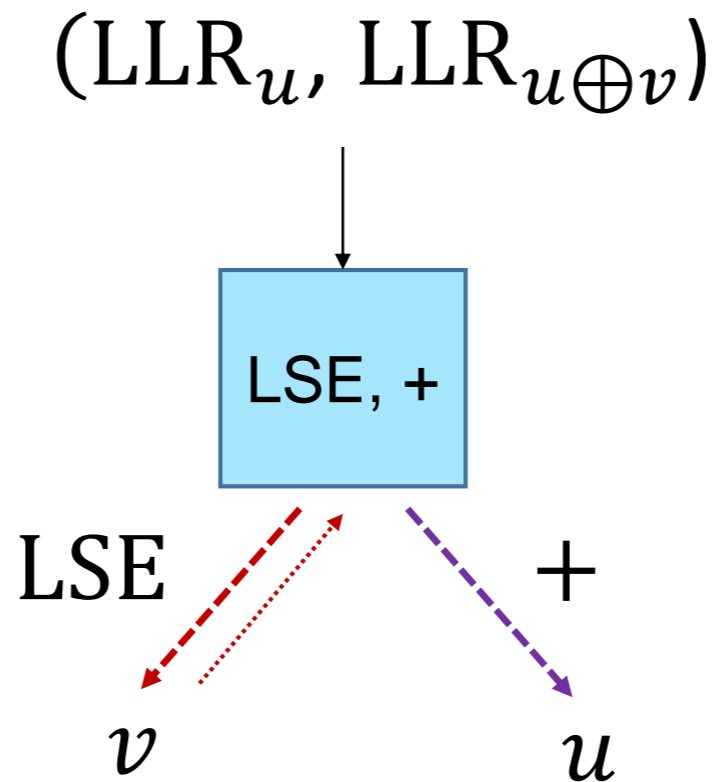


Dumer, 2004-06  
Arikan, 2009

# SC decoder



# SC decoder



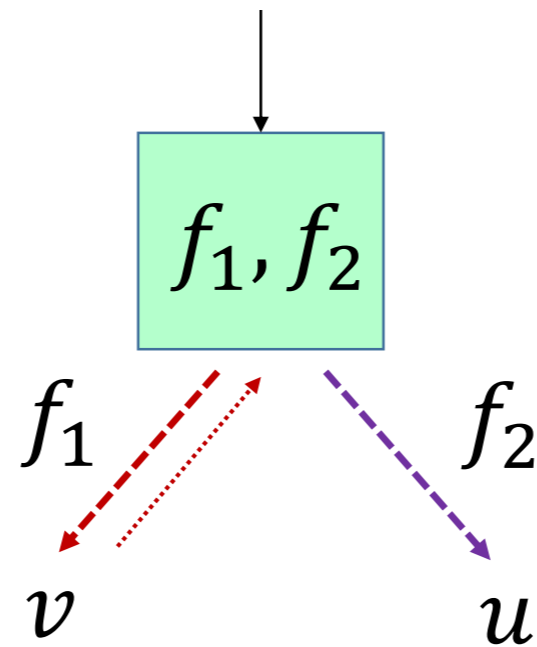
First Decode  $v$

Next Decode  $u$



# KO decoder

$(\text{LLR}_u, \text{LLR}_{g(u,v)})$

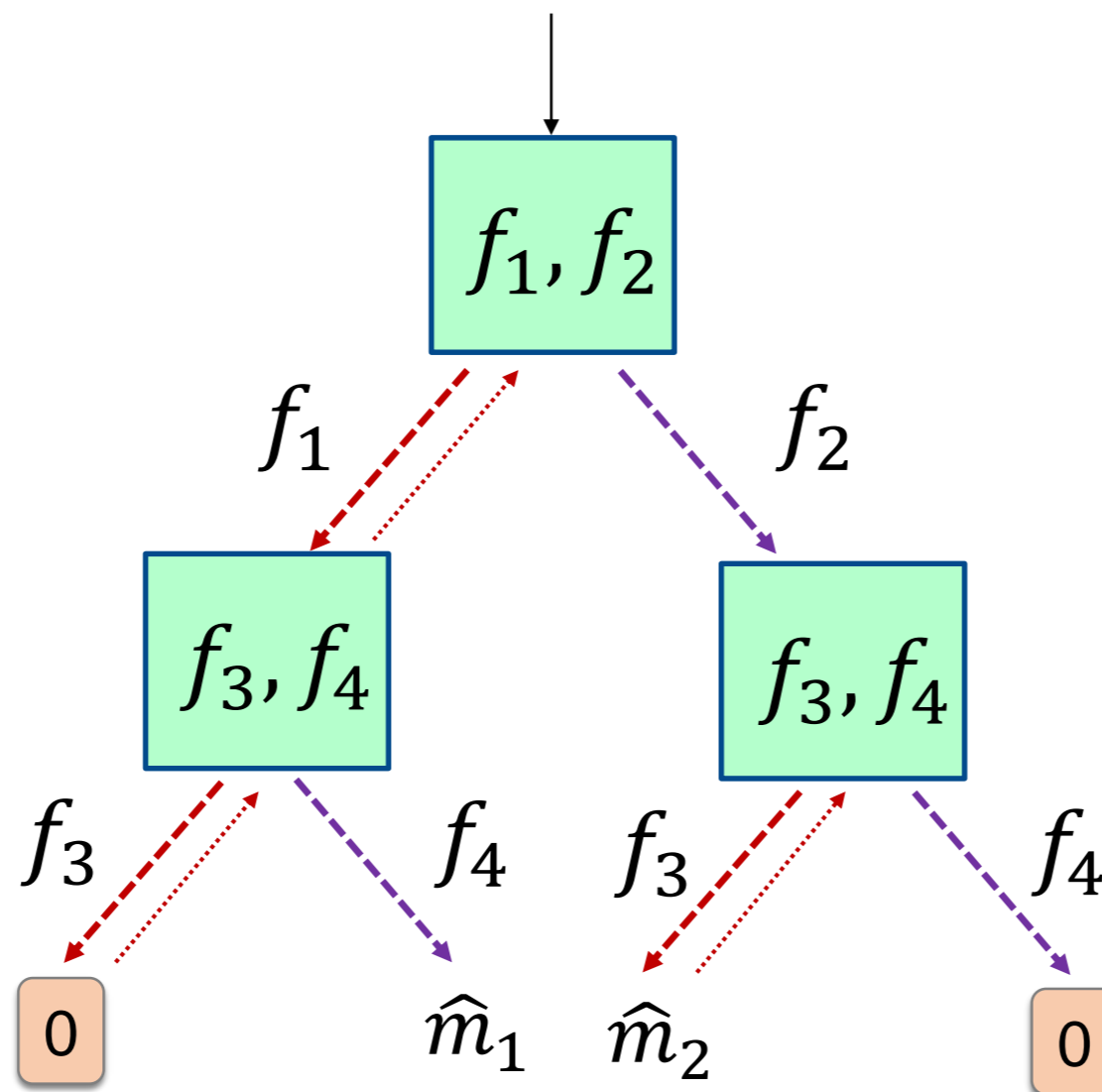


First Decode  $v$

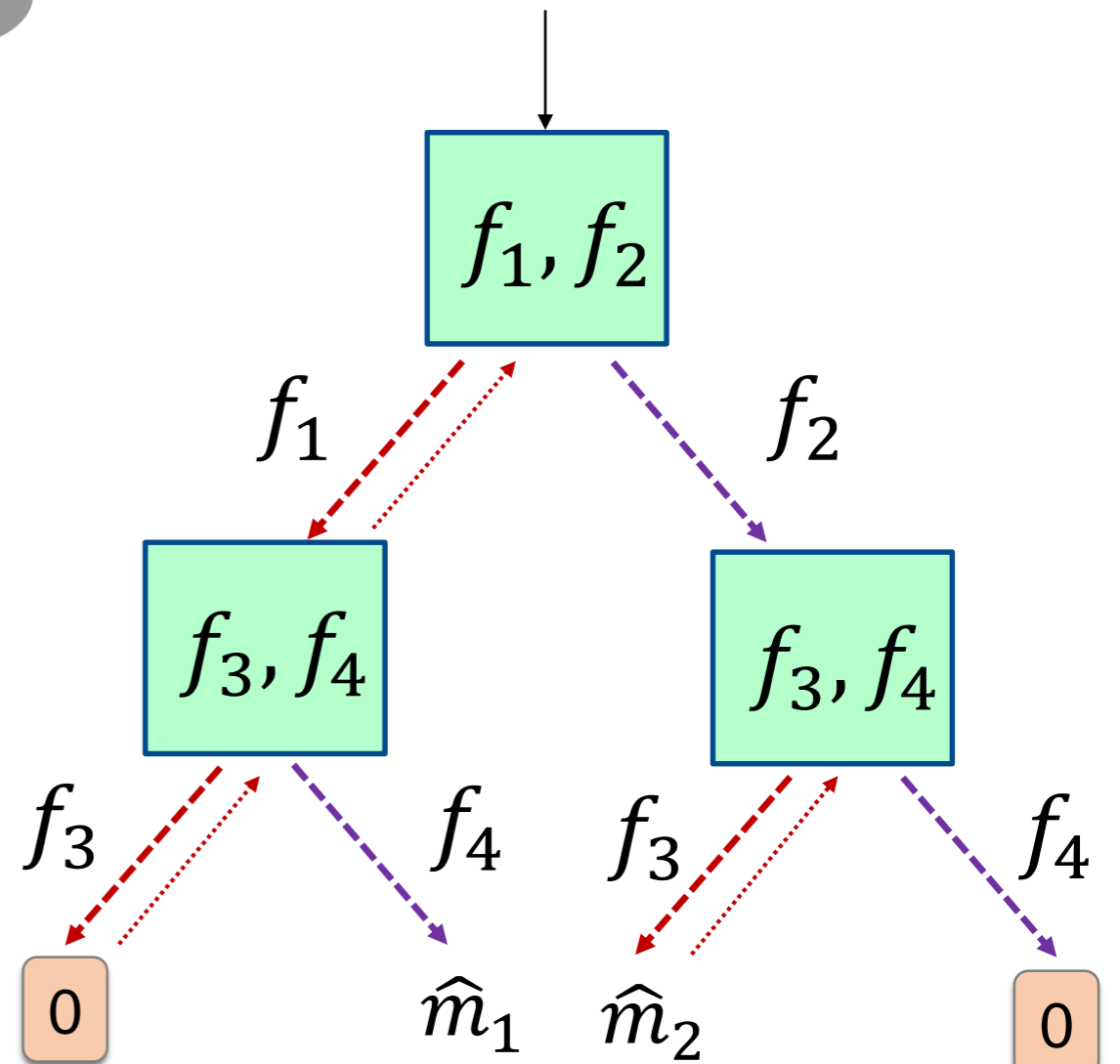
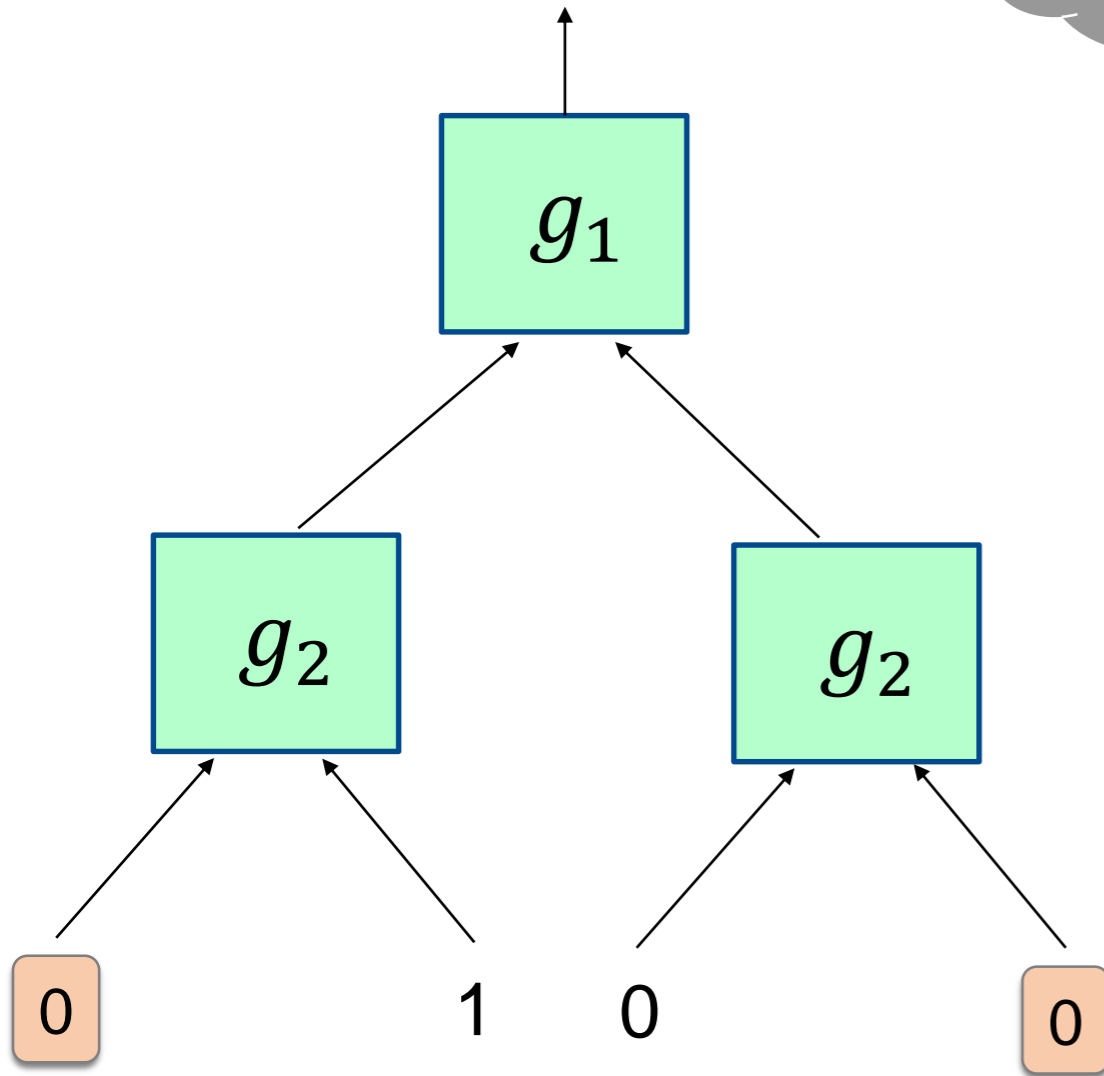
Next Decode  $u$

# KO decoder

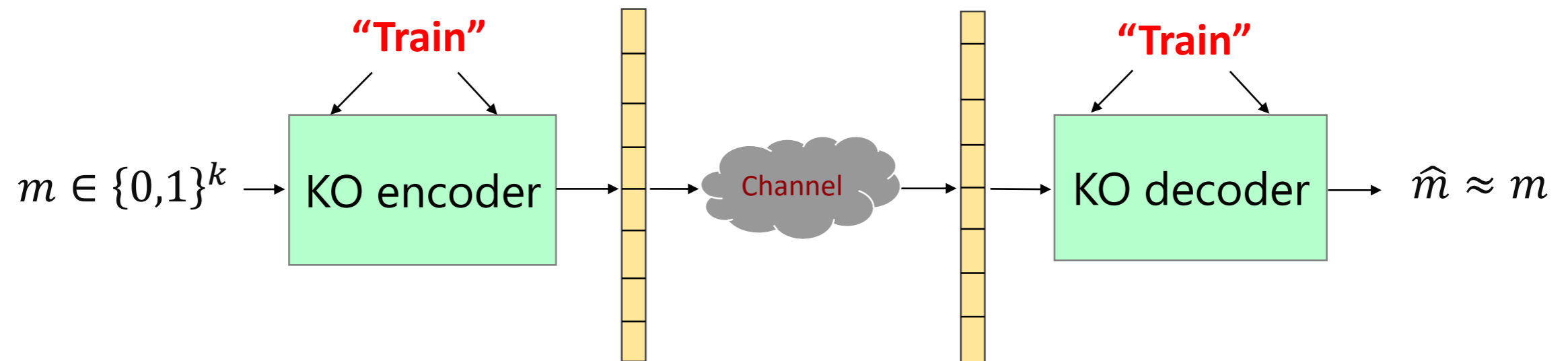
Noisy codeword



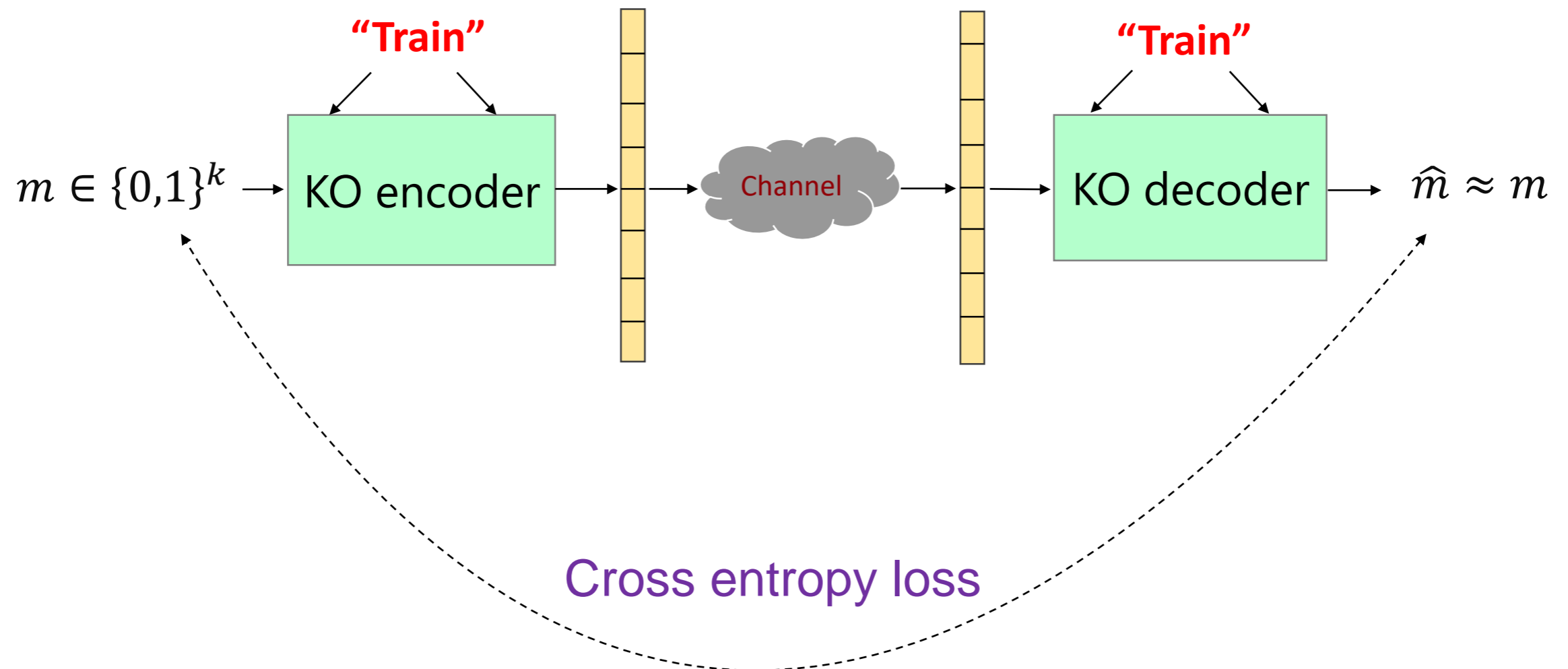
# KO (encoder, decoder)



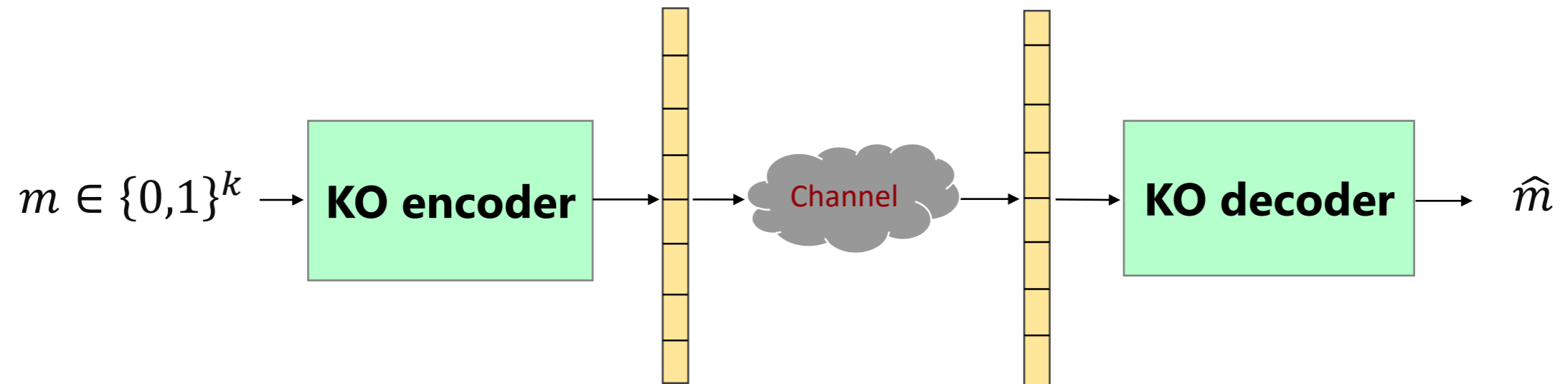
# Training KO Codes



# Training KO Codes



# Testing



# Performance metrics

- Reliability
- Computational complexity

# Baselines

- KO codes vs. RM codes
- KO codes vs. Polar codes



# Setup

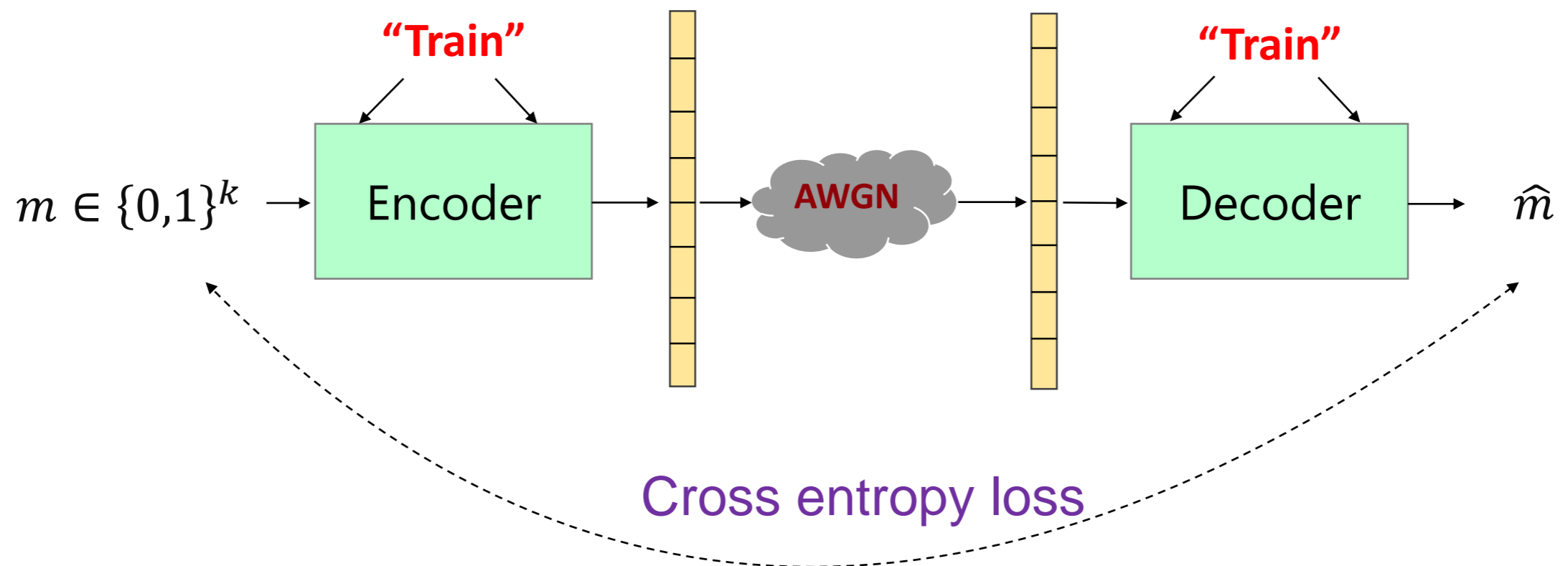
- Train and test on the same channel
  - AWGN
  
- Robustness: Train and test on different channels
  - Rayleigh fading

# Setup

- Train and test on the same channel
  - AWGN
- Robustness: Train and test on different channels
  - Rayleigh fading

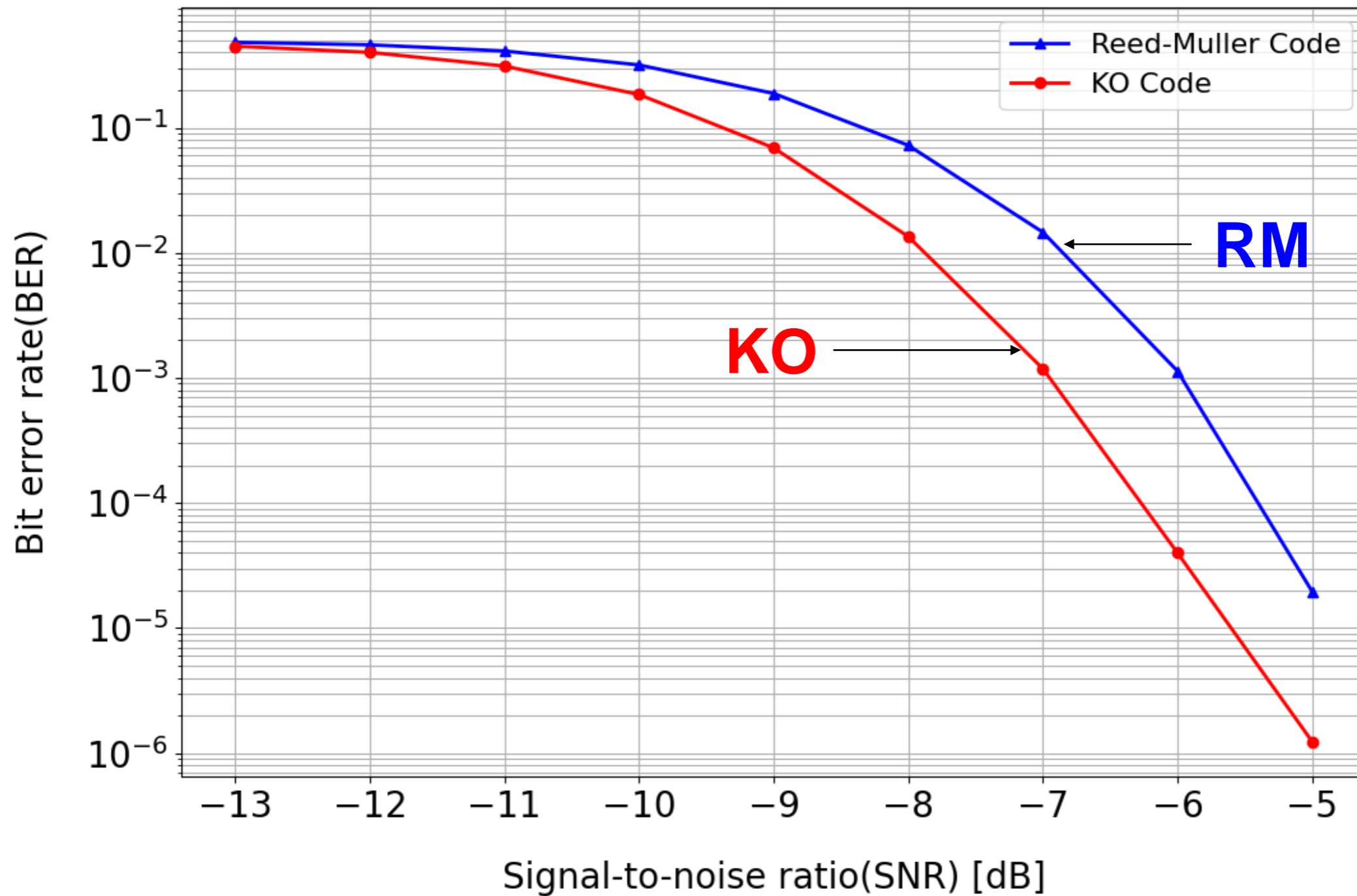
# Setup #1: AWGN

- Train and test on AWGN



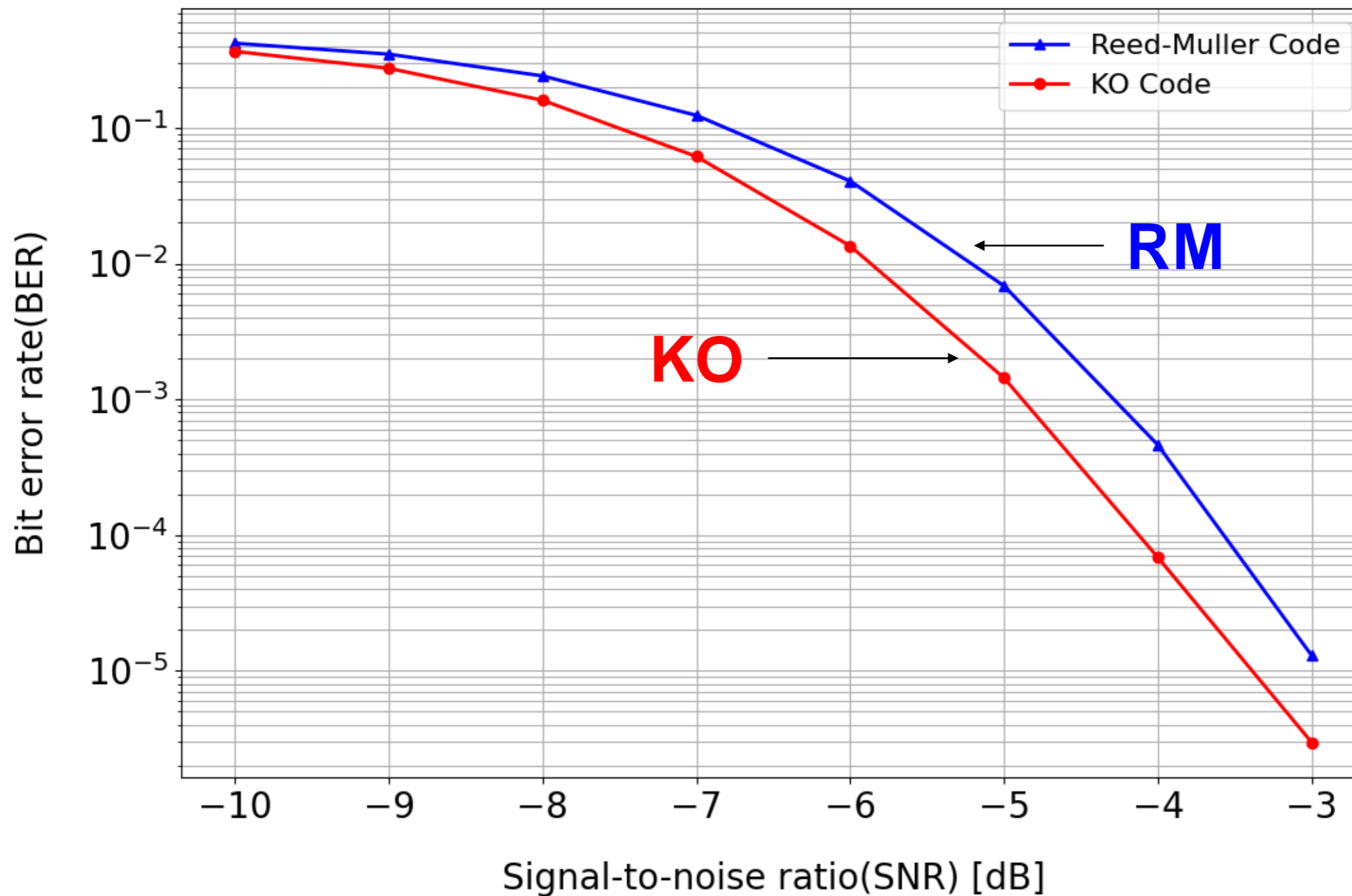
# KO codes beat RM

Code-dimension=46, Block length = 512



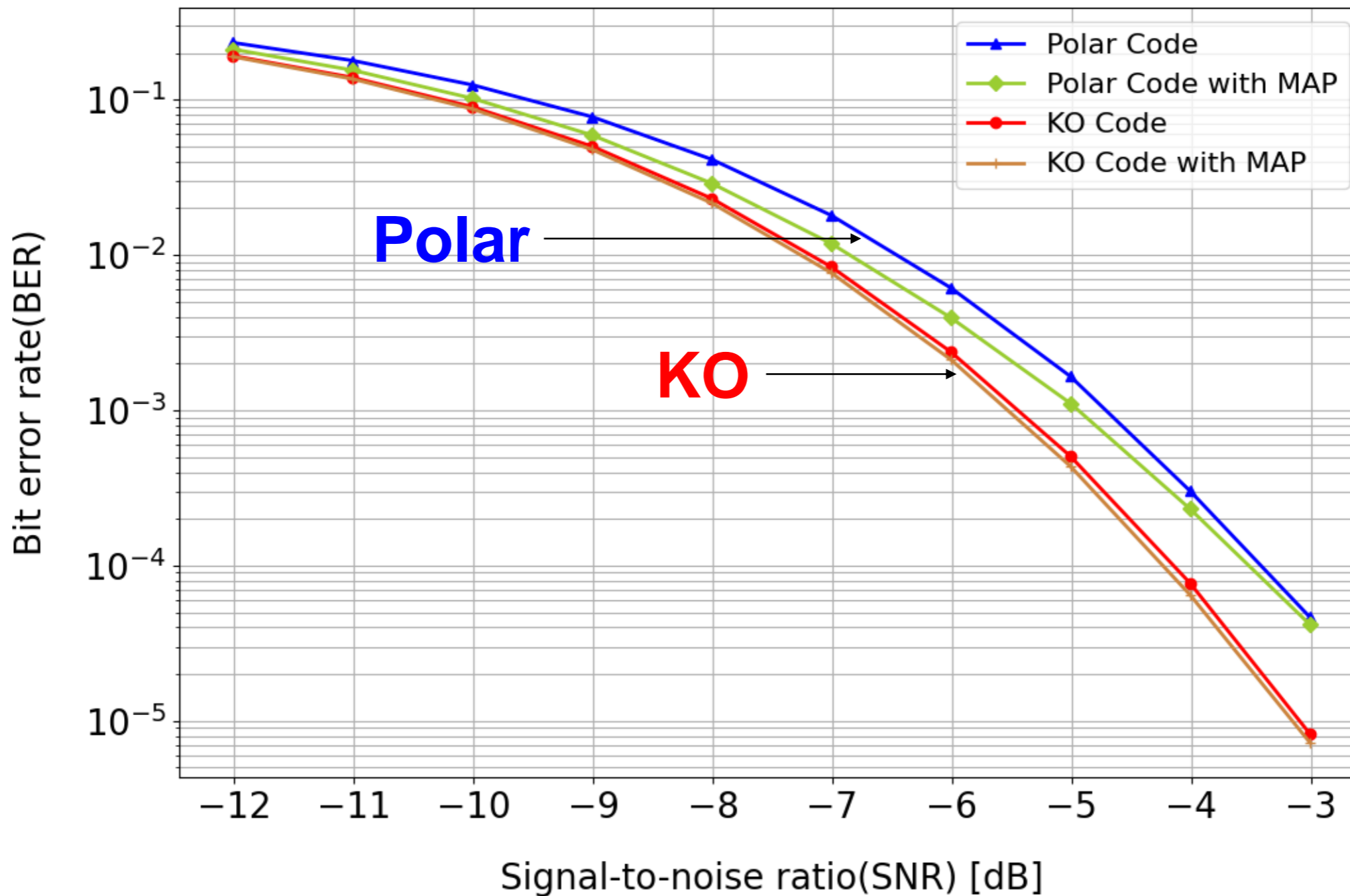
# KO codes beat RM

Code-dimension=37, Block length = 256



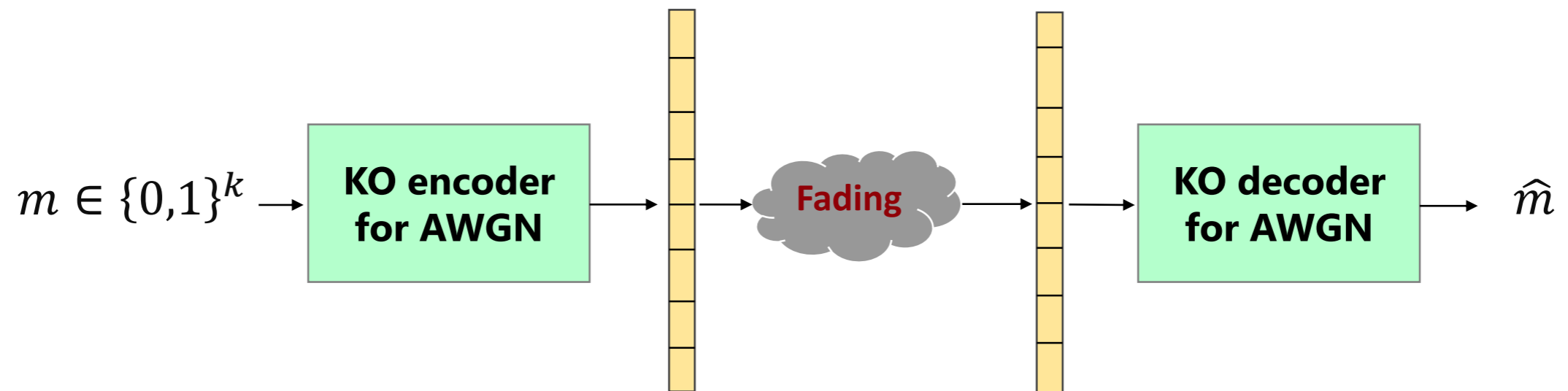
# KO beats Polar

Code-dimension=7, Block length = 64

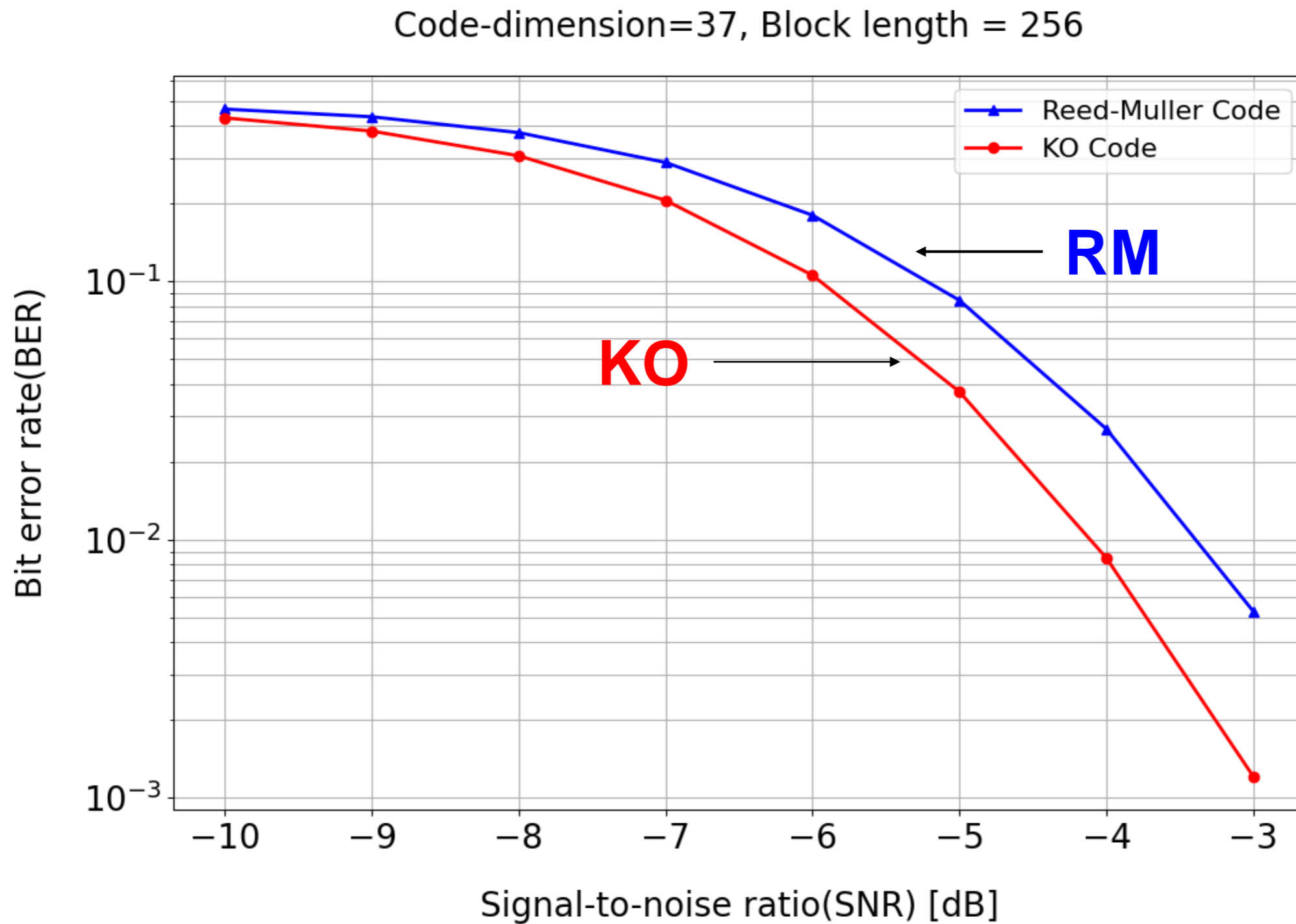


# Setup #2: Robustness

- Train on **AWGN** → Test under **Rayleigh fading**





# Robustness: Fading channel





# Setup

- Train and test on the same channel
  - AWGN
- Robustness: Train and test on different channels
  - Rayleigh fading

# Complexity

- Computational complexity:  $O(n \log n)$ 
  - KO codes  $\approx$  RM codes
- Number of operations
  - RM codes (11k)  $\ll$  KO codes (550k)

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- Computational complexity:  $O(n \log n)$ 
  - KO codes  $\approx$  RM codes
- Number of operations
  - RM codes (11k)  $\approx$  Tiny KO (44k)  $\ll$  KO codes (550k)

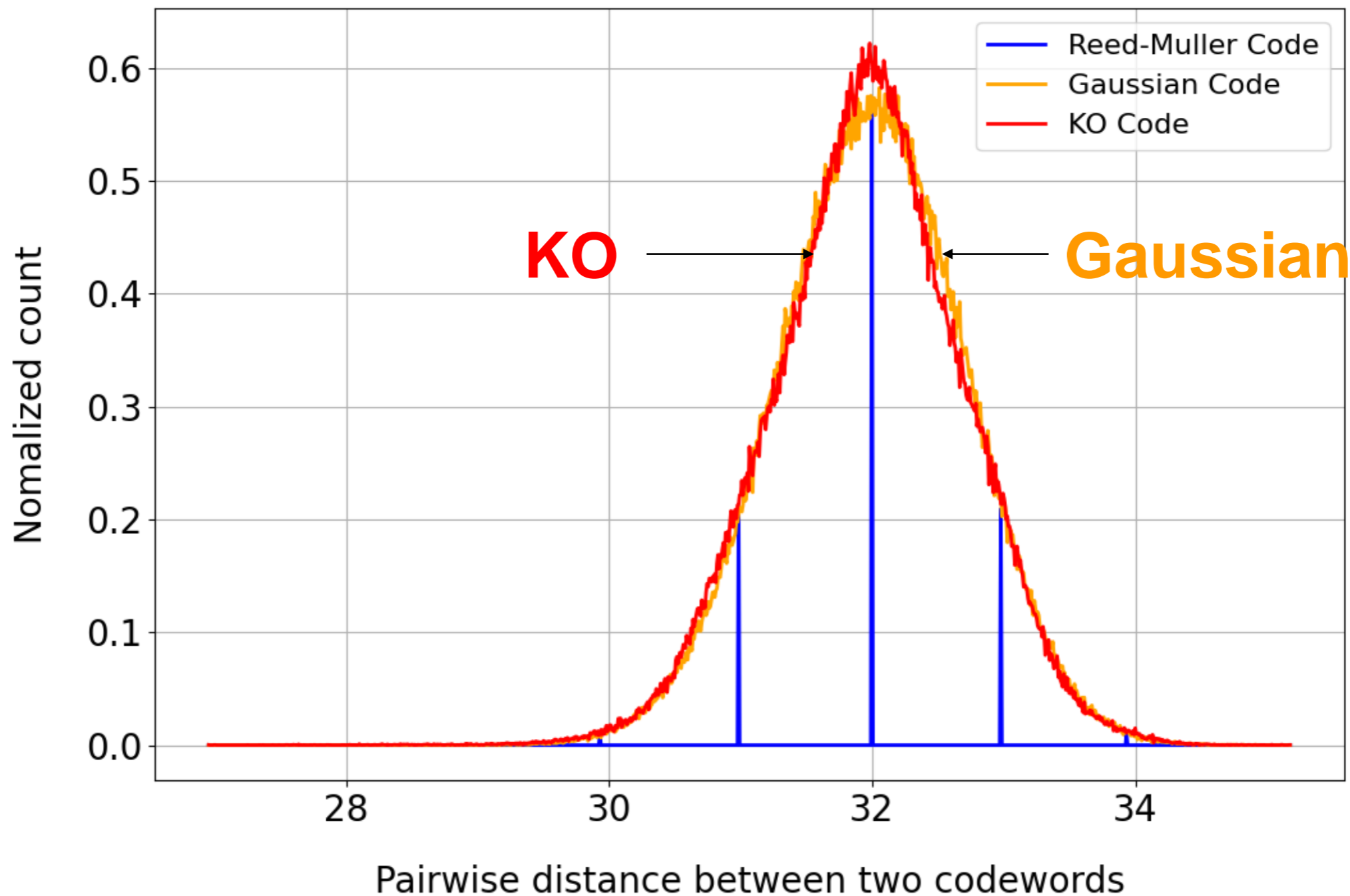
# Why are KO codes good

# Why are KO codes good

- Surprising resemblance to Gaussian codes!

# Gaussian like!

Code-dimension=46, Block length = 512



# Future directions

- Training with complex decoding algorithms
  - Recursive Projection Aggregation (RPA)
  - SC + list decoder

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- Learning the frozen bits: Liao et al, 2020



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- Training with complex decoding algorithms
  - Recursive Projection Aggregation (RPA)
  - SC + list decoder
- Learning the frozen bits: Liao et al, 2020
- Commercialization
  - Hardware implementation

# Long-term directions

- Discover new coding structures
  - Recursive: this work
  - Graph: LDPC
  - Sequential: convolutional
  - What is the best structure?
- Theoretical analysis
- Beyond point-to-point: Network coding

# Collaborators





**La Fin**

**Thank you!**