Polynomial Paradigm in Algorithms **Winter 2020**

Problem Set 1

Deadline: Jan 27th in Canvas

1) Prove that for any n and k the elementary symmetric polynomial

$$e_k(z_1, \ldots, z_n) = \sum_{S \in \binom{n}{k}} z^S$$

is real stable.

2) Let $p \in \mathbb{R}[z_1, \ldots, z_n]$ be a homogeneous real stable polynomial. Prove that either all coefficients of p are positive or they are all negative.

3) Let $p \in \mathbb{R}[z_1, z_2] = a + bz_1 + cz_2 + dz_1z_2$ be a real stable polynomial. Prove that $bc \geq ad$.

4) Prove that the following polynomial is *not* real stable:

$$\sum_{M : |M| = k} \prod_{i \text{ sat in } M} z_i,$$

i.e., there exists a graph $G = (V, E)$ and an integer k such that the above polynomial is not real stable.