Polynomial Paradigm in Algorithms **Winter 2020**

Problem Set 2

Deadline: Feb 26th in Canvas

1) Let $p \in \mathbb{R}[z_1, \dots, z_n]$ be a d-homogeneous polynomial hyperbolic with respect to $e \in \mathbb{R}^n$. Prove that $D_e p$ is also hyperbolic with respect to e. Recall that $D_e p = \sum_{i=1}^{n} e_i \partial_{z_i} p$.

2) Recall that a polynomial $p \in \mathbb{R}_{\geq 0}[z_1, \dots, z_n]$ is log-concave if for any $x, y \in \mathbb{R}^n_{\geq 0}$ and any $0 < \alpha < 1$,

$$p(\alpha x + (1-\alpha)y) \geq p(x)^{\alpha} \cdot p(y)^{1-\alpha}.$$

For $A \in \mathbb{R}^{n \times m}_{\geq 0}$ and $b \in \mathbb{R}^n_{\geq 0}$, let $T : \mathbb{R}^m \to \mathbb{R}^n$ defined as $y \mapsto Ay + b$. For a log-concave polynomial $p \in \mathbb{R}_{\geq 0}[z_1, \dots, z_n]$, prove that $p(T(y_1, \dots, y_m))$ has non-negative coefficients and is log-concave.

3) Prove the basis generating polynomial of any matroid with at most 5 elements is real stable.

4) Let $p(z_1, z_2) = a + bz_1 + cz_2 + dz_1 z_2$ with $a, b, c, d \geq 0$. Prove that p is completely log-concave iff $2bc \geq da$.

5) Let $p \in \mathbb{R}_{\geq 0}[z_1, \dots, z_n]$ be a homogeneous multilinear log-concave polynomial. Prove that p is completely log-concave.