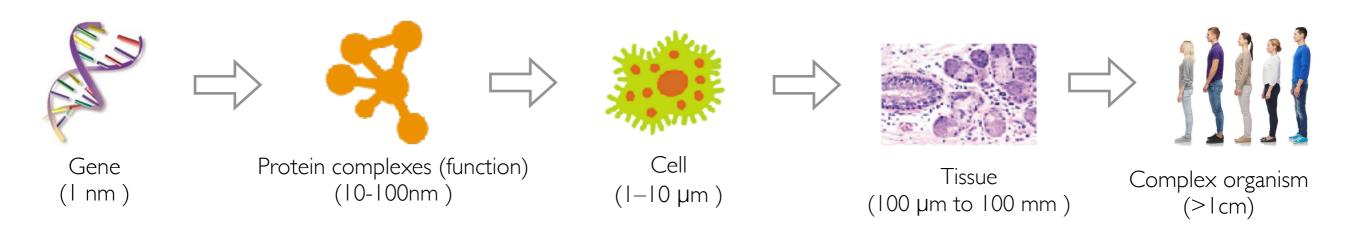
## CSE 427 Computational Biology

## Lecture I: Introduction

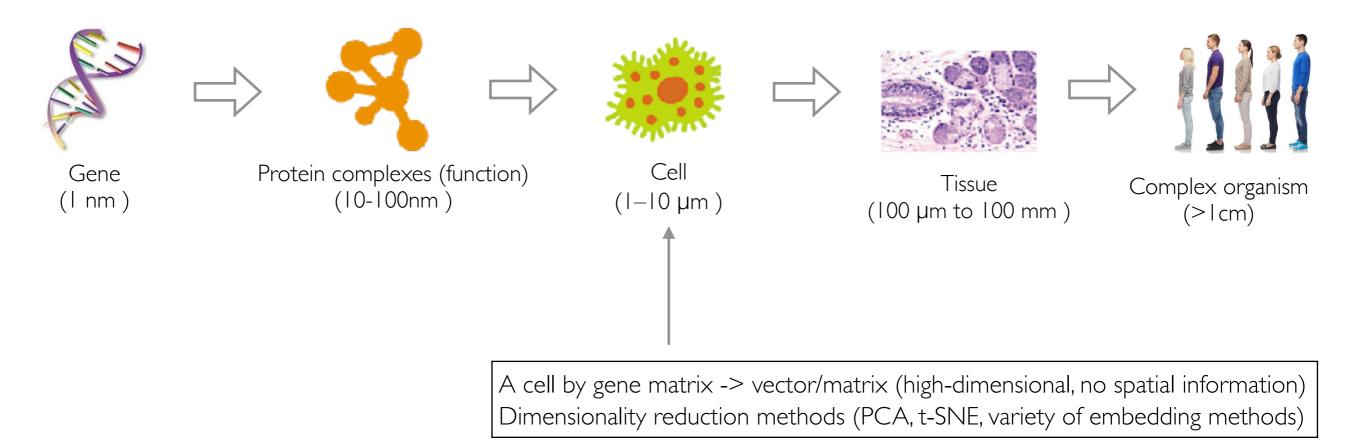
# CSE427: Computational methods for biology at different scales



A rich hierarchy of biological subsystems at multiple scales: genotypic variations in nucleotides (1 nm scale) -> proteins (1–10 nm) -> protein complexes (10–100 nm), cellular processes (100 nm) -> phenotypic behaviors of cells (1–10  $\mu$ m), tissues (100  $\mu$ m to 100 mm), -> complex organisms (>1 m).

source: Yu, Michael Ku, et al. "Translation of genotype to phenotype by a hierarchy of cell subsystems." *Cell systems* 2.2 (2016): 77-88.

## Data structure for each scale: cell

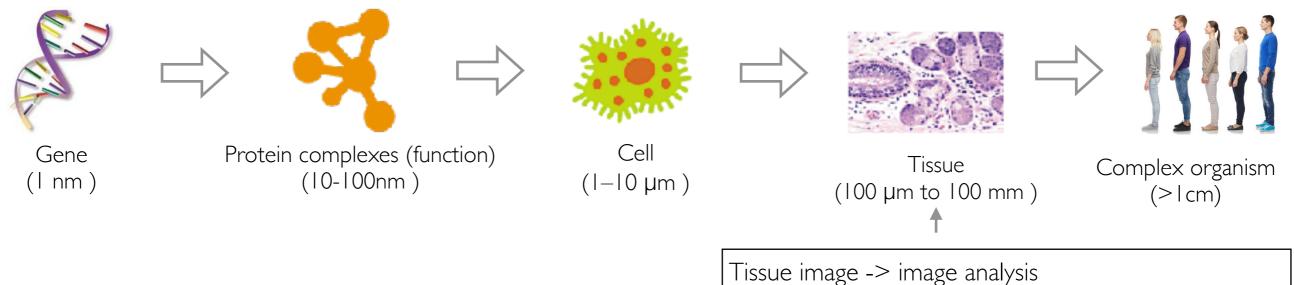


### High-dimensional, noisy, large-scale

## Single cell RNA sequencing (scRNA-seq)

- What is scRNA-seq?
  - A technique that can measure the gene expression vector of each cell
- What is the data structure?
  - A 2D array. Rows are cells. Columns are genes.
  - Lots of rows (millions of cells)
  - ~20k columns for human
- Analogy in other applications?
- What is the research question here?
  - Machine learning: dimensionality reduction, clustering, classification.

## Data structure for each scale: tissue

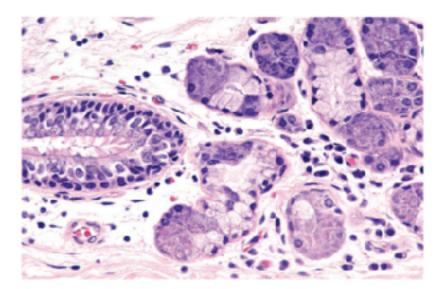


I issue image -> image analysis Image analysis (segmentation, detection, CNN)

Image analysis, lack of high-quality annotations

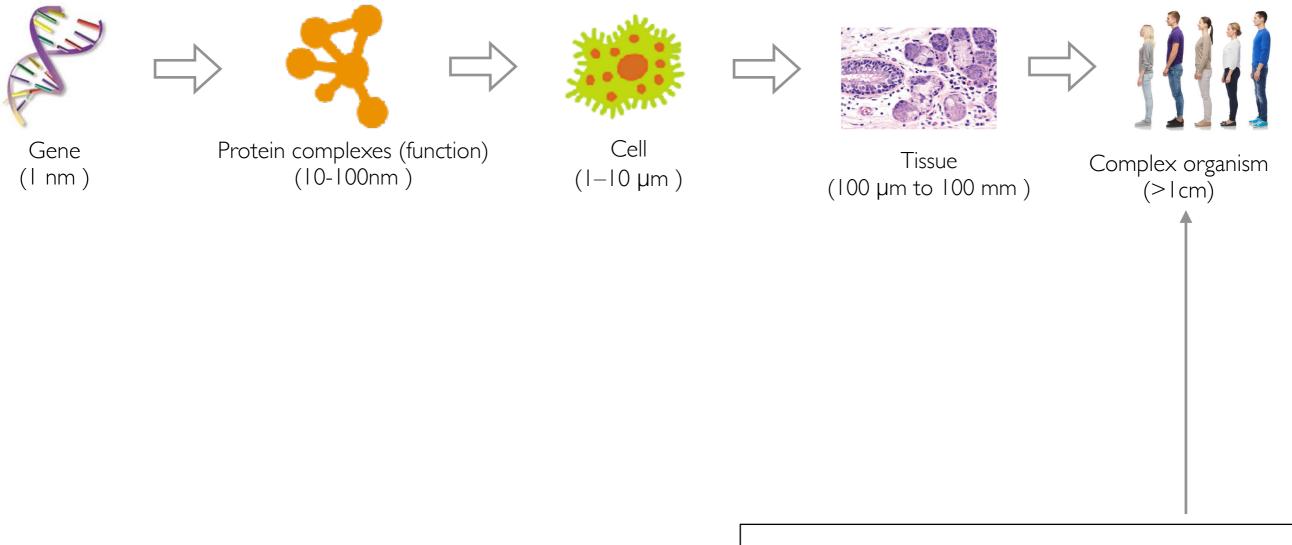
## Medical imaging technology

- What is the data structure?
  - One image for a small part of the tissue
- Analogy in other applications?
  - Image analysis
- What is the research question here?
  - Machine learning: image segmentation (which region is tumor), image classification (tumor v.s. healthy)



Tumor tissue image

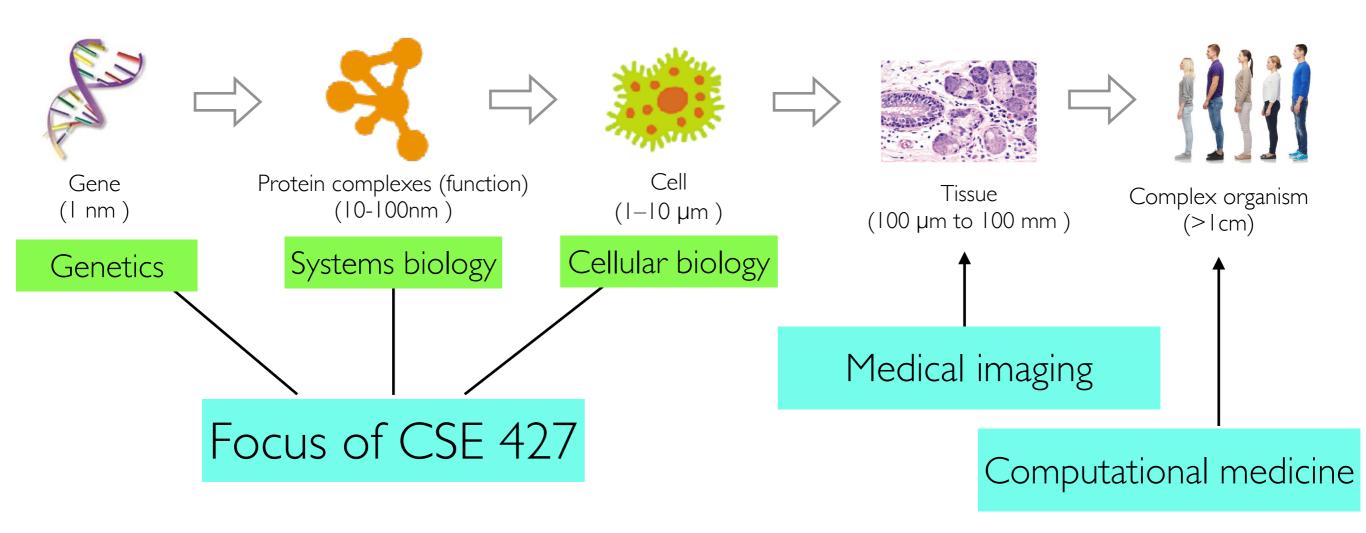
## Data structure for each scale: organism



Disease mechanisms -> Multimodality Integration of information from sequences, networks, images and matrixes

Multi-modality and heterogeneous

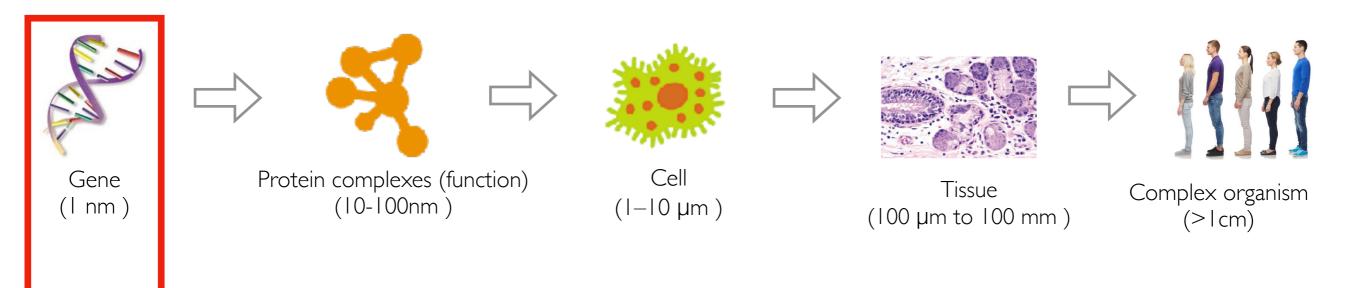
# Computational methods for biology at different scales



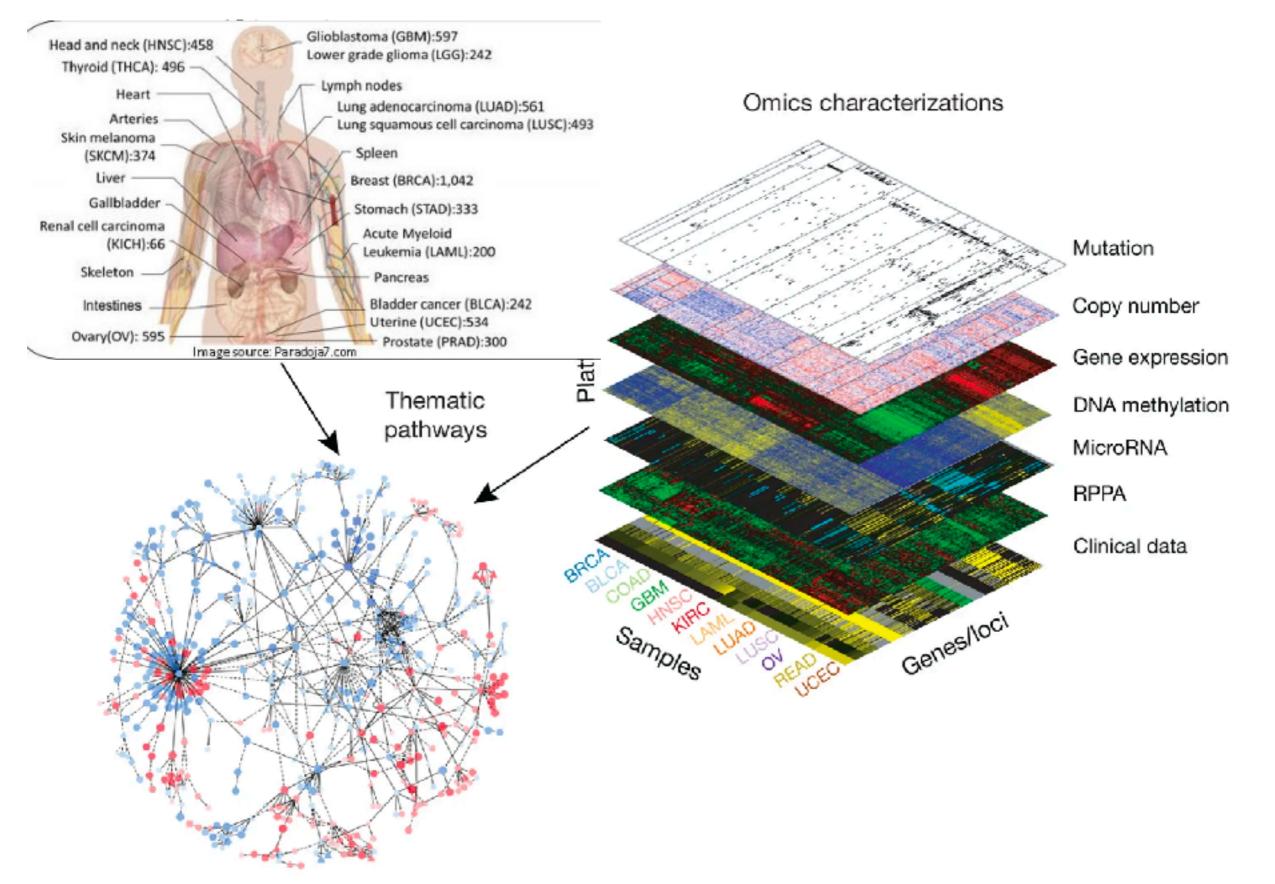
## Real world research question: how to measure the similarity between two patients

- We will have
  - DNA sequences of these two persons
  - A protein-protein interaction network
  - Gene expression matrix of cells in each person
  - Tissue image
  - Other datasets...
- Which of these data should we use?
- How should we integrate these multiple datasets?

# Computational methods for biology at different scales



## A concrete example: The Cancer Genome Atlas Program



## DNA sample analysis by 23andMe





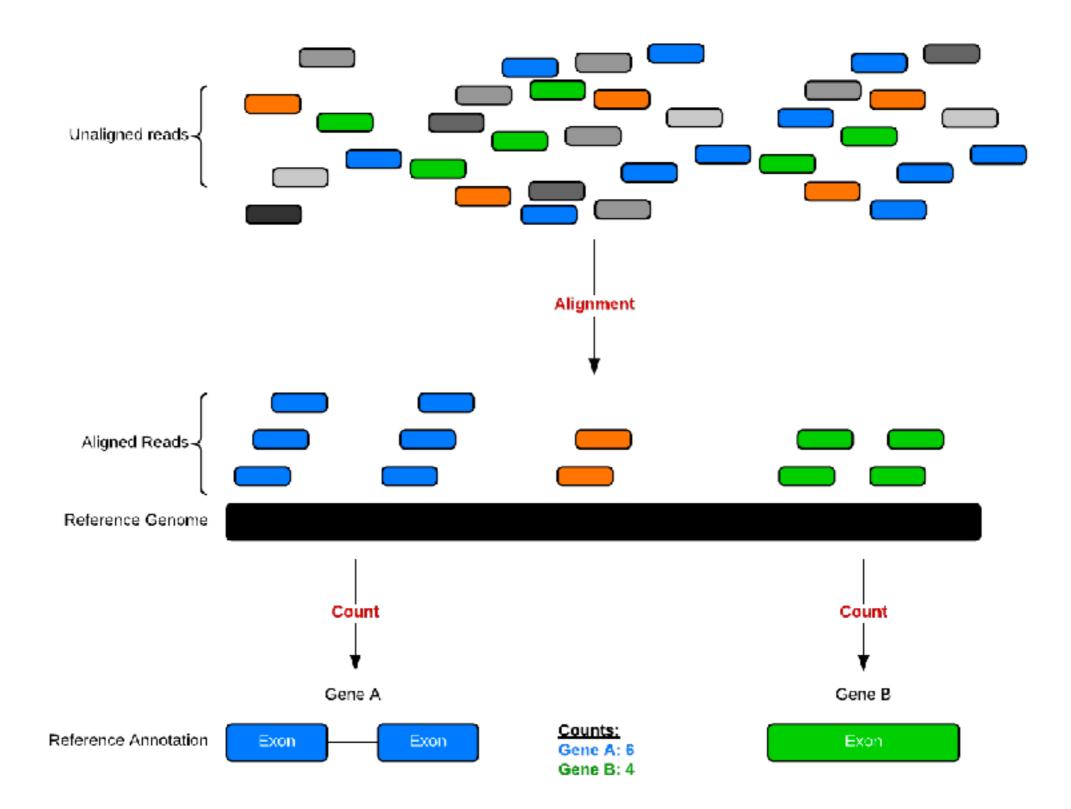
DNA sample

## How did they do this?



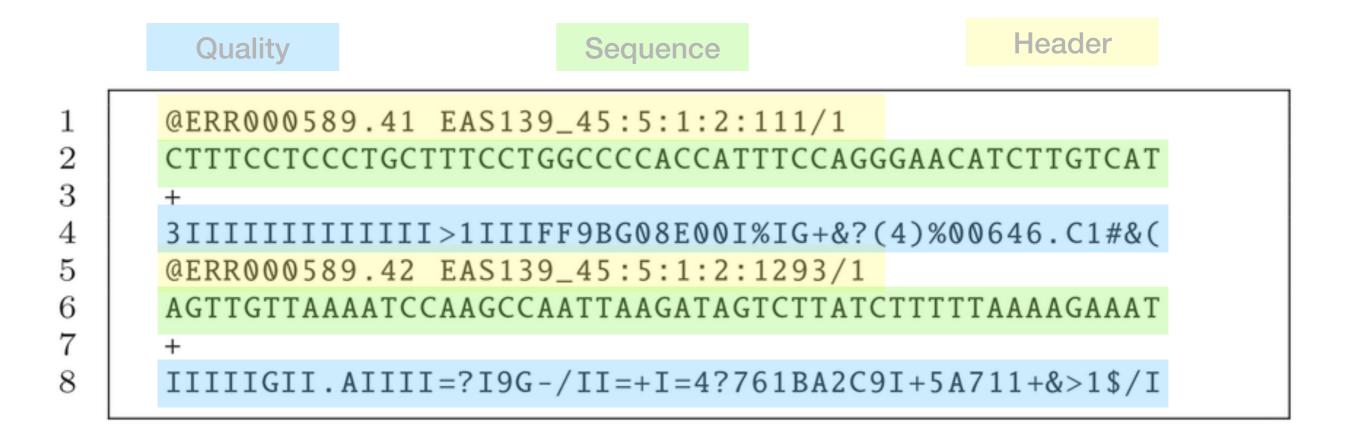
Our job as a computer scientist: analyze \*.fastq file

## Process raw data using sequence alignment (dynamic programming)



source: https://bioconnector.github.io/bims8382/r-rnaseq-airway.html

## What does a fastq file look like?



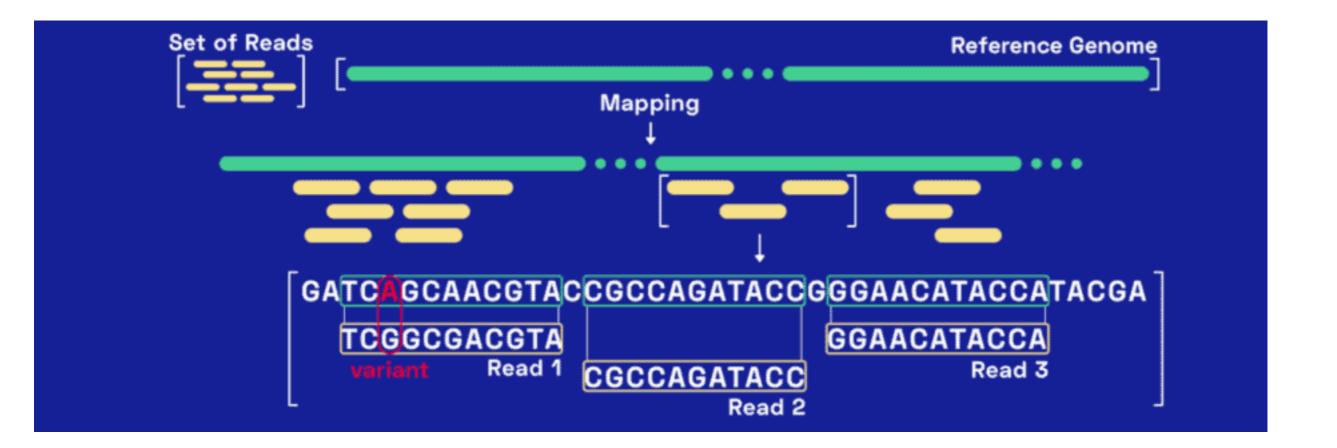
Very large! ~30000000 lines Quality: ASCII chars

What should we do? Map each short sequence (we call it read) to the entire human genome

## What does a fastq file look like?

Reference genome: "average" human genome.

Most widely used human genome GRCh38: derived from 13 thirteen anonymous volunteers



### Processed data

## countData

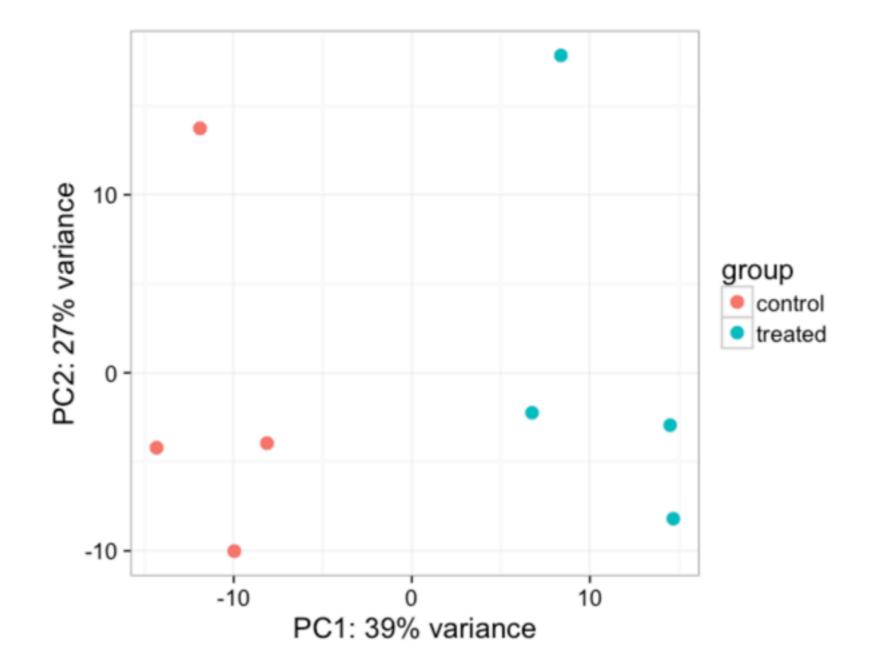
	ctrl_1	ctrl_2	exp_1	exp_1
geneA	10	11	56	45
geneB	0	0	128	54
geneC	42	41	59	41
geneD	103	122	1	23
geneE	10	23	14	56
geneF	0	1	2	0

## colData

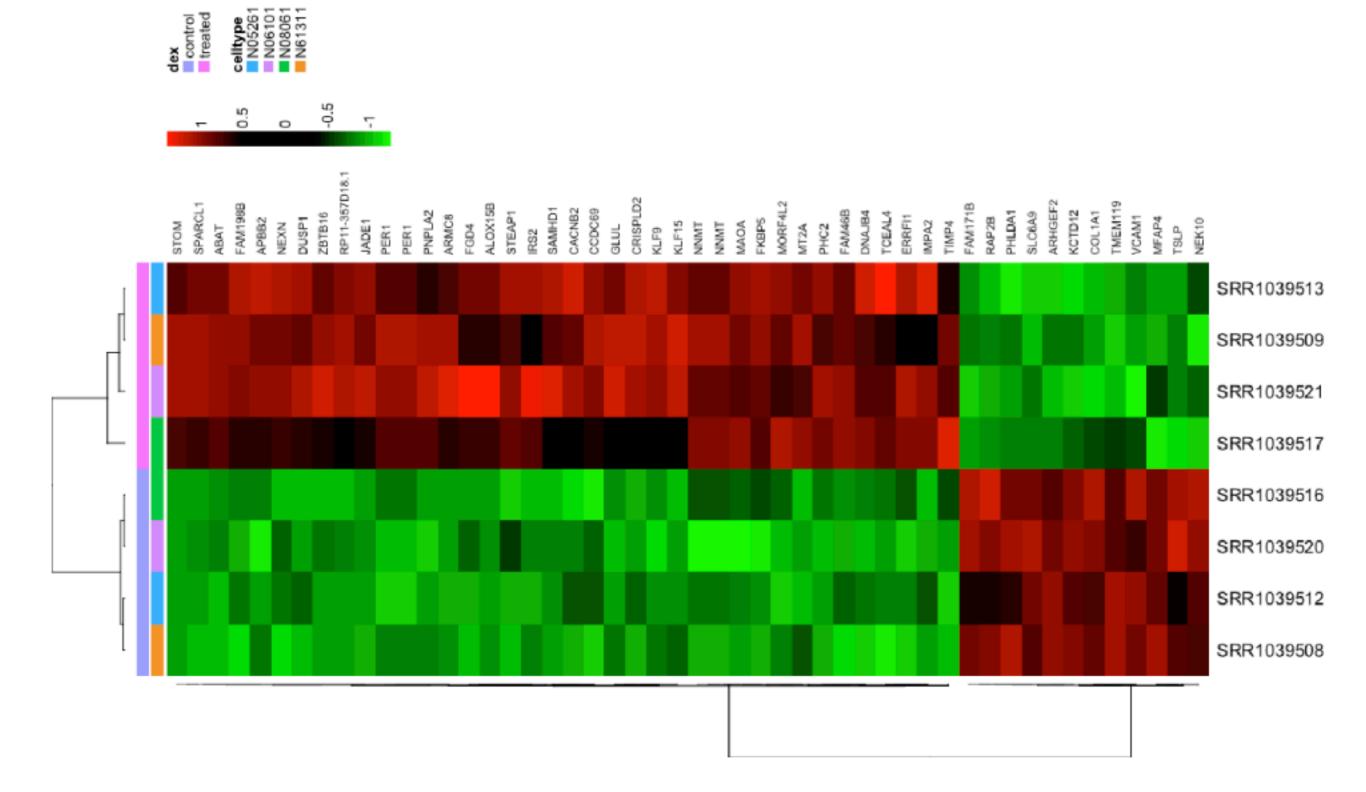
	treatment	sex
ctrl_1	control	male
ctrl_2	control	female
exp_1	treatment	male
exp_2	treatment	female

Sample names: ctrl\_1, ctrl\_2, exp\_1, exp\_2

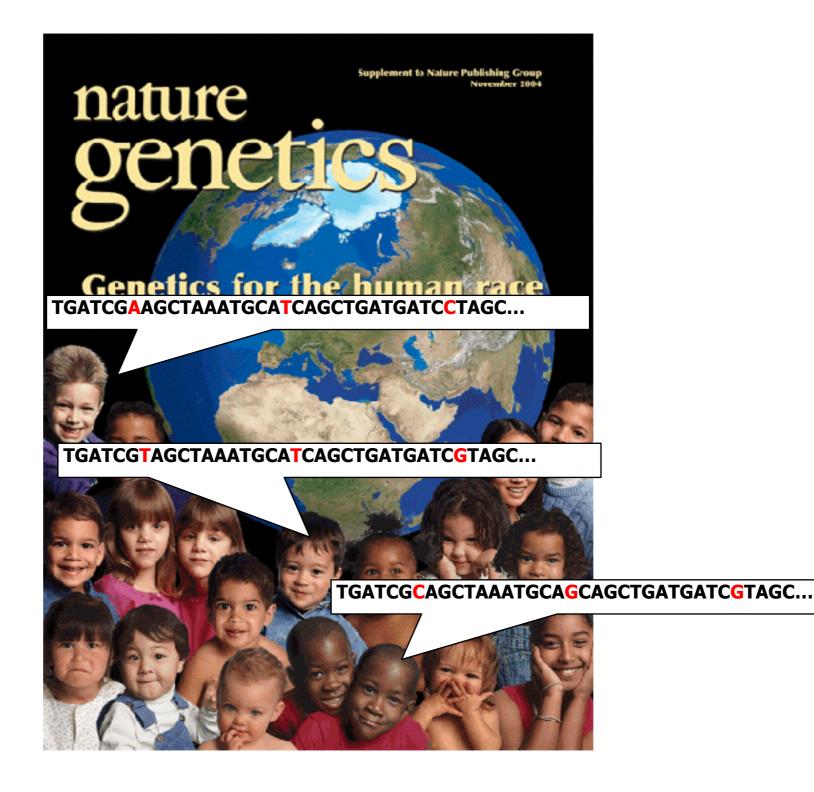
## Clustering analysis using dimensionality reduction



## Heatmap for visualization



# Each individual has a slightly different version of the DNA sequence



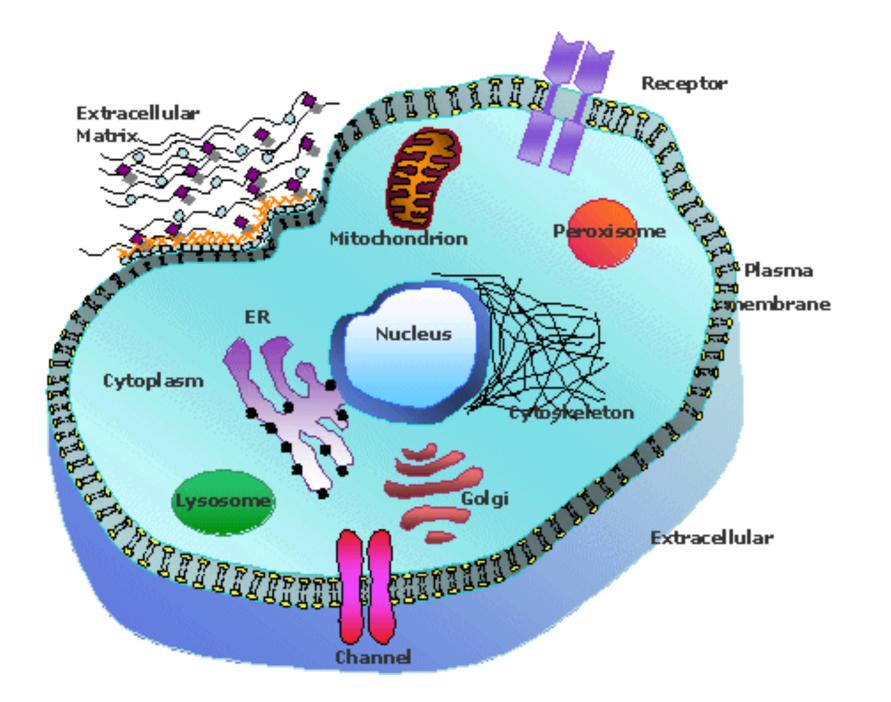
## DNA: "Blueprints" for a cell

- Genetic information encoded in long strings of double-stranded DNA (Deoxyribo Nucleic Acid)
- DNA comes in only four flavors: Adenine, Cytosine, Guanine, Thymine
  - In human, DNA is a 3 billion-long string of As, Cs, Gs and Ts
- DNA acts as the "brain" of the cell, telling the cell how to properly grow and work



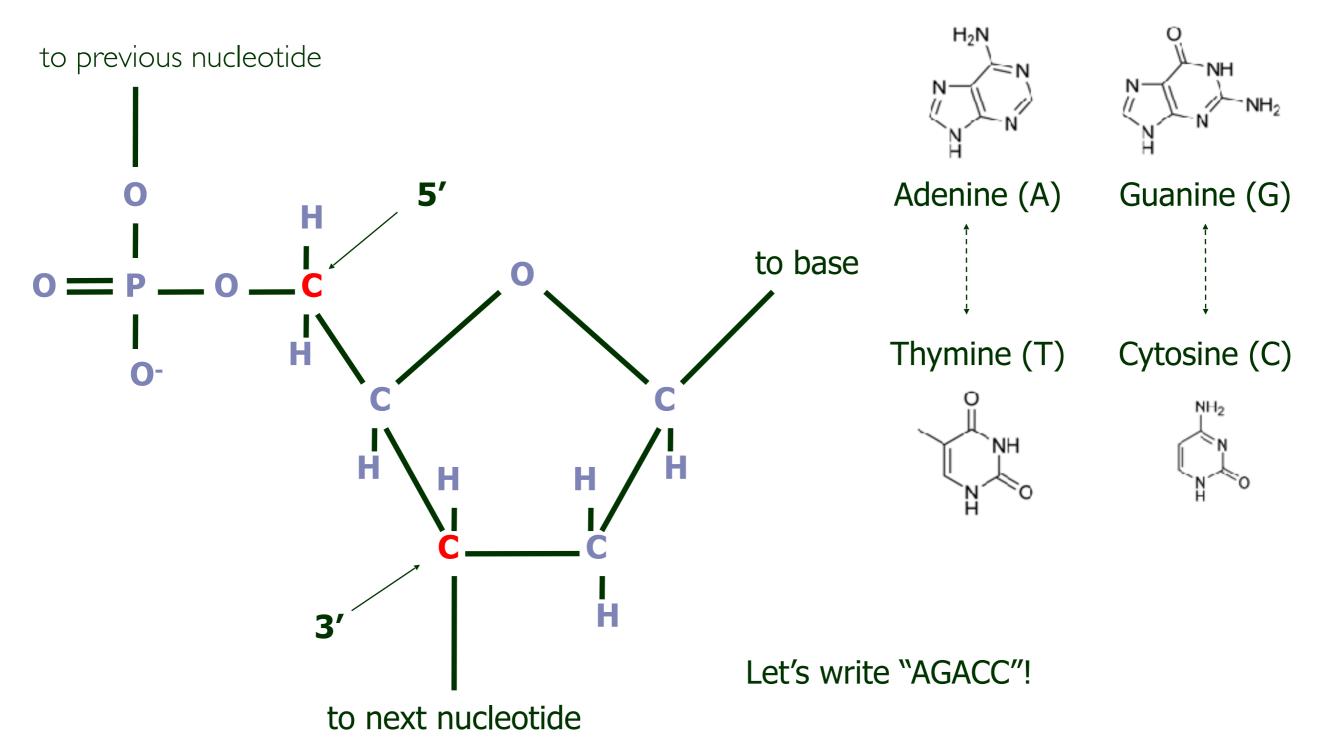
Cell

### Cell, nucleus, cytoplasm, mitochondrion

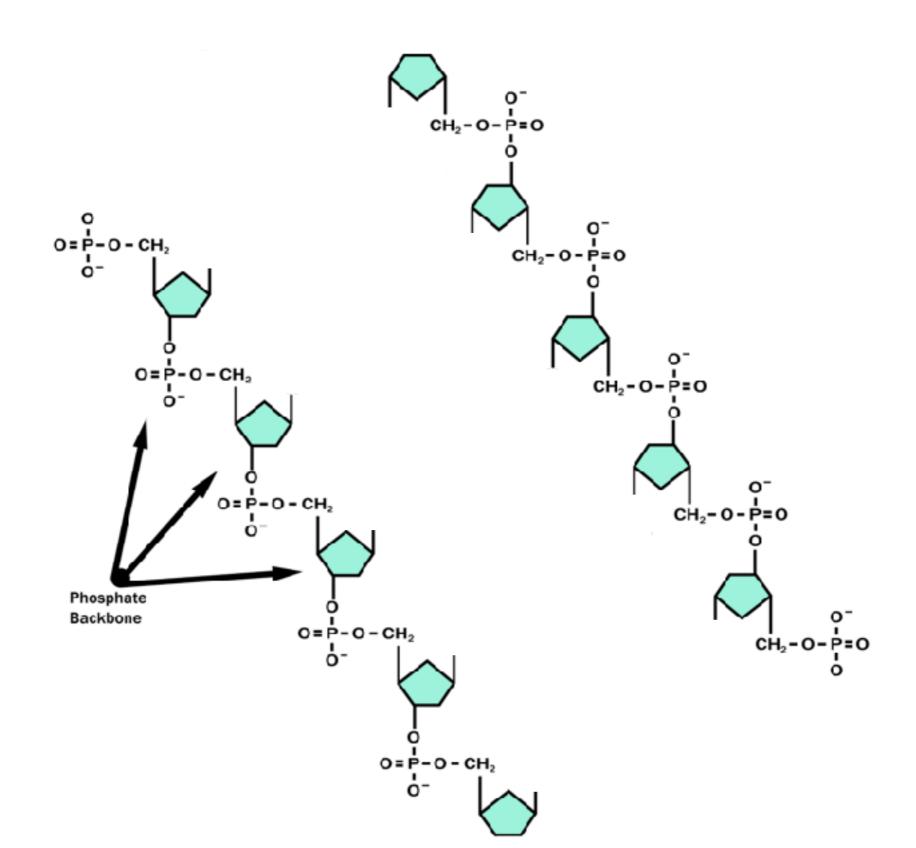


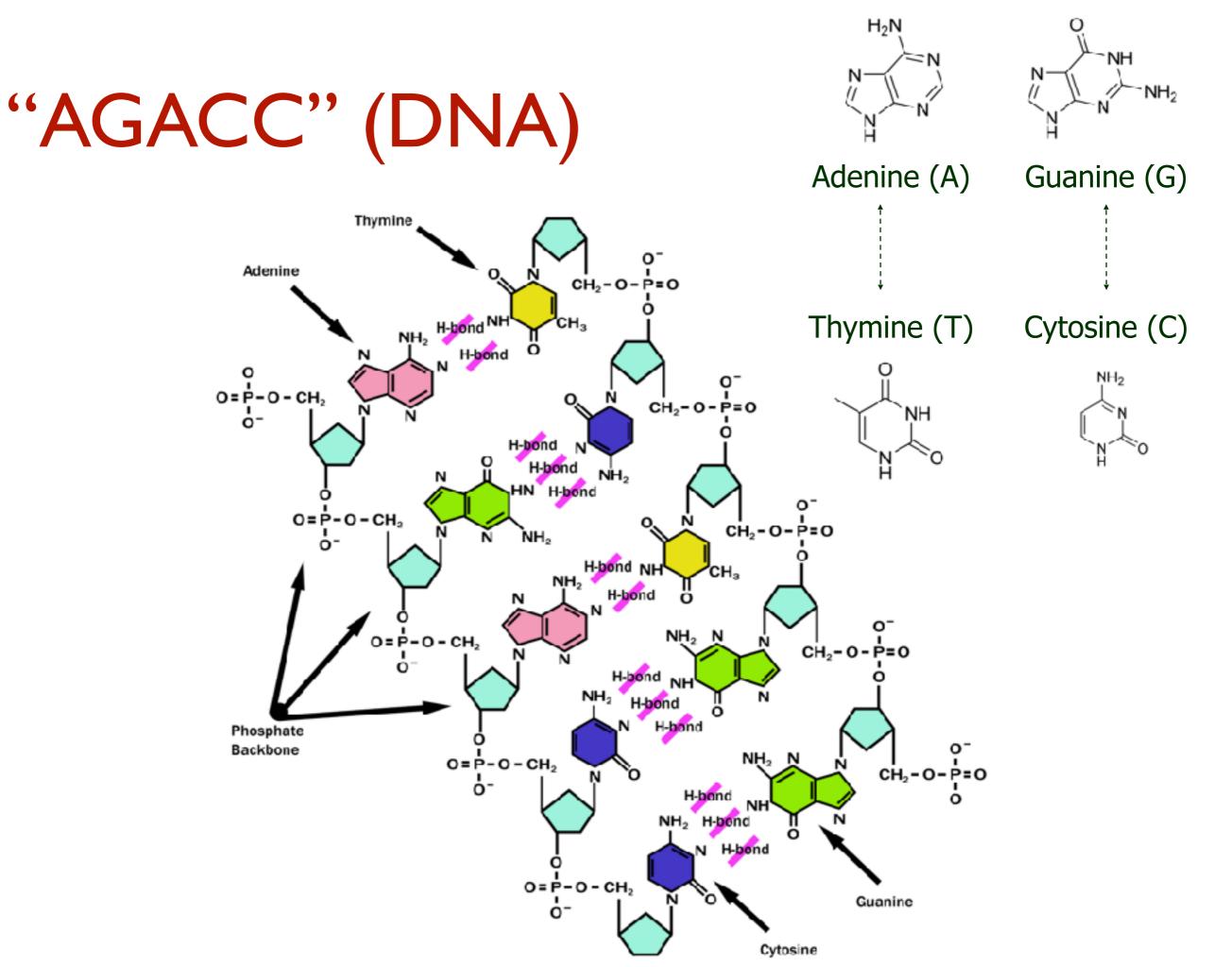
## Nucleotide

### Nucleotide, base, A, C, G, T, 3', 5'



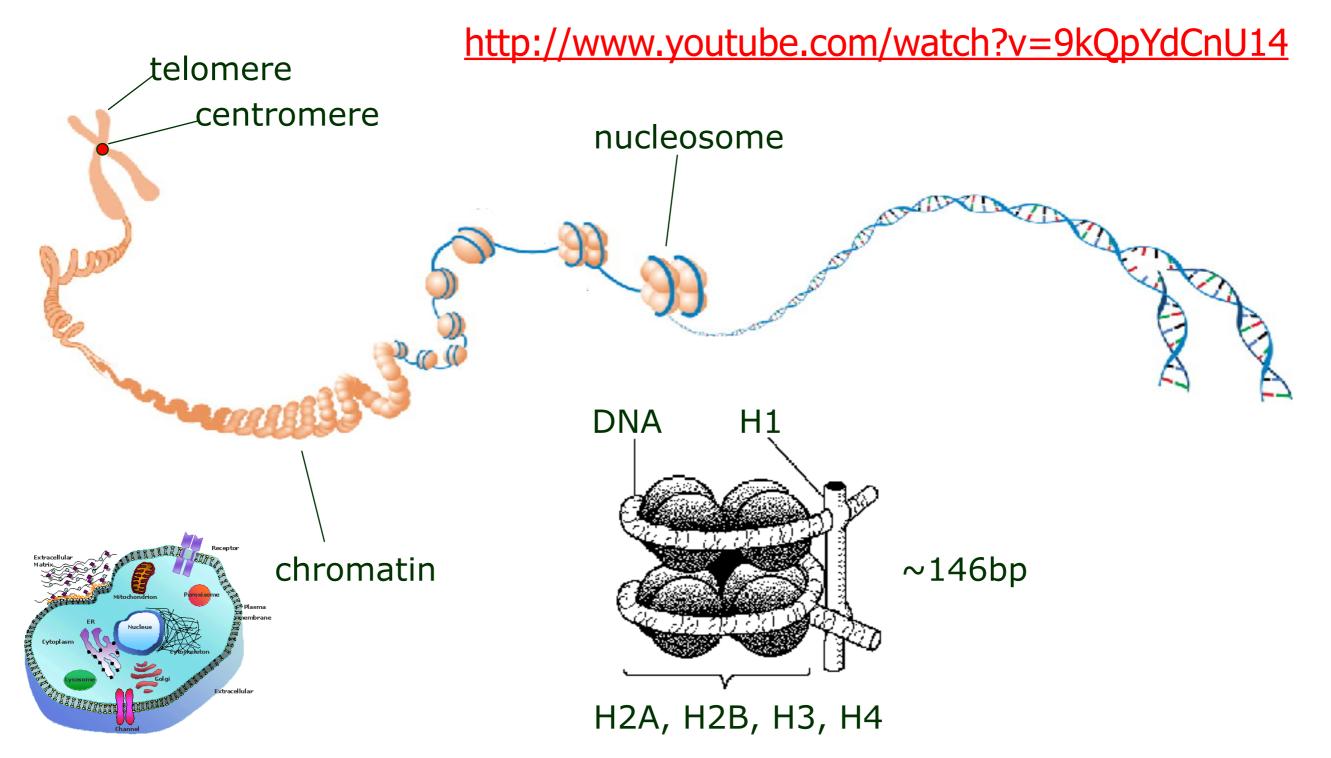
## "AGACC" (backbone)



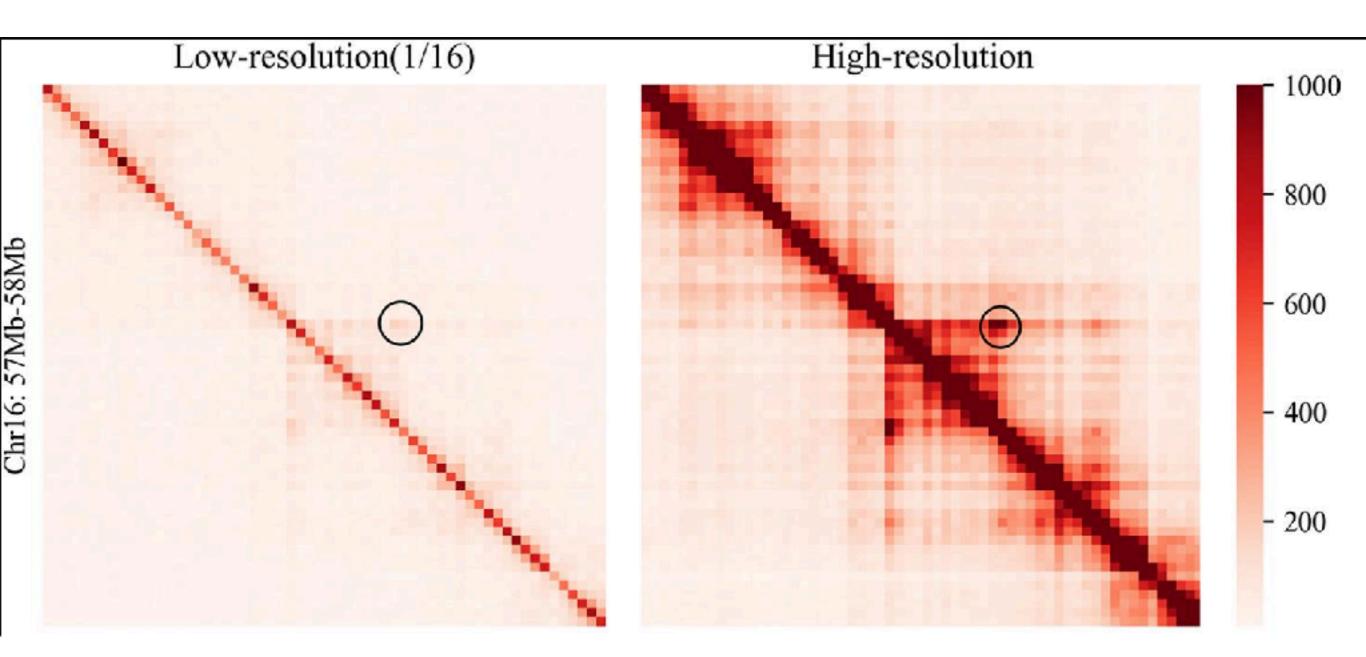


## DNA packaging (DNA is 6 feet long!)

Histone, nucleosome, chromatin, chromosome, centromere, telomere

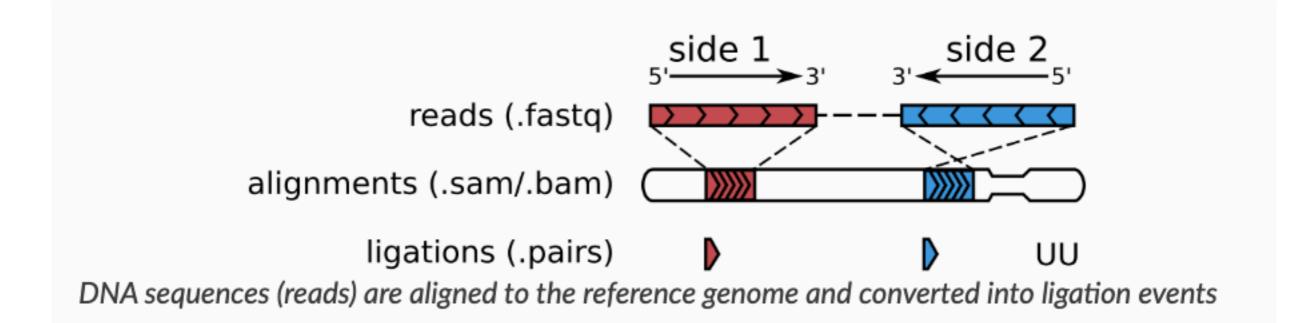


# Data structure and computational problem



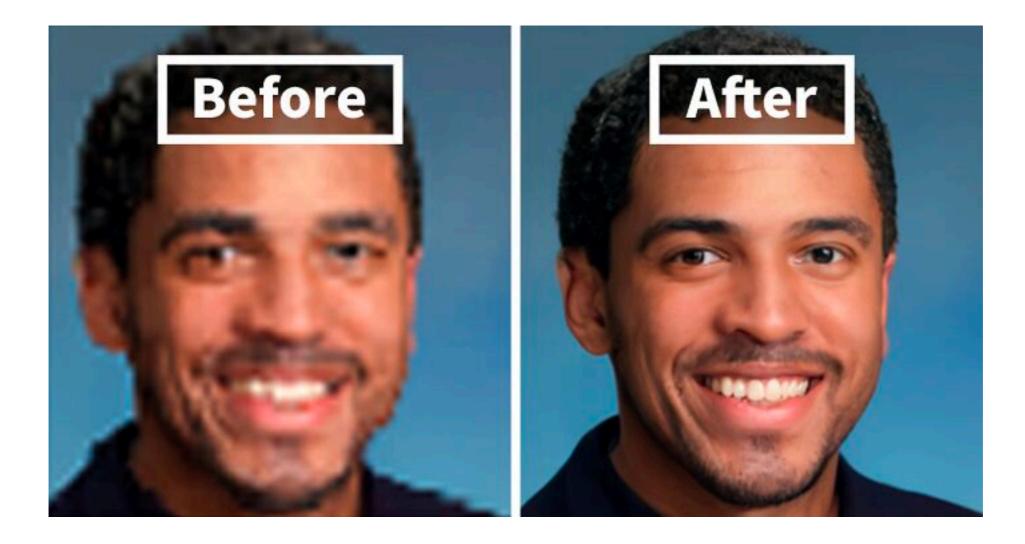
source: SRHiC: A Deep Learning Model to Enhance the Resolution of Hi-C Data

## What will the data look like? Two .fastq files. Lines correspond to each other



bowtie2 -p 20 -x hg38index -U hicExp1\_R1\_fastq.trimmed > hicExp1\_R1.hg38.sam bowtie2 -p 20 -x hg38index -U hicExp1\_R2\_fastq.trimmed > hicExp1\_R2.hg38.sam

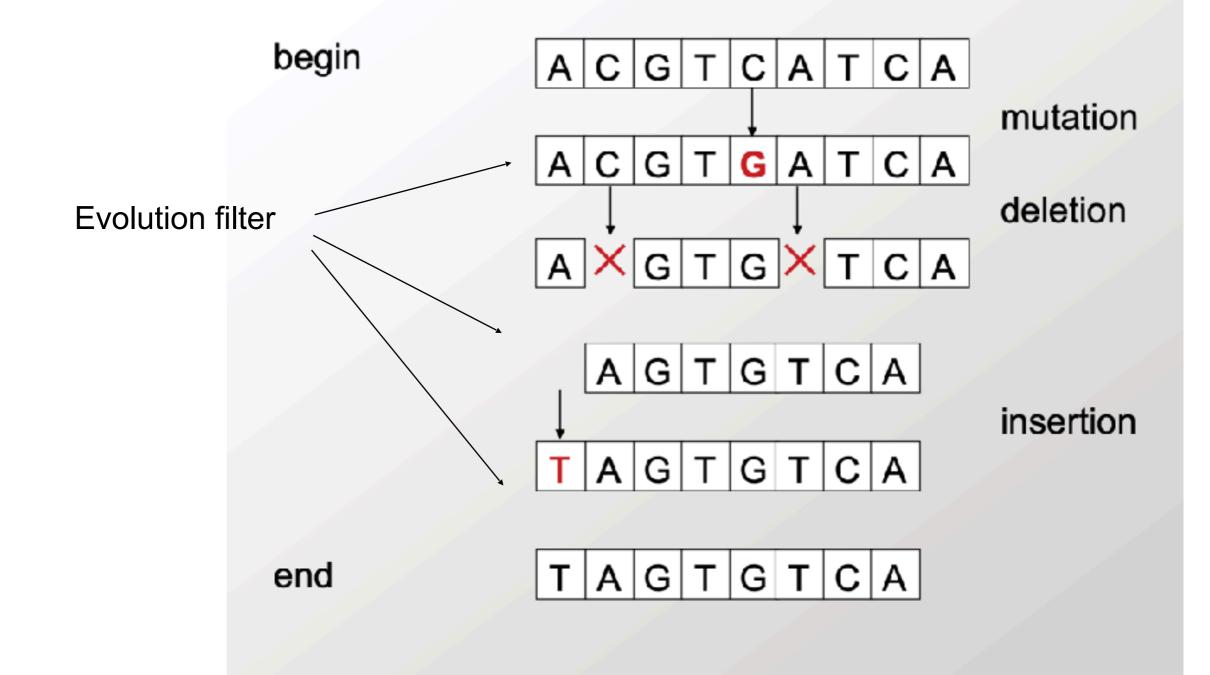
## Computer vision-based solution



source:https://www.boredpanda.com/google-ai-amazing-image-enhancement/

### Nothing in biology makes sense except in the light of evolution --Theodosius Dobzhansky

### **Genomes change over time**



### That is why we want to compare sequences

### Partial CTCF protein sequence in 8 organisms:

H .	sapiens	-EDSSDS-ENAEPDLDDNEDEEEPAVEIEPEPEPQPVTPA
Ρ.	troglodytes	-EDSSDS-ENAEPDLDDNEDEEEPAVEIEPEPEPQPVTPA
С.	lupus	-EDSSDS-ENAEPDLDDNEDEEEPAVEIEPEPEPQPVTPA
B .	taurus	-EDSSDS-ENAEPDLDDNEDEEEPAVEIEPEPEPQPVTPA
M.	musculus	-EDSSDSEENAEPDLDDNEEEEEPAVEIEPEPEPQPQPPPPPQPVAPA
R .	norvegicus	-EDSSDS-ENAEPDLDDNEEEEEPAVEIEPEPEPQPQPQPQPQPQPVAPA
G.	gallus	-EDSSDSEENAEPDLDDNEDEEETAVEIEAEPEVSAEAPA
D.	rerio	DDDDDDSDEHGEPDLDDIDEEDEDDL-LDEDQMGLLDQAPPSVPIP-APA

- Identify important sequences by finding conserved regions.
- Find genes similar to known genes.
- Understand evolutionary relationships and distances (D. rerio aka zebrafish is farther from humans than G. gallus aka chicken).
- Interface to databases of genetic sequences.
- As a step in genome assembly, and other sequence analysis tasks.
- Provide hints about protein structure and function

### That is why we want to compare sequences

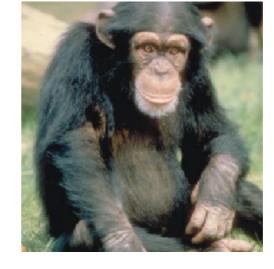
#### Partial CTCF protein sequence in 8 organisms:

- H. sapiens
- P. troglodytes
- C. lupus
- B. taurus
- M. musculus
- R. norvegicus
- G. gallus
- D. rerio

-EDSSDS-ENAEPDLDDNEDEEEPAVEIEPEPE-----PQPVTPA -EDSSDS-ENAEPDLDDNEDEEEPAVEIEPEPE----PQPVTPA -EDSSDS-ENAEPDLDDNEDEEEPAVEIEPEPE----PQPVTPA -EDSSDS-ENAEPDLDDNEEEEPAVEIEPEPE---PQPQPPPPPQPVAPA -EDSSDS-ENAEPDLDDNEEEEEPAVEIEPEPEPQPQPQPQPQPQPQPVAPA -EDSSDS-ENAEPDLDDNEEEEEPAVEIEPEPEPQPQPQPQPQPQPQPVAPA -EDSSDSEENAEPDLDDNEDEEETAVEIEAEPE----VSAEAPA DDDDDDSDEHGEPDLDDIDEEDEDDL-LDEDQMGLLDQAPPSVPIP-APA









D. rerio

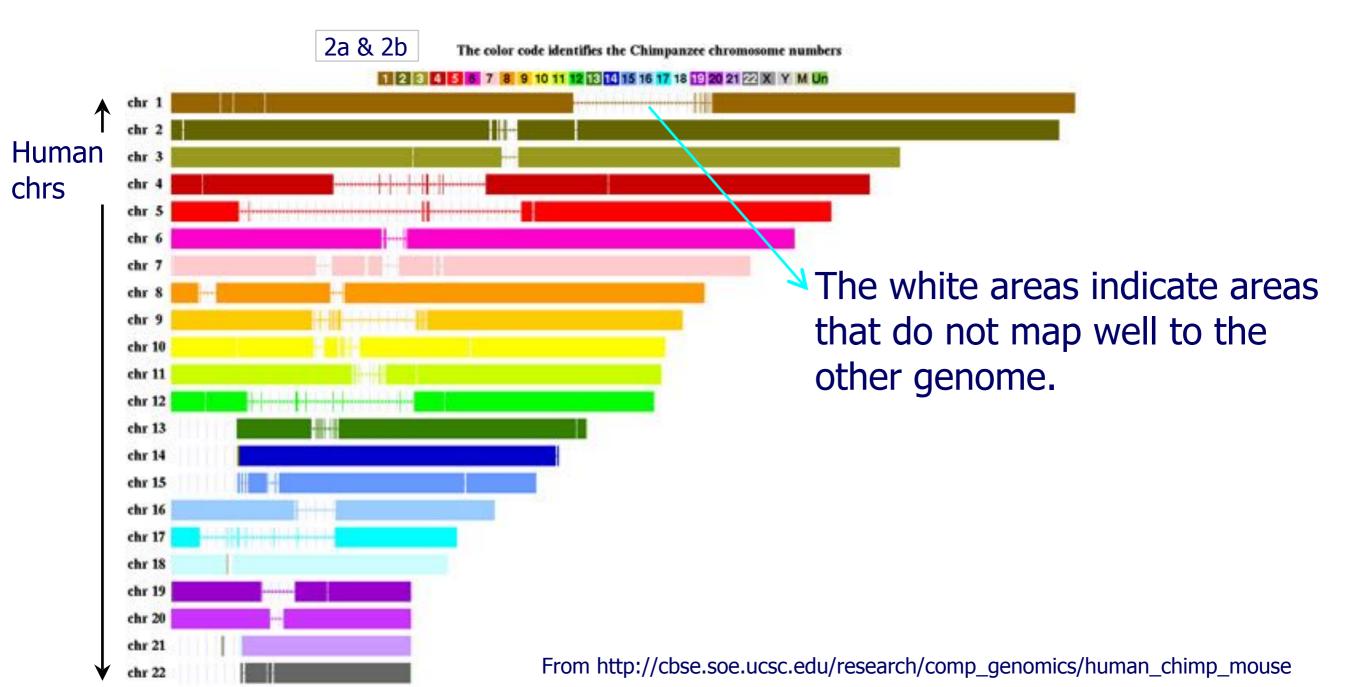
G. gallus

P. Troglodytes

C. lupus

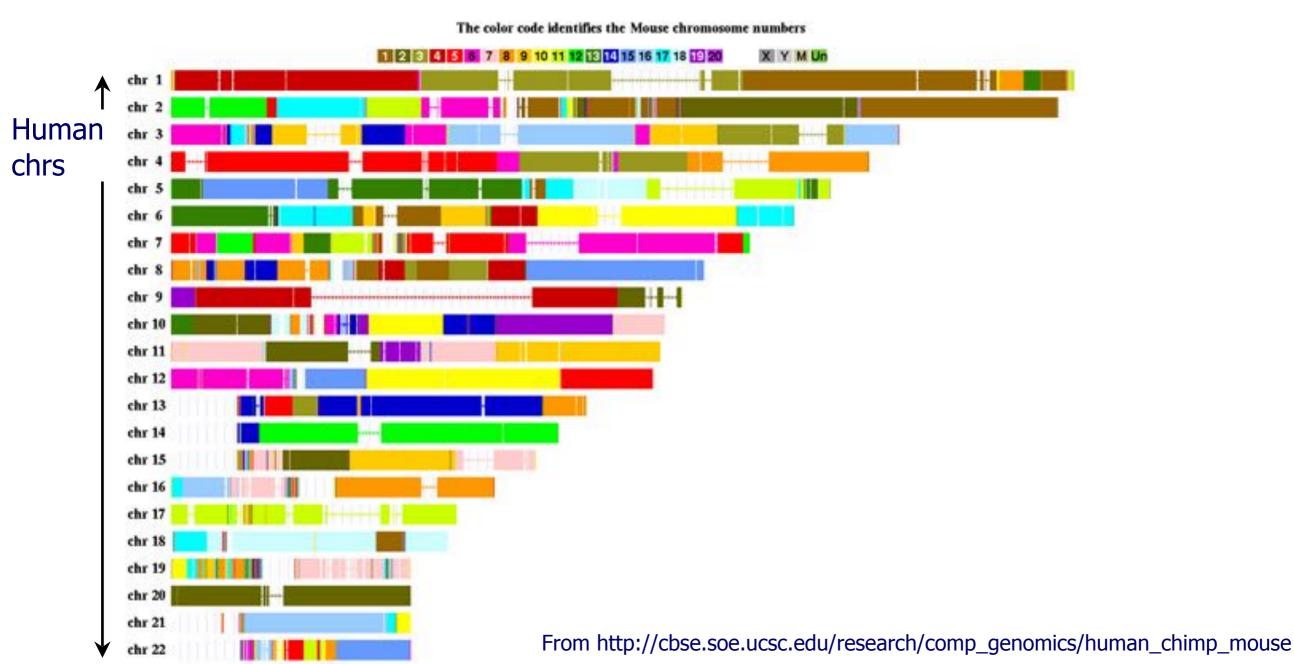
## Comparing Human, Chimp, and Mouse Genomes

95% of the chimp genome is mapped to identical sequence in the human genome.

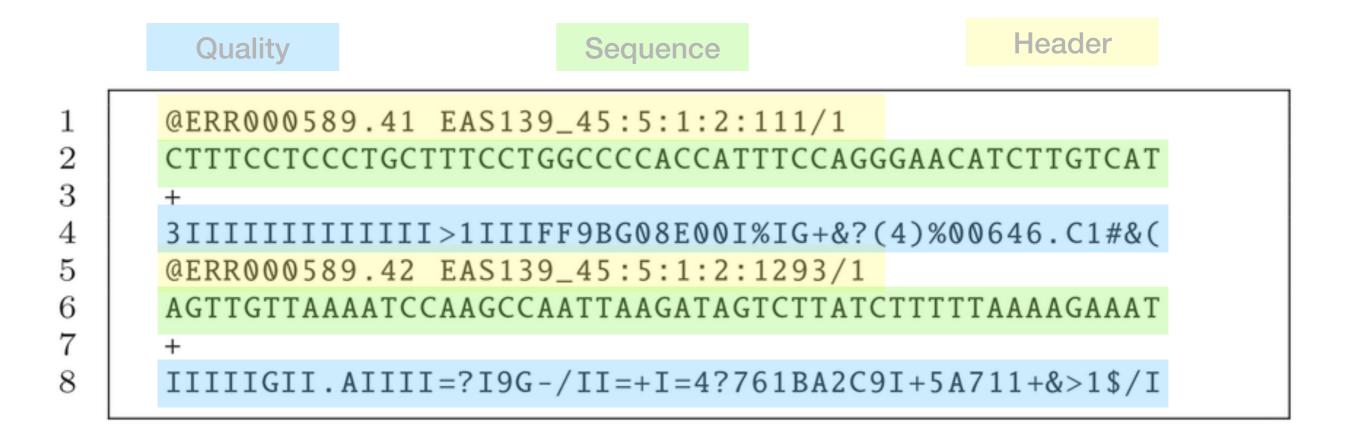


## Comparing Human, Chimp, and Mouse Genomes

 34% of the mouse genome is mapped to identical sequence in the human genome.



## What does a fastq file look like?



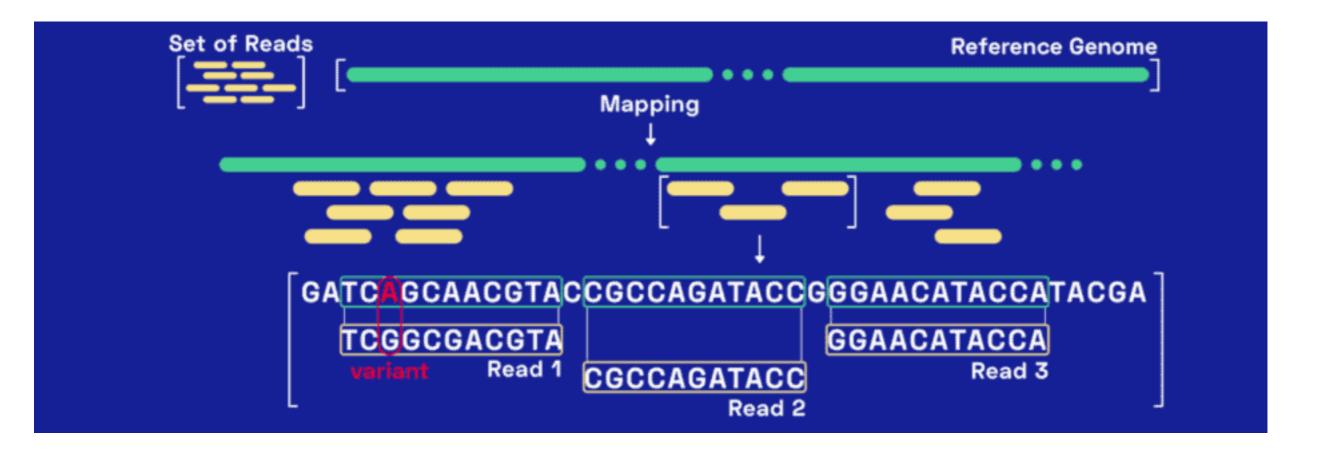
Very large! ~30000000 lines Quality: ASCII chars

What should we do? Map each short sequence (we call it read) to the entire human genome

## What does a fastq file look like?

Reference genome: "average" human genome.

Most widely used human genome GRCh38: derived from 13 thirteen anonymous volunteers



## The Simplest String Comparison Problem

```
Given: Two strings
```

 $a = a_1 a_2 a_3 a_4 \dots a_m$  $b = b_1 b_2 b_3 b_4 \dots b_n$ 

where *a<sub>i</sub>*, *b<sub>i</sub>* are letters from some alphabet like {A,C,G,T}.

**Compute** how similar the two strings are.

#### What do we mean by "similar"?

**Edit distance** between strings *a* and *b* = the smallest number of the following operations that are needed to transform *a* into *b*:

riddle  $\xrightarrow{\text{delete}}$  ridle  $\xrightarrow{\text{mutate}}$  riple  $\xrightarrow{\text{insert}}$ 

triple

- mutate (replace) a character
- delete a character
- insert a character

#### Dynamic Programming (DP)

- Dynamic programming is used to solve optimization problems, similar to greedy algorithms.
- DP problem can always be decomposed to a series of subproblems with the same structure.
  - Define proper subproblems.
  - Ensure the subproblem space is polynomial.
  - Define a table (matrix), called DP table, to store all the optimal score for each subproblem.
  - Need a traversal order. Subproblems must be ready (solved) when they are needed, so computation order matters.
  - Determine a recursive formula: A larger subproblem is typically solved as a function of its subparts.
  - Remember choices or the solution of each subproblem.

#### Dynamic Programming (DP)

- Once dynamic programming is setup, computation is typically straight-forward:
  - Systematically fill in the table of results (and usually traceback pointers) and find an optimal score.
  - Traceback from the optimal score through the pointers to determine an optimal solution.

- Example: Fibonacci Numbers
  - The Fibonacci sequence is recursively defined as F(0) = F(1) = 1, F(n) = F(n-1) + F(n-2) for  $n \ge 2$ .

#### Local and Global Alignment

Sometimes we need to choose whether we want to align the entire sequence.

А	Т	Α	С	G	Т	С	Т

 A
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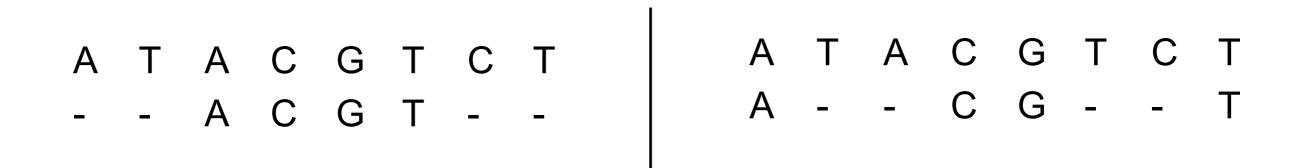
Local alignment: Smith-Waterman algorithm

Global alignment: Needleman-Wunsh algorithm

- They both contain four align positions and four gaps. Which one should we choose?
- Criteria
  - Do we want to check the whole sequence or a local region?
  - Is there a big length difference between two sequences?
  - Are the sequences distantly related during evolution?
  - Is your job about finding motifs, conserved domains?

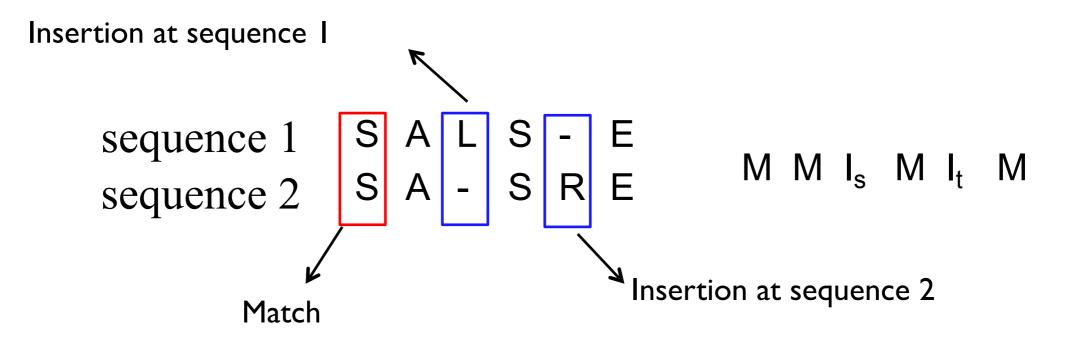
### Key difference

Sometimes we need to choose whether we want to align the entire sequence.



We don't want to punish the gap at the two ends!

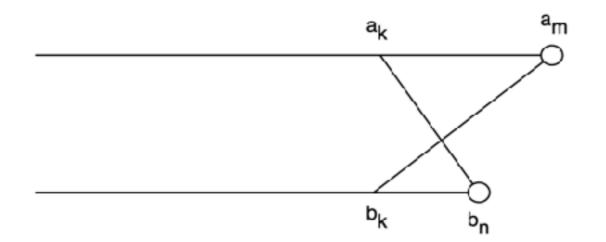
#### We need to assign a score for each alignment



The score of an alignment is equal to the sum of the score contributed by each position.

Several rules must hold:

- Each position on sequence I can only be aligned to one position on sequence 2
- No crossing rule:



## Sequence alignment

AGGCTATCACCTGACCTCCAGGCCGATGCCC TAGCTATCACGACCGCGGTCGATTTGCCCGAC

-AGGCTATCACCTGACCTCCAGGCCGA--TGCCC---TAG-CTATCAC--GACCGC--GGTCGATTTGCCCGAC

## What is a good alignment?

AGGCTAGTT, AGCGAAGTTT

AGGCTAGTT-AGCGAAGTTT 6 matches, 3 mismatches, 1 gap

AGGCTA-GTT-AG-CGAAGTTT 7 matches, 1 mismatch, 3 gaps

AGGC-TA-GTT-AG-CG-AAGTTT 7 matches, 0 mismatches, 5 gaps

# **Scoring Function**

<ul> <li>Sequence ed</li> </ul>	its:	AGGCCTC	
<ul> <li>Mutations</li> </ul>		AGGACTC	
<ul> <li>Insertions</li> </ul>		AGGGCCTC	
<ul> <li>Deletions</li> </ul>		AGG . CTC	
			Alternative definition:
			minimal edit distance
Scoring Func	tion:		
Match:	+m		"Given two strings x, y,
Mismatch:	-S		find minimum # of edits (insertions, deletions,
Gap:	-d		mutations) to transform one string to the other"

Score  $F = (\# matches) \times m - (\# mismatches) \times s - (\# gaps) \times d$ 

# How do we compute the best alignment?

Y

 $\rightarrow$  N bps AGTGCCCTGGAACCCTGACGGTGGGTCACAAAACTTCTGGA 12 GTGACCTGGGAAGACCCTGACCCTGGG CACAAAA OTO OTO

Every non-decreasing path from (0,0) to (M, N) corresponds to an alignment of the two sequences, and vice versa.

(exercise)

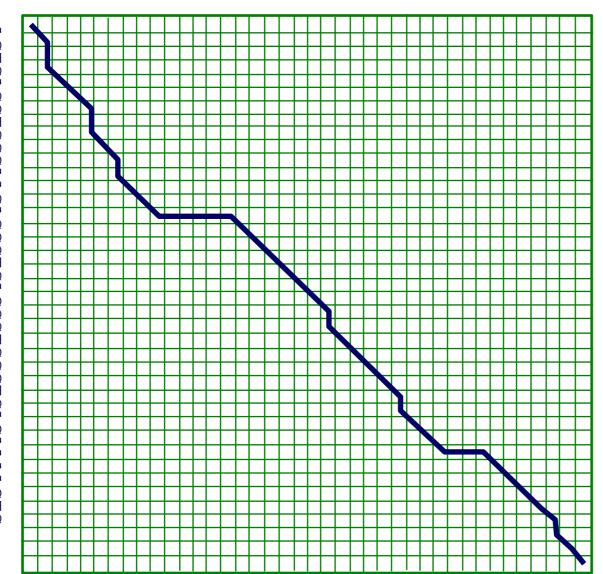
X:AGTGACCTGGGAAGA----C... Y:AG--TGC--CC-TGGAACCCT...

M base pairs (bps)

X

# How do we compute the best alignment?

AGTGACCTGGGAAGACCCTGACCCTGGGTCACAAAACTC



AGTGCCCTGGAACCCTGACGGTGGGTCACAAAACTTCTGGA

Too many possible alignments:

>> 3min(M,N)

## Alignment is additive

**Observation:** 

The score of aligning	<b>x</b> <sub>1</sub> .	X <sub>M</sub>
is additive	yı.	y <sub>N</sub>
Say that	<b>x</b> <sub>1</sub> <b>x</b> <sub>i</sub>	x <sub>i+1</sub> x <sub>M</sub>
aligns to	$\mathbf{y}_1 \dots \mathbf{y}_j$	<b>у</b> <sub>j+1</sub> у <sub>N</sub>

The two scores add up:

F(x[1:M], y[1:N]) = F(x[1:i], y[1:j]) + F(x[i+1:M], y[j+1:N])

# Dynamic Programming

- Consider subproblems for  $i \le M$  and  $j \le N$ 
  - Align  $x_1...x_i$  to  $y_1...y_j$
- Original problem is one of the subproblems
  - Align  $x_1...x_M$  to  $y_1...y_N$
- Each subproblem is easily solved from smaller subproblems
  We will show next
- Then, we can apply Dynamic Programming!!!

Let F(i, j) = optimal score of aligning $x_1....x_i$  $y_1....y_j$ 

F is the DP "Matrix" or "Table"

"Memorization"

# **Scoring Function**

<ul> <li>Sequence ed</li> </ul>	its:	AGGCCTC	
<ul> <li>Mutations</li> </ul>		AGGACTC	
<ul> <li>Insertions</li> </ul>		AGGGCCTC	
<ul> <li>Deletions</li> </ul>		AGG . CTC	
			Alternative definition:
			minimal edit distance
Scoring Func	tion:		
Match:	+m		"Given two strings x, y,
Mismatch:	-S		find minimum # of edits (insertions, deletions,
Gap:	-d		mutations) to transform one string to the other"

Score  $F = (\# matches) \times m - (\# mismatches) \times s - (\# gaps) \times d$ 

# Dynamic Programming (cont'd)

Notice three possible cases:

I.  $x_i$  aligns to  $y_i$  $X_1 \dots X_{i-1} \quad X_i$  $y_1, \dots, y_{j-1}, y_j$ 

$$F(i, j) = F(i - 1, j - 1) + \begin{cases} m, \text{ if } x_i = y_j \\ -s, \text{ if not} \end{cases}$$

1

- 2.  $x_i$  aligns to a gap  $X_1 \dots X_{i-1} \quad X_i$ y<sub>1</sub>.....y<sub>i</sub> -
- 3. y<sub>i</sub> aligns to a gap X<sub>1</sub>.....X<sub>i</sub>  $y_1, \dots, y_{j-1}, y_j$

$$F(i, j) = F(i - 1, j) - d$$

$$F(i, j) = F(i, j - 1) - d$$

# Dynamic Programming (cont'd)

How do we know which case is correct?

Inductive assumption:

F(i, j - 1), F(i - 1, j), F(i - 1, j - 1) are optimal

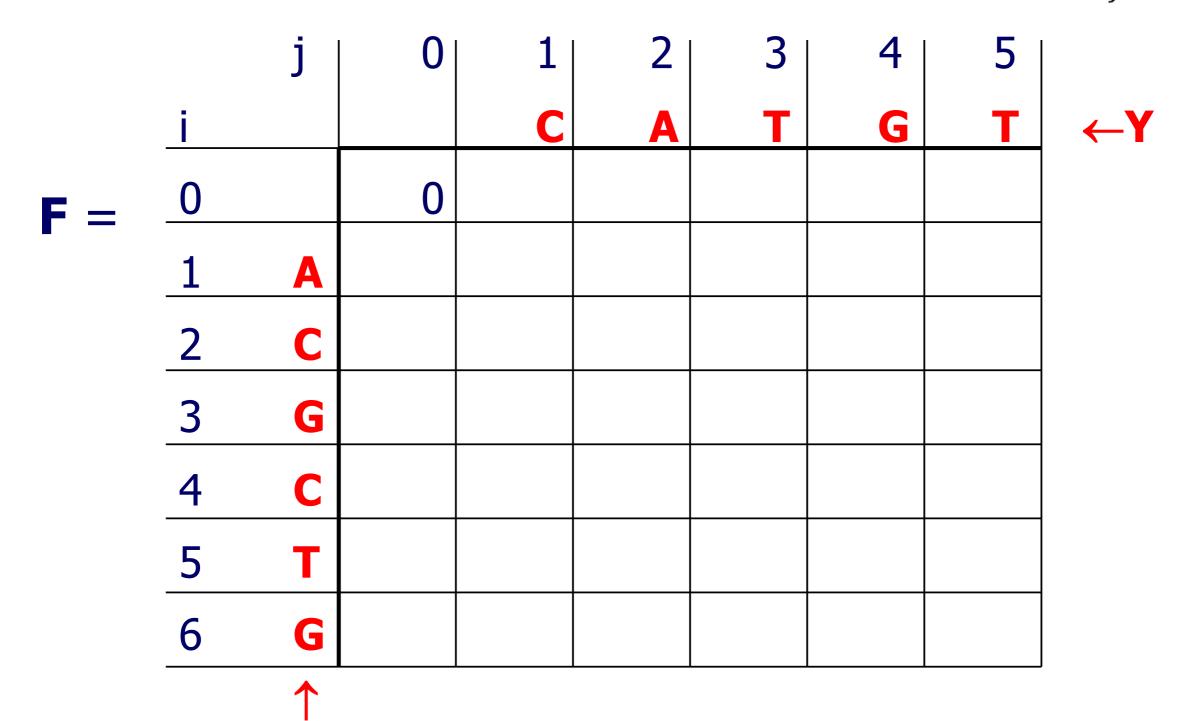
Then,

F(i, j) = max 
$$\begin{cases} F(i - 1, j - 1) + s(x_i, y_j) \\ F(i - 1, j) - d \\ F(i, j - 1) - d \end{cases}$$

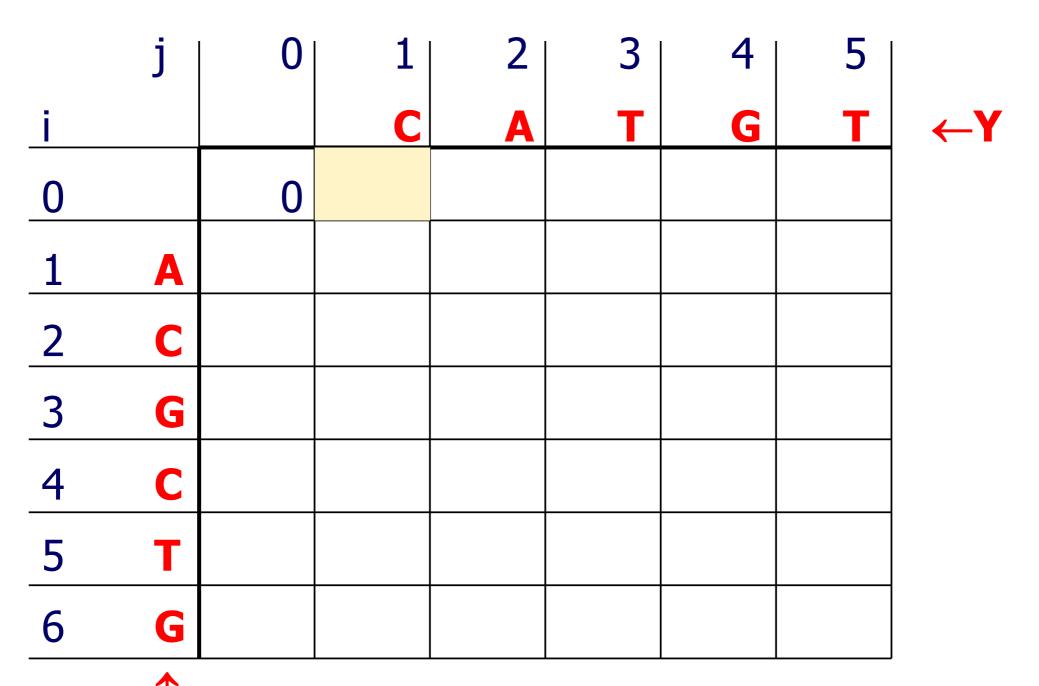
where

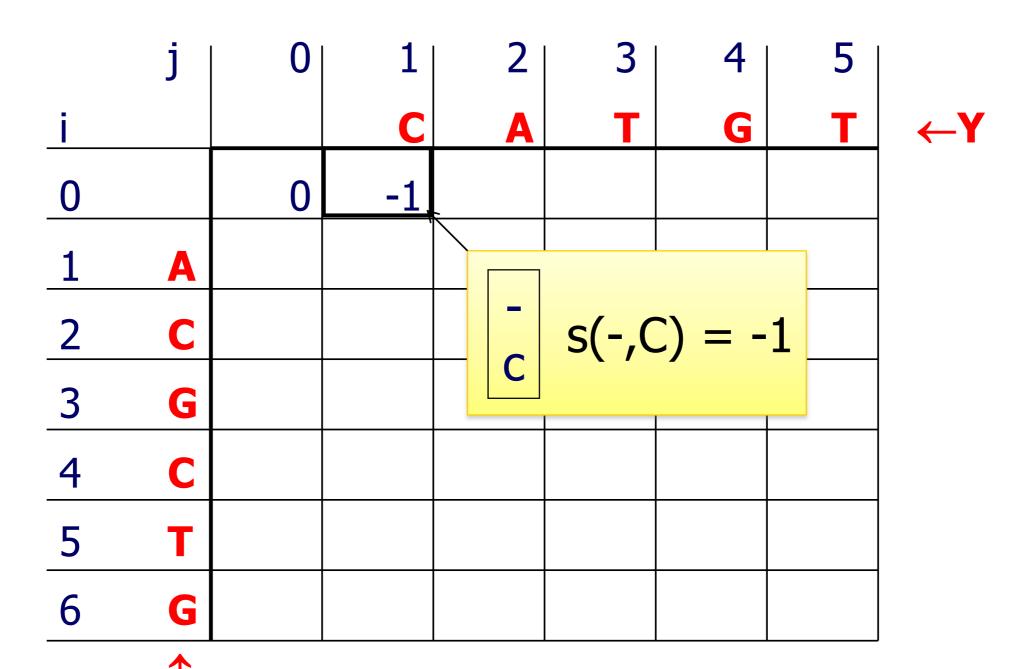
 $s(x_i, y_j) = \begin{cases} m, \text{ if } x_i = y_j \\ -s, \text{ if not} \end{cases}$ 

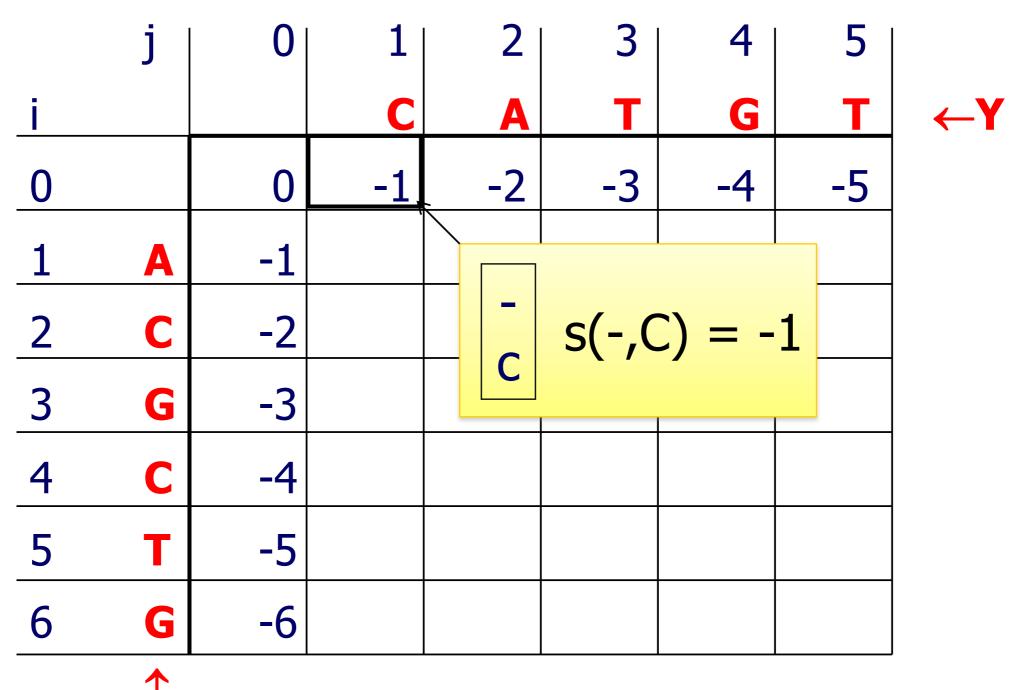
F(*i*, *j*) = optimal score of aligning  $x_1, ..., x_i$  to  $y_1, ..., y_j$ 



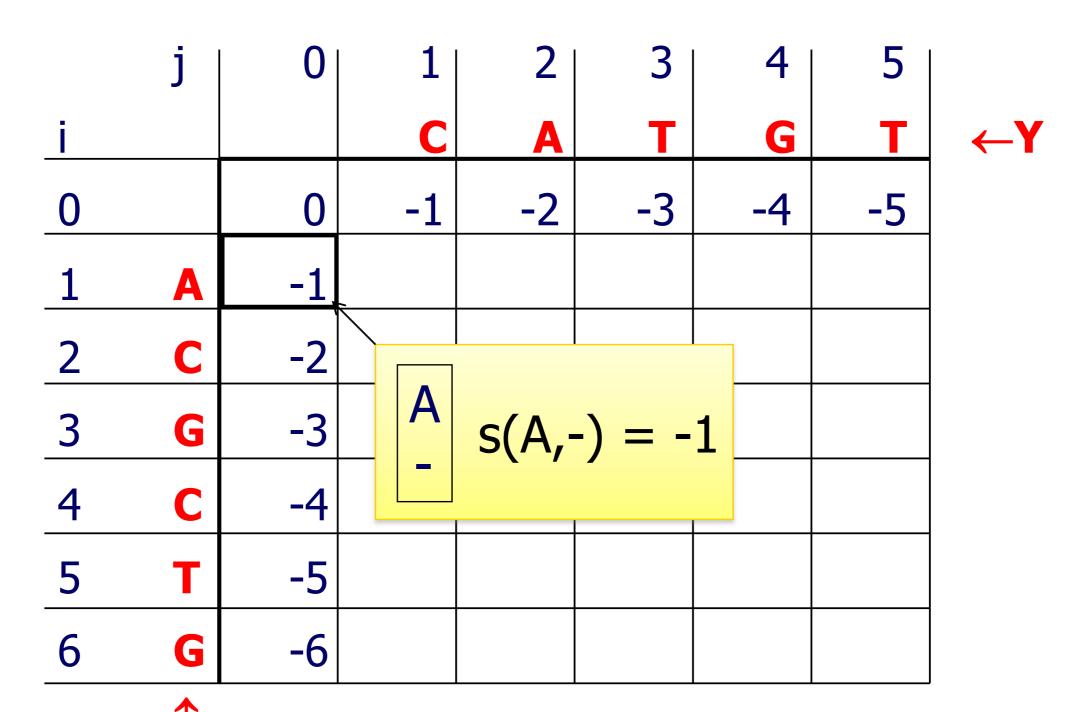
x = ACGCTGmatch: +2 mismatch, gap: -1 y = CATGT



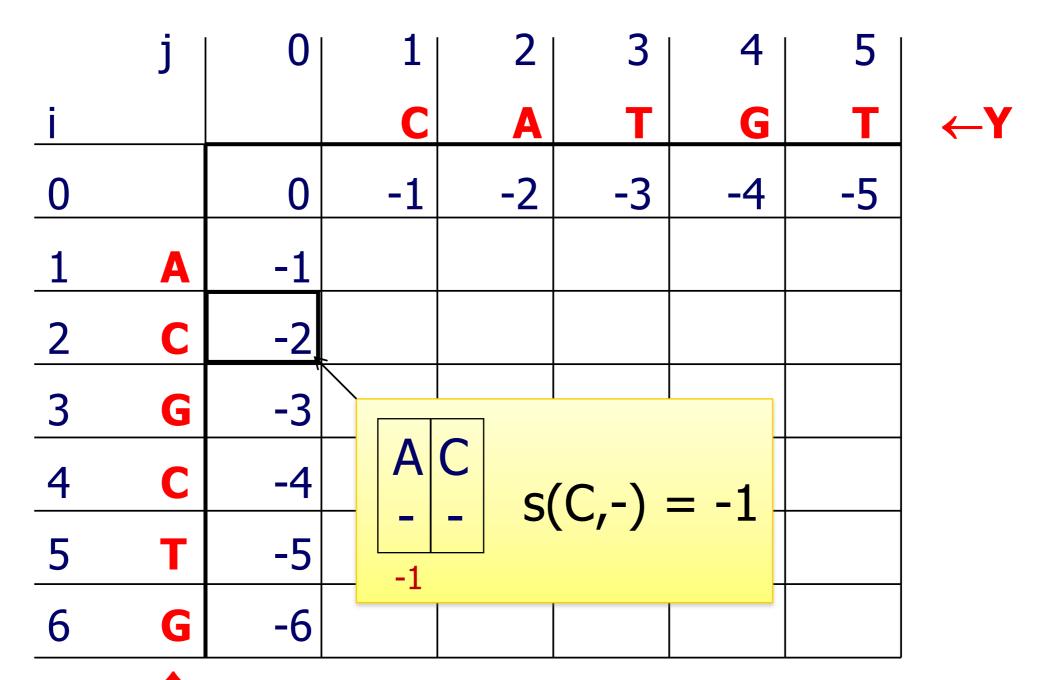


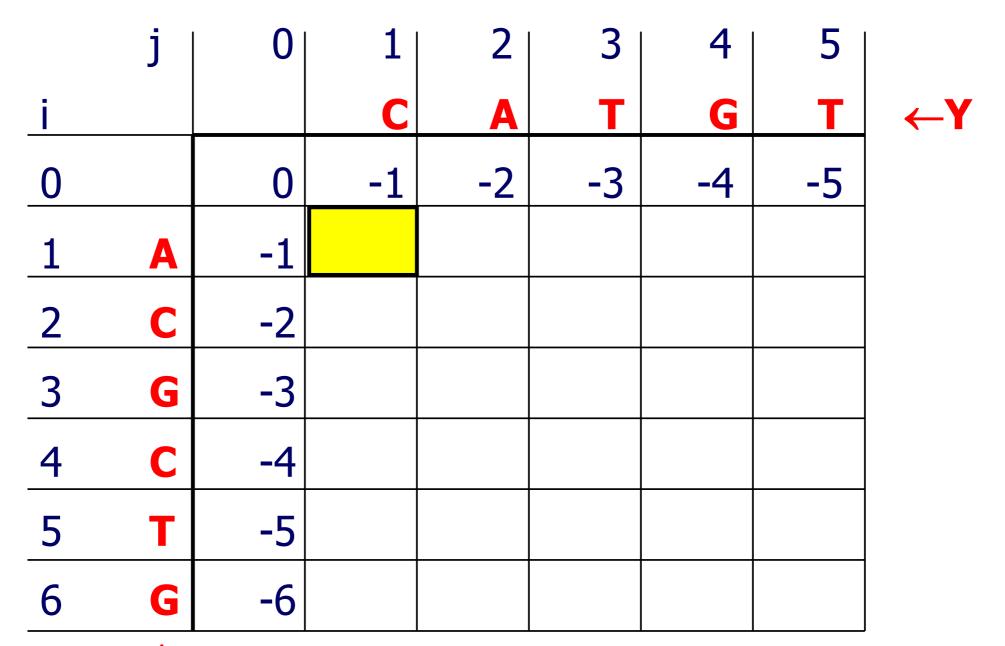


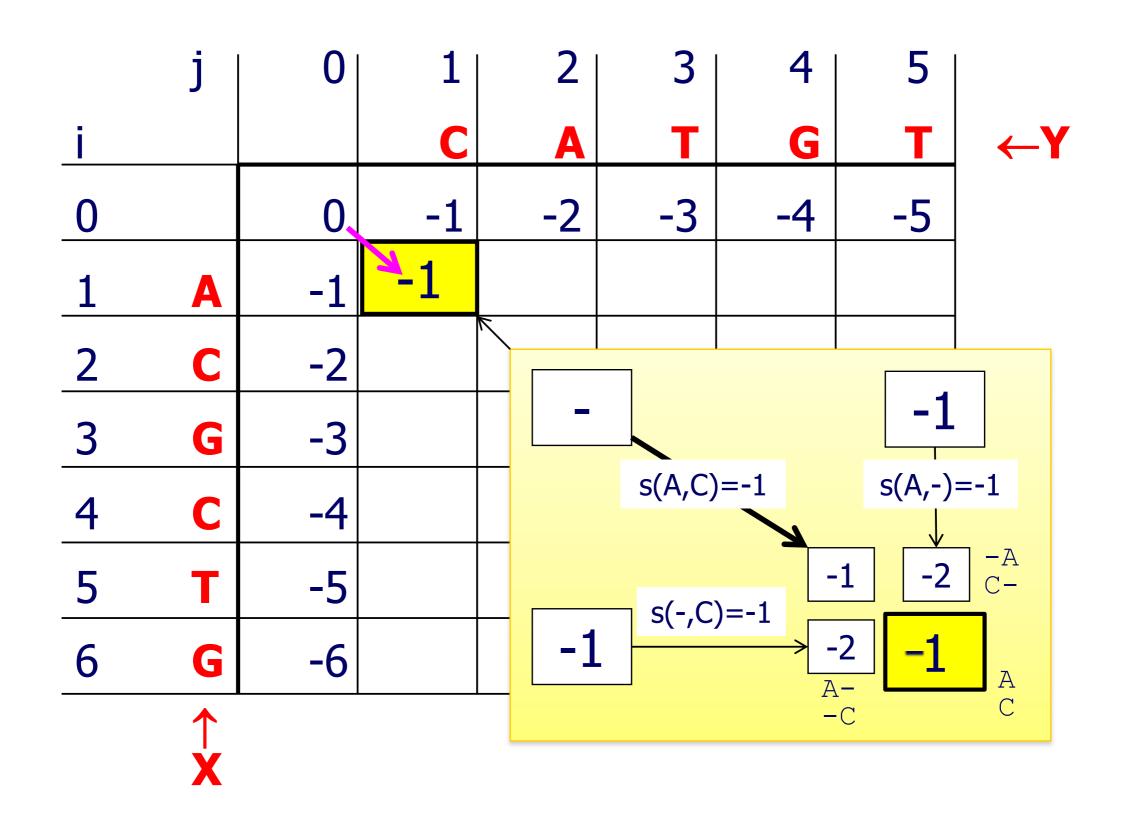
x = ACGCTG	match: +2	2
y = CATGT	mismatch, ga	ap: -1

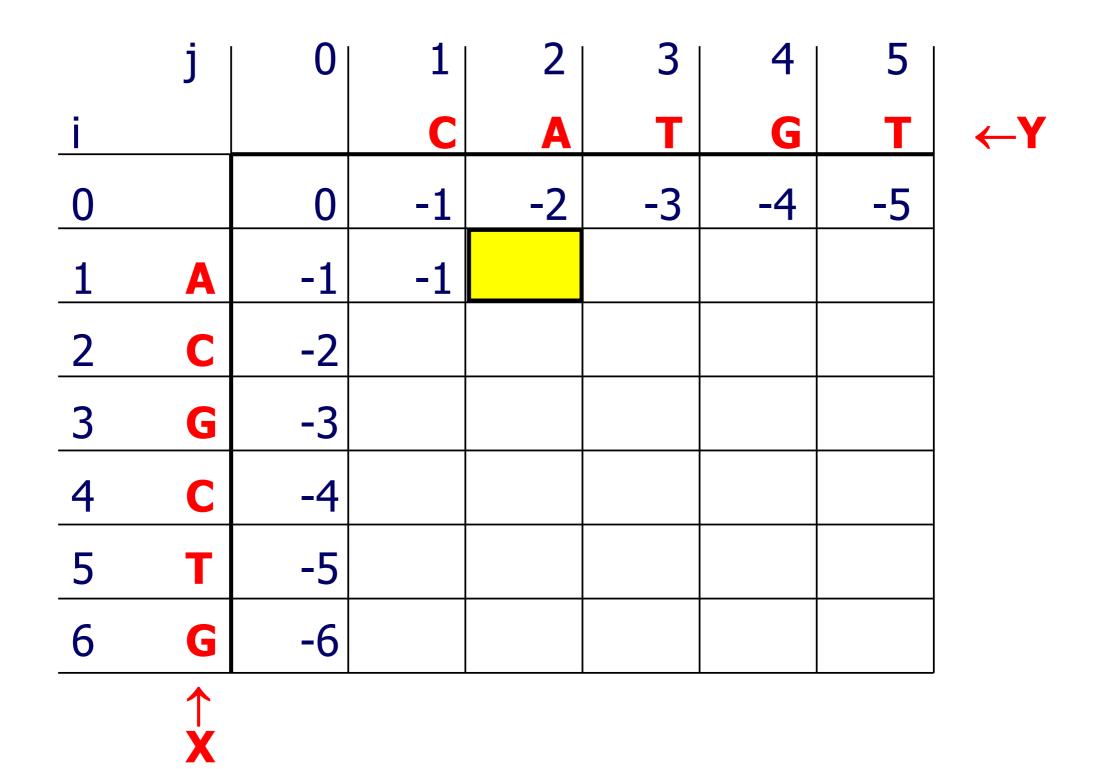


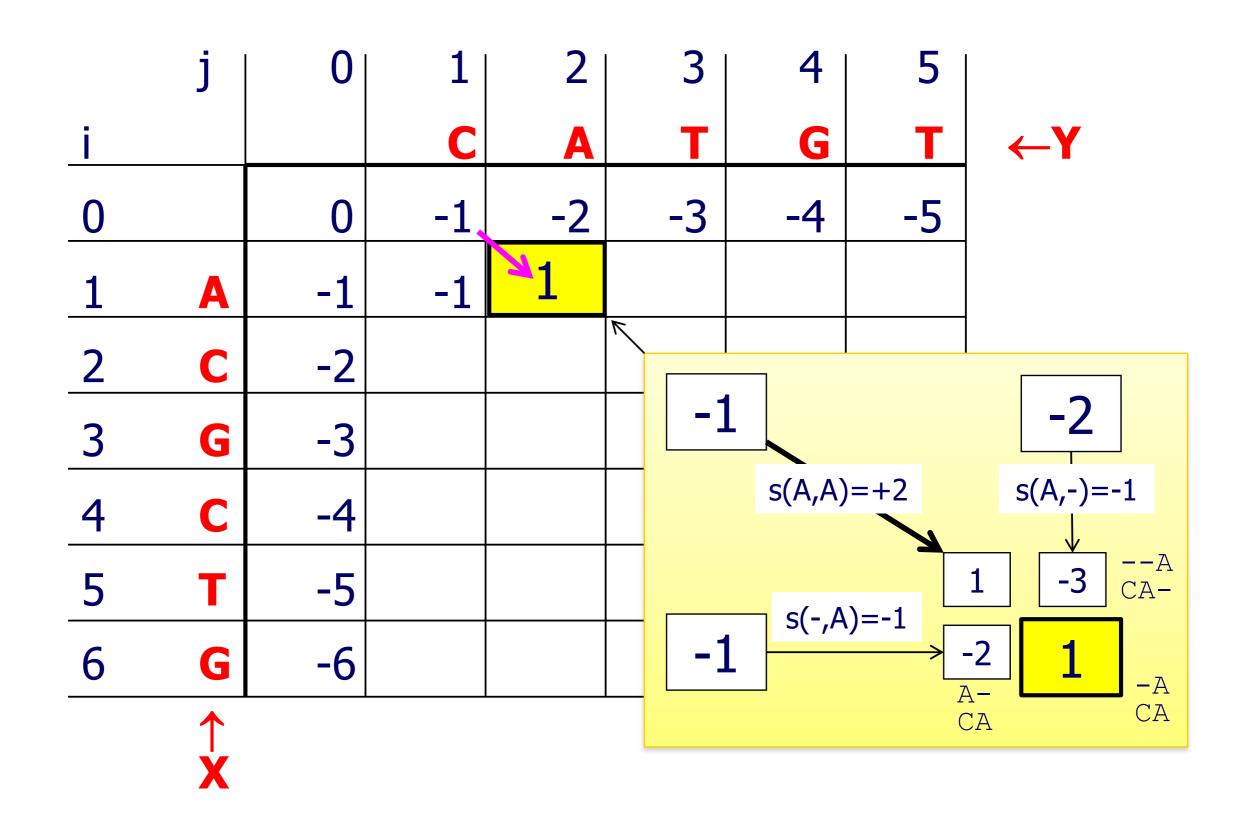
x = ACGCTG	match:	+2
y = CATGT	mismatch	, gap: -1

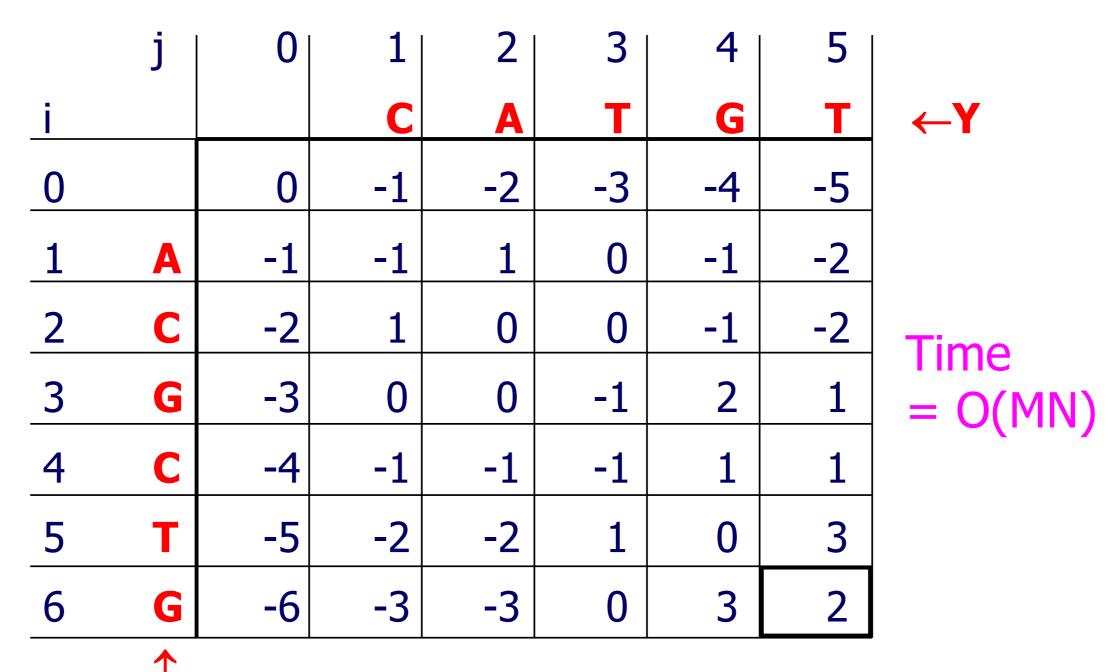






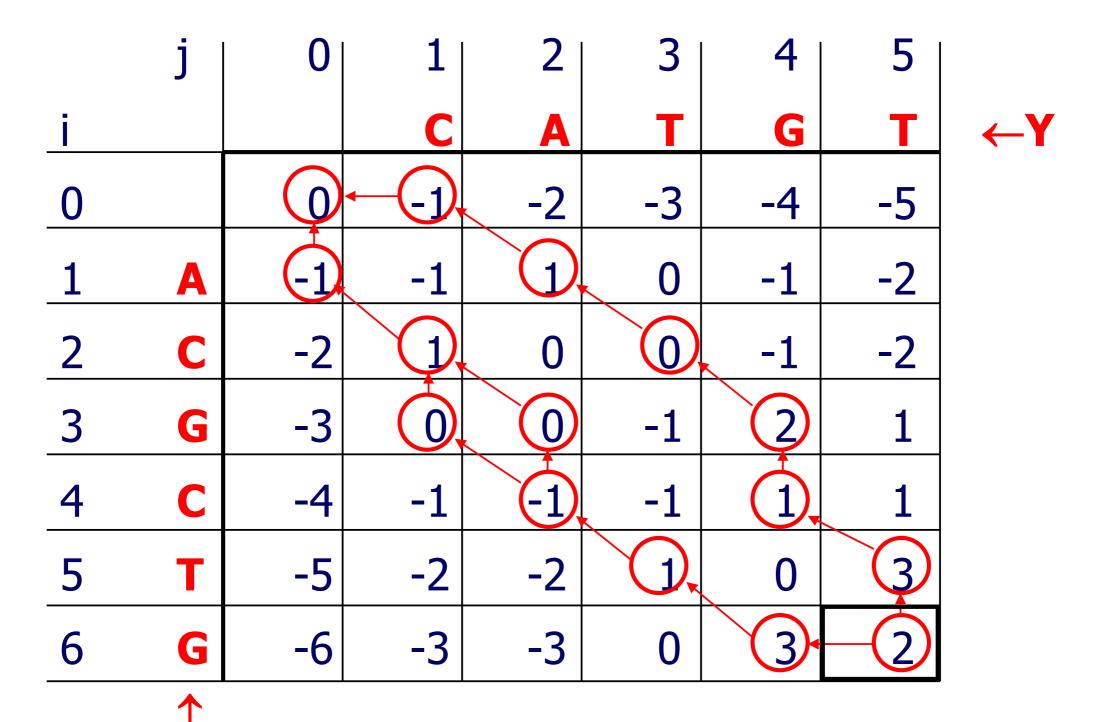




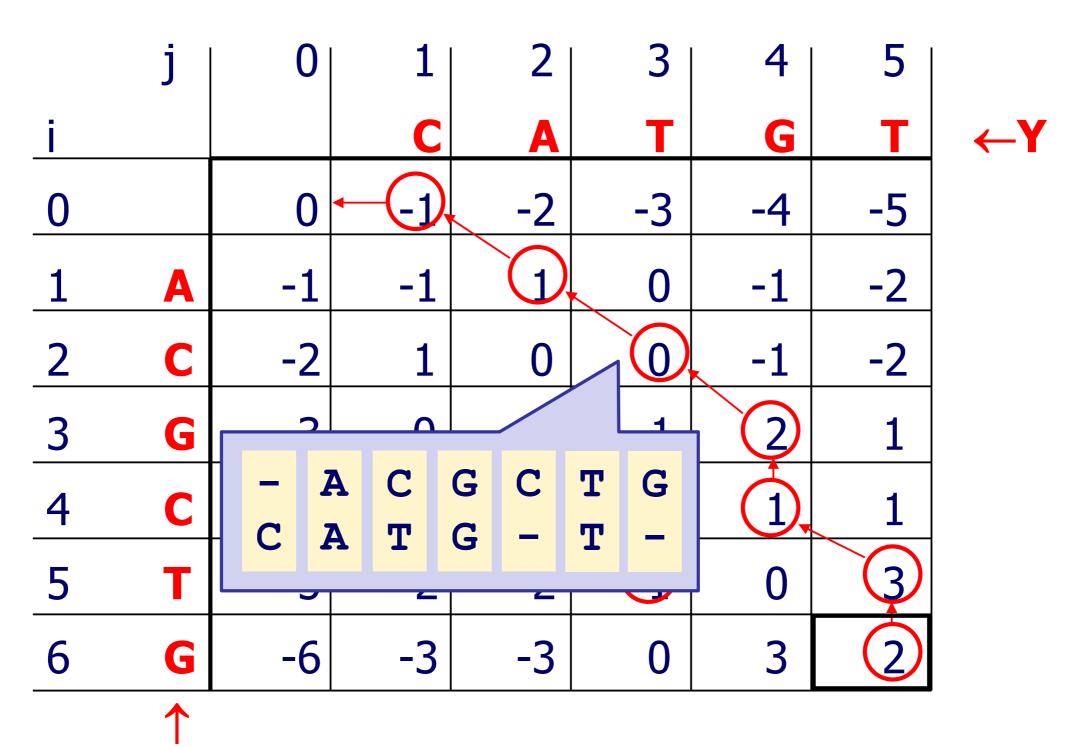


# Finding alignments: trace back

Arrows = (ties for) max in F(i,j); 3 LR-to-UL paths = 3 optimal alignments



# Finding alignments: trace back



X

## The Needleman-Wunsch Algorithm

- 1. <u>Initialization</u>.
  - a. F(0, 0) = 0b.  $F(0, j) = -j \times d$
  - c.  $F(i, 0) = -i \times d$
- 2. <u>Main Iteration.</u> Filling-in partial alignments
  - For each i = 1....NFor each j = 1....N  $F(i, j) = \max \begin{cases} F(i - 1, j - 1) + s(x_i, y_j) & [case 1] \\ F(i - 1, j) - d & [case 2] \\ F(i, j - 1) - d & [case 3] \end{cases}$  $Ptr(i, j) = \begin{cases} DIAG, & if & [case 1] \\ UP, & if & [case 2] \\ LEFT, & if & [case 3] \end{cases}$
- 3. <u>Termination</u>. F(M, N) is the optimal score, and from Ptr(M, N) can trace back optimal alignment

### **Global Alignment**

#### Needleman-Wunsch algorithm

Initialization:

F(0, 0) = 0

Iteration:

F(i, j) = max

$$\begin{cases}
F(i - 1, j) - d \\
F(i, j - 1) - d \\
F(i - 1, j - 1) + s(x_i, y_j)
\end{cases}$$

Termination:

Bottom right

Termination:

Anywhere

Local alignment

#### Smith-Waterman algorithm

Initialization:

Iteration:

VS.

 $F(i, j) = max \begin{cases} F(i - 1, j) - d \\ F(i, j - 1) - d \\ F(i - 1, j - 1) + s(x_i, y_j) \end{cases}$ 

F(0, j) = F(i, 0) = 0

## Performance

Time:

O(NM)

Space:

O(NM)

### **Global Alignment**

#### Needleman-Wunsch algorithm

Initialization:

F(0, 0) = 0

Iteration:

F(i, j) = max

$$\begin{cases}
F(i - 1, j) - d \\
F(i, j - 1) - d \\
F(i - 1, j - 1) + s(x_i, y_j)
\end{cases}$$

Termination:

Bottom right

Termination:

Anywhere

Local alignment

#### Smith-Waterman algorithm

Initialization:

Iteration:

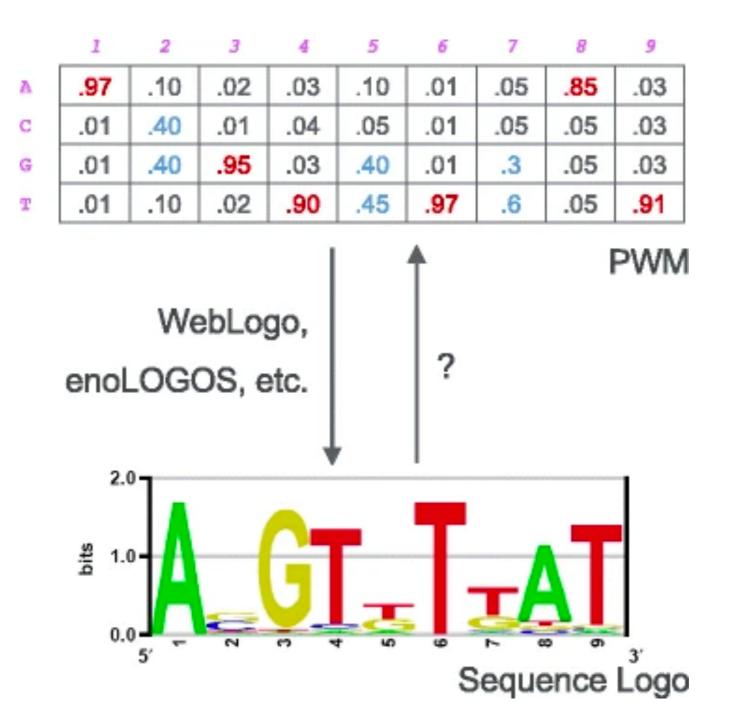
VS.

 $F(i, j) = max \begin{cases} F(i - 1, j) - d \\ F(i, j - 1) - d \\ F(i - 1, j - 1) + s(x_i, y_j) \end{cases}$ 

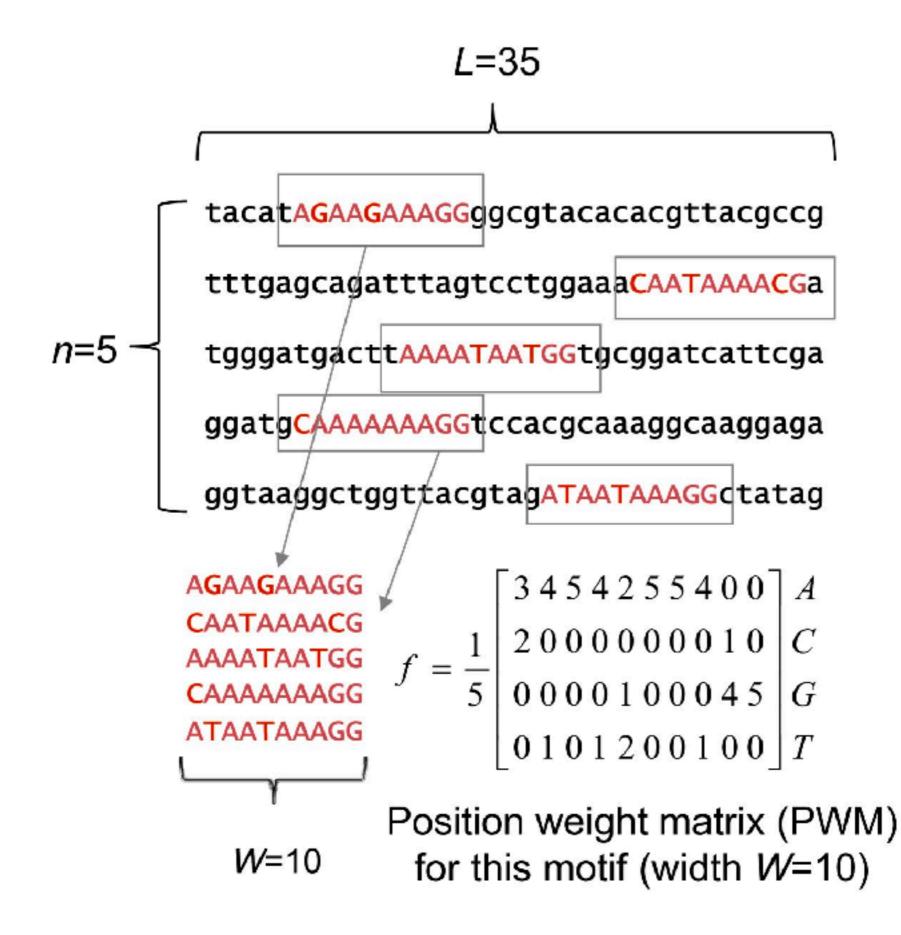
F(0, j) = F(i, 0) = 0

- What if we only penalize the gap at the beginning
- What if we only penalize the gap at the end

# Motif: probabilistic representation of a sequence



https://bmcgenomics.biomedcentral.com/articles/10.1186/s12864-017-4023-9/figures/1



For example, given the following DNA sequences:

GAGGTAAAC	
TCCGTAAGT	
CAGGTTGGA	
ACAGTCAGT	
TAGGTCATT	
TAGGTACTG	
ATGGTAACT	
CAGGTATAC	
TGTGTGAGT	
AAGGTAAGT	

The corresponding PFM is:

 $M = egin{array}{c} A \ C \ G \ G \ T \ 1 \ 1 \ 7 \ 10 \ 0 \ 1 \ 1 \ 2 \ 6 \ \end{array} egin{array}{c} 0 & 0 & 6 & 7 & 2 & 1 \ 2 & 2 & 1 & 0 & 0 & 2 & 1 & 1 & 2 \ 1 & 1 & 7 & 10 & 0 & 1 & 1 & 5 & 1 \ 4 & 1 & 1 & 0 & 10 & 1 & 1 & 2 & 6 \ \end{array} egin{array}{c} . \end{array}$ 

Therefore, the resulting PPM is:[1]

$$M = \begin{bmatrix} A \\ C \\ G \\ T \end{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.1 & 0.0 & 0.0 & 0.6 & 0.7 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0.1 & 0.0 & 0.0 & 0.2 & 0.1 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.7 & 1.0 & 0.0 & 0.1 & 0.1 & 0.5 & 0.1 \\ 0.4 & 0.1 & 0.1 & 0.0 & 1.0 & 0.1 & 0.1 & 0.2 & 0.6 \end{bmatrix}$$

https://en.wikipedia.org/wiki/Position\_weight\_matrix#:~:text=A%20position%20weight%20matrix%20(PWM,represented%20graphically%20as%20sequence%20logos.

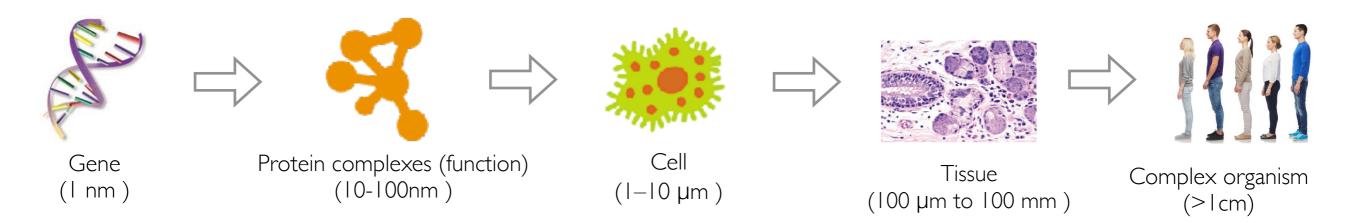
$$M = \begin{bmatrix} A \\ C \\ G \\ T \end{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.1 & 0.0 & 0.0 & 0.6 & 0.7 & 0.2 & 0.1 \\ 0.2 & 0.2 & 0.1 & 0.0 & 0.0 & 0.2 & 0.1 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.7 & 1.0 & 0.0 & 0.1 & 0.1 & 0.5 & 0.1 \\ 0.4 & 0.1 & 0.1 & 0.0 & 1.0 & 0.1 & 0.1 & 0.2 & 0.6 \end{bmatrix}.$$

the probability of the sequence S = GAGGTAAAC given the above PPM M

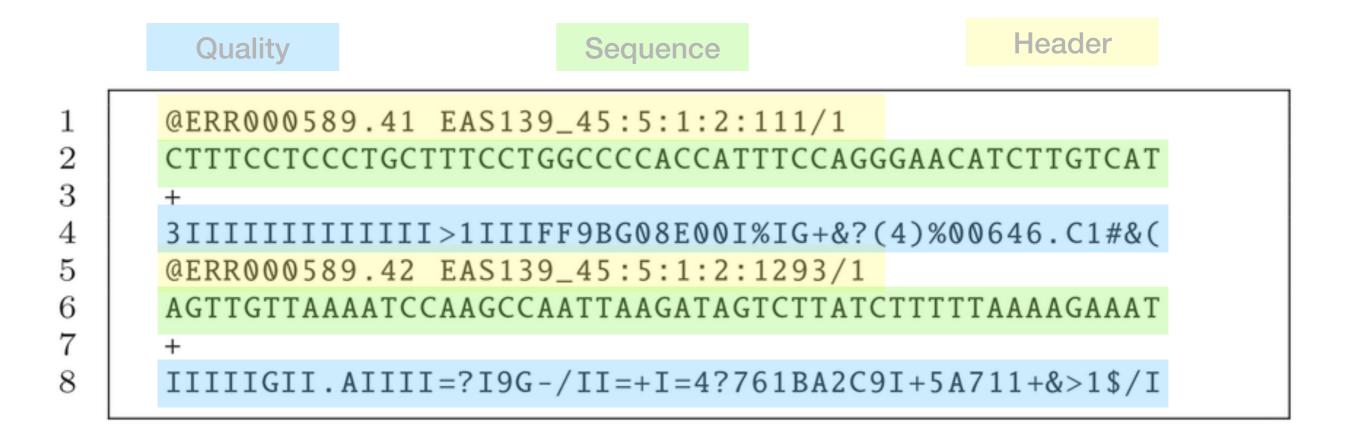
 $p(S|M) = 0.1 \times 0.6 \times 0.7 \times 1.0 \times 1.0 \times 0.6 \times 0.7 \times 0.2 \times 0.2 = 0.0007056.$ 

https://en.wikipedia.org/wiki/Position\_weight\_matrix#:~:text=A%20position%20weight%20matrix%20(PWM,represented%20graphically%20as%20sequence%20logos.

# Computational methods for biology at different scales



## What does a fastq file look like?



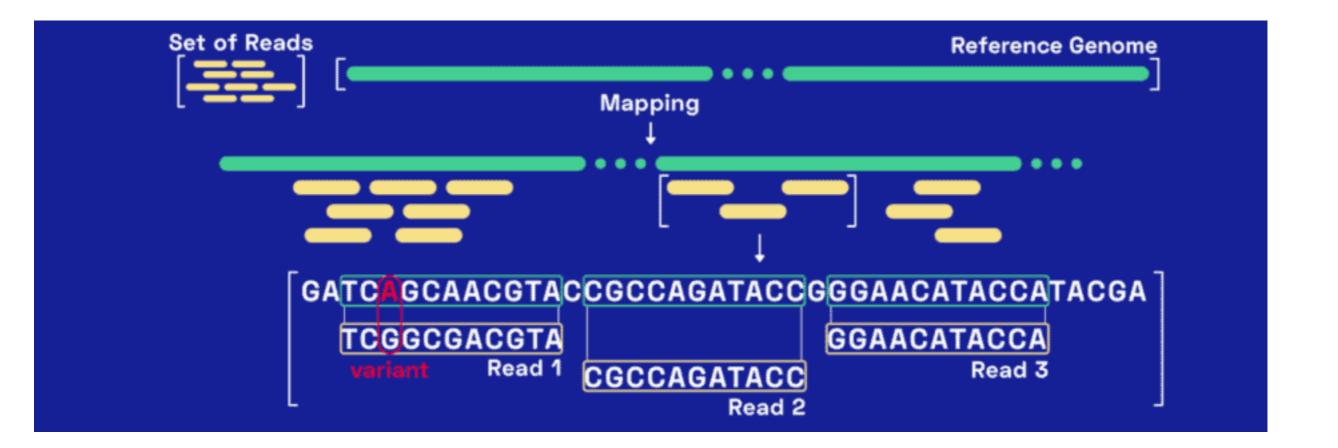
Very large! ~30000000 lines Quality: ASCII chars

What should we do? Map each short sequence (we call it read) to the entire human genome

## What does a fastq file look like?

Reference genome: "average" human genome.

Most widely used human genome GRCh38: derived from 13 thirteen anonymous volunteers



### Processed data

### countData

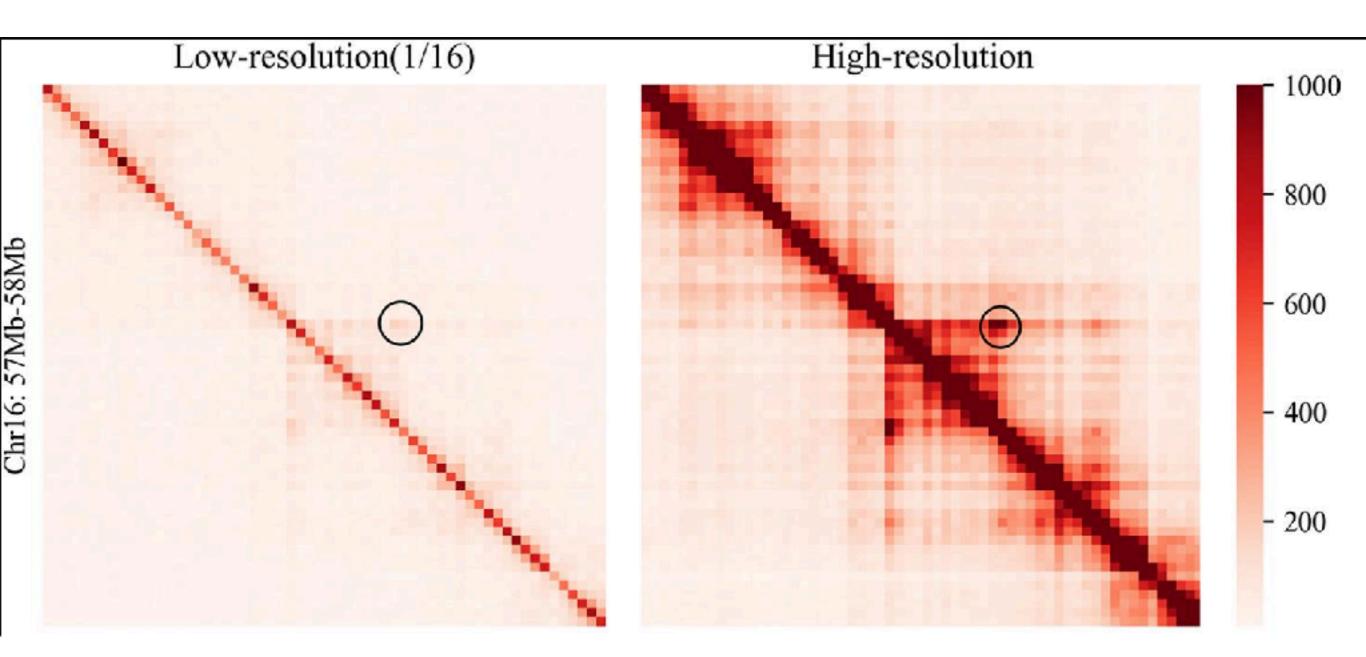
	ctrl_1	ctrl_2	exp_1	exp_1
geneA	10	11	56	45
geneB	0	0	128	54
geneC	42	41	59	41
geneD	103	122	1	23
geneE	10	23	14	56
geneF	0	1	2	0

## colData

	treatment	sex
ctrl_1	control	male
ctrl_2	control	female
exp_1	treatment	male
exp_2	treatment	female

Sample names: ctrl\_1, ctrl\_2, exp\_1, exp\_2

# Data structure and computational problem



source: SRHiC: A Deep Learning Model to Enhance the Resolution of Hi-C Data

# Finding alignments: trace back

Arrows = (ties for) max in F(i,j); 3 LR-to-UL paths = 3 optimal alignments

