Figures of Merit: The Sequel

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Science has marched on despite the appearance of the original "Figures of Merit" [18]. The purpose of this survey is to bring the community up to date on the most recent bounds, so that we may collaborate to improve them.

1 The Spotlight Factor

Recall the definition of the spotlight factor for first authors:

Definition: In a collaboration of alphabetized coauthors $X_0 < X_1 < \cdots < X_k$, the *spotlight factor* of X_0 is

$$\star(X_0) = (1 - .X_0)^k,$$

where the notation "$.X$" is the radix 27 fraction, where $a = 1$, $b = 2$, ..., $z = 26$, and blanks and punctuation represent 0.

In words, the spotlight factor is the probability that k coauthors chosen uniformly and independently at random will all have surnames later in the alphabet than X_0; the lower the spotlight factor, the more impressive the achievement of the first author in attaining first authorship.

The best previous bound [18] on the spotlight factor arose from the collaboration

Santoro, Sidney J., Sidney S., and Urrutia [15]

whose spotlight computation goes as follows:

$$\begin{aligned}\star(\text{Santoro}) &= (1 - .\text{Santoro})^3 \\ &= \left(1 - \left(\frac{19}{27^1} + \frac{1}{27^2} + \frac{14}{27^3} + \cdots\right)\right)^3 \\ &\approx 0.0255\end{aligned}$$

This record has been dented by the collaboration

Kaklamanis, Karlin, Leighton, Milenkovic, Raghavan, Rao, Thomborson, and Tsantilas [9]

for which ★(Kaklamanis) ≈ 0.0251. A cynic might wonder whether some authors did this calculation themselves in order to know just how many coauthors to invite. At one point a preliminary version of their paper had a ninth coauthor whose surname, incredibly, also began with a letter later than K in the alphabet. This would have been worth a spotlight factor of approximately 0.0148.

2 The Coefficient of Obliviousness

A second figure of merit from [18] was the coefficient of obliviousness:

Definition: In a collaboration of $X_0 < X_1 < \cdots < X_k$, the *coefficient of obliviousness* of X_i is

$$¿(X_i) = (.X_i - .X_0)^i,$$

for $1 \leq i \leq k$.

In words, the coefficient of obliviousness is the probability that i coauthors chosen uniformly and independently at random will all have surnames that precede X_i as narrowly as does X_0; the lower the coefficient, the more oblivious X_i is to the fame of being first author.

The record for coefficient of obliviousness from [18] was held by

Plumstead B. and Plumstead J. [14]

for which ¿(Plumstead J.) $\approx 1.53 \times 10^{-15}$. There was some grumbling about the fact that many of the most oblivious collaborations in [18] came from familial ties, and so were not random at all. The suggestion was that one should measure *nonnepotistic* obliviousness, for which the best example from [18] was

Brassard and Bratley [2]

with ¿(Bratley) $\approx 1.40 \times 10^{-6}$.

This record of nonnepotistic obliviousness is beaten, however, by

Goldreich, Goldwasser, and Micali [7]

for which ¿(Goldwasser) $\approx 3.39 \times 10^{-7}$. This even edges out the familial obliviousness of

Yao A. and Yao F. [20]

for which ¿(Yao F.) $\approx 3.58 \times 10^{-7}$.

But the most astonishing find is the collaboration

Smith J., Smith K., and Smith R. [16]

for which ¿(Smith R.) $\approx 5.84 \times 10^{-19} \ll$ ¿(Plumstead J.), shattering all previous records. Moreover, the text of this paper asserts that the authors are unrelated, satisfying the nonnepotistic condition as well.

3 Monotone Erdös Number

The greatest strides have occurred in the important subfield of monotone Erdös numbers. Define a directed graph $G = (V, E)$, where V is the set of all researchers, and $(u, v) \in E$ if and only if there is some publication in which u appears earlier in the list of coauthors than v.

Definition: The *monotone Erdös number* of X is the length of a longest directed path in G between Paul Erdös and X.

In [18] it was shown that Wigderson's monotone Erdös number was 5, and conjectured that this was the best bound possible. Table 1, however, resoundingly refutes this conjecture, by producing a researcher whose monotone Erdös number is 12.

1.	Erdös and *Freiman* [4]
2.	Chaimovich, Freiman, and *Galil* [3]
3.	Galil, *Kannan*, and Szemerédi [6]
4.	Frieze, Kannan, and *Lagarias* [5]
5.	Lagarias, *Lenstra*, and Schnorr [11]
6.	Lenstra, Lenstra, and *Lovász* [12]
7.	Karmarkar, Karp, Lipton, Lovász, and *Luby* [10]
8.	Luby, *Micali*, and Rackoff [13]
9.	Goldwasser, Micali, and *Rivest* [8]
10.	Blum, Floyd, Pratt, Rivest, and *Tarjan* [1]
11.	Tarjan and *Vishkin* [17]
12.	Vishkin and *Wigderson* [19]

Table 1: Monotone Erdös Numbers

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Avi Wigderson refused to collaborate with anyone later in the alphabet, even in the interest of scientific advancement.

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