Solving Relational MDPs with First-Order Machine Learning[*]

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Abstract

We present a new formulation of Relational Markov Decision Processes (RMDPs) which is simpler than the situation-calculus approach of Boutilier, Reiter and Price. In addition, we describe our initial efforts developing a novel, machine-learning based method for computing an RMDP's policy. Our technique instantiates the RMDP into a number of propositional MDPs, which are then solved for their value functions. First-order regression techniques are then used to learn a value function for the complete RMDP. This value function may then be used to produce a policy for huge decision-theoretic planning problems, outputting compact solutions without actually requiring explicit state space enumeration. Finally, we extend our RMDP formalism to cover the case of a dynamic universe, i.e. in which action effects may create new objects or destroy existing ones.

Introduction

Two communities of researchers have looked at planning problems involving uncertainty. One group ("neo-classical planning") has started from algorithms for solving deterministic STRIPS problems, and extended them to handle *disjunctive uncertainty* (Peot & Smith 1992; Etzioni *et al.* 1992; Pryor & Collins 1996; Goldman & Boddy 1994; Smith & Weld 1998; Weld, Anderson, & Smith 1998; Cimatti & Roveri 2000; Bertoli & Cimatti 2002) (or less frequently a probabilistic representation (Kushmerick, Hanks, & Weld 1995; Draper, Hanks, & Weld 1994; Blythe 1998). Disjunctive uncertainty denotes a qualitative representation that captures the different possible values for state variables without using numeric measures of likelihood such as probabilities. This community has typically focussed on generating a plan from some initial belief state which is guaranteed to achieve a goal, rather than attempting to maximize expected utility. Researchers have developed *conformant planning* algorithms, which generate straight-line plans in the case of zero observability, and *contingent planners*, which generate branched plans in the case of partial observability.

In contrast, the "UAI community" starts from the premise that a probabilistic representation is more powerful. Uncertain actions are represented with a probability distribution over possible resulting states, and the planning problem is formulated as a Markov decision process (MDP) or partially-observable Markov decision process (POMDP).

There are many clear differences in the approaches adopted by the two communities, and these complicate comparison and combination of the methods each has developed.

- **Representation:** disjunctive or probabilistic.
- **Output:** the neoclassists generate straight-line plans or branching plans, while the UAI community typically generates a *policy* — a mapping from states to appropriate actions.
- **Objective:** the neoclassists have mostly considered binary goals which must be achieved (as much as possible) by the end of the plan. In contrast the UAI community usually focuses on the expected discounted reward achieved by an agent, acting over an infinite timeline, gaining utility and incurring costs with each action. Although single shot goals can be simulated by infinite horizon reward model by having each sub goal as a state variable, note that each additional sub-goal doubles the size of the state space making the computation over it slower.
- **World Specification:** the neoclassists define planning problems in first-order terms: with a set of objects, their attributes and relations, and a set of parameterized action schemata. The UAI community, on the other hand, typically considers state spaces and transition matrices, or factors them into dynamic Bayesian networks (DBNs) which are inherently propositional.
- **Problem Measurement:** The neoclassists consider both the length of the plan generated and the complexity of the problem, which is estimated in terms of the number of actions or objects present. In contrast, the UAI community focusses on the size of the state space.

This last distinction is especially important because it has hindered each community from telling how the other community's techniques compared to their own. This gap can be filled if the two communities adopt a more unified measurement scheme. For example, the noeclassists could report the size of the state space and the UAI community could mention the complexity of the policy generated. Indeed, research in each community has resulted in significant progress. First consider advances which might be said to stem from the neoclassical camp. First-order representations

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greatly facilitate large problem specification. Domains with dynamic object creation (Weld 1994) and unbounded information gain (Golden, Etzioni, & Weld 1994) have been tackled. Algorithmic advances such as planning graphs (Blum & Furst 1997; Blum & Langford 1998), reachability analysis, and powerful heuristics have led to huge performance gains. Boolean decision diagrams (BDDs) (Bryant 1986) have been used to compactly represent regions of the search space (Cimatti & Roveri 2000).

On the other hand, the UAI community has powerful computational techniques for manipulating probabilities, e.g. graphical models, dynamic Bayesian networks (DBN), value iteration, policy iteration and policy search. Fast linear solvers on factored value functions have also been tried (Koller & Parr 2000). Using an extension of BDDs, called ADDs, to compactly represent transition and value functions has been shown to increase performance (Hoey *et al.* 1999). Reachability analysis has also been adapted to MDPs with some success (Boutilier, Brafman, & Geib 1998; Feng & Hansen 2002). Both communities have considered abstraction and approximation techniques.

Overview

Our long-term objective is to better understand the strengths and weaknesses of the different approaches. Largely we are motivated by recent progress on *probabilistic relational models* (PRMs) (Friedman *et al.* 1999; Getoor *et al.* 2001) and other techniques for combining first-order representations with probabilistic methods (*e.g.,* Bayesian logic programs (Kersting & Raedt 2001a; 2001b) and its precursors (Ng & Subrahmanian 1992; Wellman, Breese, & Goldman 1992; Haddawy 1994), stochastic logic programs (Muggleton 1995), RMMs (Anderson, Domingos, & Weld 2002) and DPRMs (Sanghai, Domingos, & Weld 2003)). Specifically, we wish to extend the work by Boutilier, Reiter, Price on relational MDPs (Boutilier, Reiter, & Price 2001). Their notion of *decision theoretic regression* is appealing, since it enables policy construction in time which is completely independent of the size of the state space. Unfortunately, their methods (while elegant) are too slow for practice.

In this paper we describe our initial work developing an alternative method, for solving relational MDPs (RMDPs). First, instantiate the RMDP into a number of propositional MDPs, which are then solved for their value functions. Second, use machine learning methods (*e.g.,* relational regression) to learn a value function for the complete RMDP. Third, use the value function to produce a policy for new decision-theoretic planning problems, outputting compact solutions without actually requiring explicit state space enumeration. In addition, we extend the RMDP formalism to cover the case of a dynamic universe, i.e. in which action effects may create new objects or destroy existing ones.

Road-map

The rest of the paper is organised as follows. In the background section, we discuss the fundamentals of MDPs, various techniques to solve them, and the previous work on relational MDPs. In the next two sections, we provide a novel formulation of relational MDPs, and present our algorithm to learn the value function for them. In the section on Dynamic Object RMDPs, we extend RMDPs to handle actions whose effects create new objects. We end with our contributions and future work in the conclusions section.

Background

A Markov Decision Process is a tuple <$S, A, T, \Re$>, where

- S is the set of states.
- A is the set of actions.
- T is the transition function $A \times S \times S \rightarrow [0, 1]$ which takes an action, the current state and the next state and gives the probability of this transition.
- $\Re$ is the reward model which is a mapping from a state to a real number. Intuitively, $\Re(s)$ is the reward the agent would get when it reaches a particular state s.

We consider MDPs for which the objective is to maximise the expected discounted reward gathered by the agent acting over infinite time. Thus we also include a $\gamma \in [0, 1)$ as the discount factor. Since we are in the fully observable case, we need to output a policy $\pi : S \rightarrow A$ which maximises our objective function. Note that such an MDP satisfies the Bellman backup equation given by $V^*(s) = \Re(s) + \max_{a \in A} \left\{ \gamma \left[\sum_{s' \in S} T(a, s, s') V^*(s') \right] - cost(a) \right\}$. Here *cost* represents the cost of performing an action and $V^*(s)$ is defined as the maximum expected discounted reward accumulated starting from a given state s.

In many cases, it is possible to factor the MDP. In this, a state is defined in terms of a set of state variables ($X = \{X_1, X_2, \ldots X_n\}$), which are often Boolean. Thus one assignment of values to state variables represents one state. Assuming Boolean state variables, one can see that $|S| = 2^{|X|}$. In this framework, the transition function for each action is best described using a DBN and even the reward model may be described using the state variables.

Considerable work has been done in developing methods for solving these propositional planning problems under uncertainty, especially assuming full observability. Most approaches have used dynamic programming of the Bellman equation in either the value iteration or policy iteration framework. Some have used heuristics to reduce the computational requirements. In the case where the states are non-factored, Bonet and Geffner (Bonet & Geffner 2000) use heuristic search over the state space. However, in the propositionally factored case, SPUDD (Hoey *et al.* 1999) uses algebraic decision diagrams to do symbolic reasoning over the state space. This method proves to be fairly fast on many reasonable sized problems. Feng and Hansen (2002) observe that the information of an initial state could be utilised to speed up the whole procedure. They adapt SPUDD to add state reachability information by doing alternative rounds of dynamic programming and reachable state space expansion, and achieve much better results. Alternatively, Koller and Parr (Guestrin, Koller, & Parr 2001; Koller & Parr 2000; 1999) factor the value function as linear combination of small basis functions and then solve the MDP approximately by doing closed form computations.

However all this is in the realm of propositional planning that is defined in terms of ground actions and ground state variables. But, in real life, objects are divided into various classes. For example, in the coffee-robot domain there could be many users and many robots that would behave similarly. In a standard (propositional) MDP, one would have to specify each ground action and state variable separately. If we wish to succinctly represent such domains, we would have to consider classes of objects, and define state variables and actions over classes (or ordered tuples of classes), and provide a list of ground objects separately. To expand such a description into actual propositional planning domain we would have to parametrise all these state variables and actions over all possible legal combinations from the list of objects. In other words, actions and state variables would be terms in a first order language in which action schemas and object classes were relations.

This idea, termed as Relational MDPs (RMDPs), was introduced in a seminal paper (Boutilier, Reiter, & Price 2001) in the context of the situation calculus (we provide a new and simpler formulation of RMDPs in the following section). We know that the size of the regular MDPs blow up as the number of state variables increase. The standard MDP solvers can't even take off in the expanded RMDP problems with even a small number of domain objects, as the number of state variables grow with number of domain objects fairly fast. Thus other ways of solving RMDPs which don't directly rely on expanding an RMDP into an MDP were required. They try to utilise the fact that all objects of the same class behave similarly so that rather than taking each object separately we can deal directly with object variables which could be parametrised appropriately in the actual problem. They learn the value function and the policy in the symbolic abstract form independent of the number of domain objects by partitioning the state space based on certain properties and assigning a value to each partition. To achieve this, they use situation calculus and regress the final reward model. Although their procedure is sound and elegant, they could not generate a working implementation because of the need to use first order theorem provers to prune the number of partitions as they were blowing up very fast. On the whole their system was fairly slow and they could only show results of first iteration with one time rewards after some hand pruning.

Some progress has been made recently in solving such RMDPs. Guestrin et al. (2003) consider a class-based, approximate value function and solve it using linear programming combined with sampling over worlds. Yoon et al. (2002) describe a method which instantiates the RMDP with a small number of objects to create a tractable ground MDP, which is solved traditionally. They use the solutions as training data and use a greedy learning algorithm to induce a policy, represented as a decision list. While their results are promising and their method resembles our learning mechanisms, their approach has three main limitations. First, it is limited to binary fluents and actions with a single parameter. And while one could preprocess a domain to transform it into this form, the expansion would result in blowup that would seriously hamper their algorithm. Second, their policy representation has very high bias, and hence there are many policies which cannot be represented. Third, it seems questionable to try and learn a policy directly; decades of research on learning for two person games has shown that it is better to learn a board-evaluation function than a move selector. Given the close connections between uncertain actions and games against nature, it seems likely that value functions will be easier to learn than policies. We rectify these problems in the solution we propose.

Relational Markov Decision Process

Let us extend MDPs with full observability to RMDPs with full observability as follows:

Definition (RMDP): We formalise a relational Markov decision process as a tuple <$C, F, A, D, T, \Re$>, where C, F and A are all sets of relational schemata. In particular,

- C is a set of classes denoting the different possible types of a ground object.
- F is the set of fluent schemata. Each fluent $f \in F$ has arity $\alpha(f)$ and we assume typed logic i.e. with each fluent f is associated a function $t_f : \{1, 2, ..., \alpha(f)\} \rightarrow C$. This function represents the types of different arguments for f to be valid.
- A is a set of action schemata and as with the elements of F, there is an associated arity and a type function t_a for every $a \in A$. Also the cost of each action is a positive real number.
- D represents a set of domain objects. With each object from the set D is associated a single $type$ from C.
- T is a transition function which represents the probabilities of transition between different states (we will discuss what comprises a state, shortly).
- Finally $\Re$ is the reward model. For simplicity, we consider the model as a mapping from the set of states to real numbers. However, we could handle a more complex model which associates a real value with every tuple $(currentstate, nextstate, action)$

Example: Following the example of (Boutilier, Reiter, & Price 2001) let us consider a domain in which there are boxes in different cities and the goal is to bring one box into Paris. There are trucks which help in this transportation. To determine one state we would have to know whether a box is in a city (Bin), whether a truck is in a city (Tin) and whether a box is on a truck (On). The actions are unloading a box from the truck, loading a box onto the truck and driving the truck from a city to another. Let us formally define this RMDP.

- C : {Box, Truck, City}
- F : {Bin(Box,City), On(Box, Truck), Tin(Truck, City)}
- A : {Unload(Box,Truck,City), Load(Box, City, Truck), Drive(Truck,City,City)}
- $\Re$: if $\exists b\ Bin(b, Paris)$ then 10 else 0.
- Any number of boxes, trucks and cities will give one possible D.

- The transition function T is defined in detail in the following text.

We now define the set ($F_{D'}$) of all possible fluent tuples instantiated with elements of domain D'.

$F_{D'} = \{f(d_1, d_2, \ldots d_{\alpha(f)}) | \; f \in F, \; d_i \in D', \; t_f(i) = type(d_i)\}$.

If we expand the RMDP into a factored MDP then the state variables in the MDP are elements of F_D - the set of possible tuples comprising the fluent relations formed by instantiating the schemata in F with objects in D. Hence, the state space S of the problem is $\wp(F_D)$, the power set of F_D. We can similarly define the possible actions one can execute over some set of domain objects D' as $A_{D'} = \{a(d_1, d_2, ..., d_{\alpha(a)})\} | \; a \in A, \; d_i \in D', \; t_a(i) = type(d_i)\}$.

The transition function T is in general a mapping from $S \times S \times A_D \rightarrow [0,1]$. Note that the state space is extremely large and thus specifying a general transition function is impractical. An interesting restricted form is a relationally factored version of the transition function.

Assumption 1: We assume that an action can only affect relational fluents instantiated over its parameters.

This is a reasonable assumption for many situations since one can add an arbitrary (finite) number of parameters to an action. Thus, the assumption is akin to ruling out universally quantified effects. In such a case, we can achieve a compact specification of the transition function.

Compact specification of transition function

Let us consider an action $a^* = a(d_1, d_2, ..., d_{\alpha(a)})$ where $a \in A$ and $d_i \in D$. Define $D^* = \cup_{i=1}^{\alpha(a)} \{d_i\}$. As assumed, the action a^* can affect only d_i's, i.e. it affects the fluent tuples instantiated only by domain objects from D. Hence, the transition function associated with a^*can be thought as $T_{a^*} = \wp(F_{D^*}) \times \wp(F_{D^*}) \rightarrow [0,1]$.
We can further reduce the specification in the problems where one can assume that the value of a relational fluent in the new state is independently modified by an action irrespective of the value of other fluents in the new state i.e. depends only on the previous state. In such a case $T_{a^*} = \wp(F_{D^*}) \times F_{D^*} \rightarrow [0,1]$.
Finally if we assume that each action template, a, behaves similarly with all the similar objects (satisfying same relational fluents), then for each action we can specify this transition function as a template and we can instantiate the parameters with different domain objects to get the exact probability of a particular transition.

Example (contd): Following the previous example, the figure 1, shows the transition function in Probabilistic Strips format (Boutilier, Dean, & Hanks 1999; Hanks & McDermott 1994; Kushmerick, Hanks, & Weld 1995). Note that in our example, load and unload succeed with probability 0.8 and drive succeeds with probability 0.9. A failure happens with probabilities 0.2, 0.2 and 0.1 respectively and means that the state variable in consideration does not change its previous value. Moreover, all the variables that have not been mentioned are assumed unchanged. However to take advantage of the DBN representation (Dean & Kanazawa 1989) we could instead create a relational DBN representation where the state variables would be these parametrised relations which would be causes of other relational state variables. But recall that in standard DBN representations one must explicitly represent the causal relationship of each new state variable. However, in our case, doing this would greatly increase the size of the representation. So we adopt a DBN representation with an implicit persistence property which means that all the new variables that have not been mentioned remain unchanged. The transition function of $Unload(box, truck, city)$ as an example of this relational DBN with persistence is shown in figure 2.

```
Action : Unload(box,truck,city)
Preconditions: Tin(truck,city),
               On(box,truck)
Effects: Bin(box,city)∧
         ¬On(box,truck) p = 0.8
Action : Load(box,city,truck)
Preconditions : Bin(box,city),
                Tin(truck,city)
Effects: On(box,truck)∧
         ¬Bin(box,city) p = 0.8
Action : Drive(truck,city1,city2)
Precondition : Tin(truck,city1)
Effects: Tin(truck,city2)∧
         ¬Tin(truck,city1) p = 0.9
```

Figure 1: Transition function for actions in Probabilistic Strips representation.

As we consider full observability in our model, we assume that after each action execution the agent knows the new state achieved. To us, that means that after executing action a^*, the agent knows the value of each fluent from the set F_{D^*}.

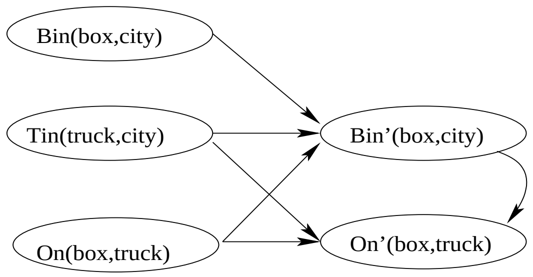
The solution of such an RMDP is similar to that of the MDP i.e. to find a policy ($\pi : S \rightarrow A_D$) which maximises the expected discounted reward over an infinite horizon. We see that the Bellman backup equations can be inherited from the MDP. The optimal value function $V^*$is defined as: $V^*(s) = \Re(s) + \max_{a \in A_D} \left\{\gamma \left[\sum_{s' \in S} T_a(s, s')V^*(s')\right] - cost(a)\right\}$

Solving RMDPs by learning the value function

In this section, we describe our approach to learn the first order value function for the RMDPs. Notice that it is straightforward to use this value function to generate the policy.

Assumption 2: We assume that the RMDP reward model is a piecewise constant function which defines a partition over state space such that each equivalence class has the same value. We also assume that each equivalence class can be defined as a quantified first order logic expression.

That is we will not handle rewards which are, for example, proportional to the number of objects satisfying certain constraints. Where (Boutilier, Reiter, & Price 2001) used deductive reasoning in generating the first order value function, we use inductive learning techniques to do the same.



Tin	On	Bin	Bin'
F	F	F	0
F	F	T	1
F	T	F	0
T	F	F	0
T	F	T	1
T	T	F	0.8

Tin	On	Bin'	On'
F	F	X	0
F	T	X	1
T	F	X	0
T	T	T	0
T	T	F	1

Figure 2: Relational DBN representation of transition function of Unload(box,truck,city). Note that all unmentioned terms (eg. Tin'(truck,city)) will remain unchanged. Note that both Bin(box,city) and On(box, truck) can't be true at the same time.

1. We first expand the RMDPs with an extremely small number of objects into a ground MDP.
2. We then use a state of the art MDP solver to compute the value function of this small MDP.
3. We repeat the above two steps to generate a suitable number of training examples.
4. We now apply learning techniques using this data to generate a value function in the symbolic form. Specifically, we use a learner which generates first order regression trees (decision trees with internal nodes having quantified logic expressions and leaves as numeric values).

An example of such a value function is shown in Figure 3. This figure can be read as follows. If there is a box in Paris the value is 10, else if there is a box on some truck and that truck is in Paris then value is 7. If that truck is elsewhere then the value is 5 and so on.

Since we wish to learn a real-valued value function, we require a learning technique which ascribes numeric values (rather than a symbolic classifications) to a partition of the state space. A variant of inductive logic programming called structural regression trees (SRT) (Kramer 1996) is tailor-made for our purposes. SRT builds a sequence of increasingly complex trees (by gradually decreasing the minimum coverage parameter) and then chooses the best according to a minimum description length (MDL) heuristic (Rissanen 1978).

Each regression tree is grown in a manner similar to the top-down induction of a decision tree. A partition is split by considering conjunctions of literals and choosing the conjunction whose split most lowers the sum of squared differences (equation 1). For example, a set of instances I might be split by a conjunction Γ into a set, I_1 that satisfies Γ and a

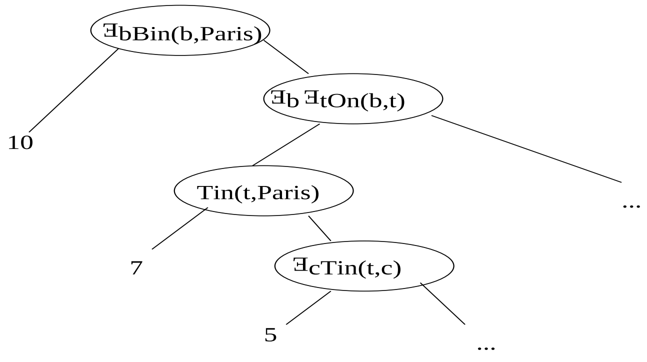


Figure 3: A value function represented as a first order regression tree. The leaf nodes are the values of the partitions. Note that all the left branches are true branches.

set, I_2 that does not. If $\overline{y_i}$ denotes the mean of the elements ($y_{i,j}$) of I_i, then the sum of squared differences is:

$$\sum_{i=1}^{2}\sum_{j=1}^{|I_i|}(y_{i,j}-\overline{y_i})^2 \qquad (1)$$

When the stopping criterion terminates tree growth, the tree may be used to predict the value for an arbitrary state by applying the tests at each node in turn until a leaf, I_1, is reached. The value assigned to the state is simply the mean value of the leaf: $\overline{y_i}$.

The major concerns in this approach are whether the assumption 2 is a good bias for learning; will the learner converge and is our method scalable. Although we don't have direct answers to these questions, our initial efforts with hand-implementation suggest that the learner will converge.

Implementation status

We tested the algorithm by hand-executing several iterations on a small number of examples, and the results seem extremely positive. This has led us to start with the implementation of the system. We use SPUDD (Hoey *et al.* 1999) as the MDP solver. We are currently modifying C4.5 (Quinlan 1993), a decision tree learner (which is written in C) to learn first order regression trees in the same fashion as SRT.

Dynamic Object Relational Markov Decision Process

Consider a factory domain, where lots of widgets are being continually produced by various mills. Periodically, we need to pack and ship them appropriately. We can think of a $produce$ action that creates a new widget and a $ship$ action that takes some already produced widgets out of the system. In order to model such a domain, we need to formalise action effects that change the set of objects D.

Note that such a system can't be modelled by traditional MDPs as the number of domain objects, and hence the number of states is unbounded. Even if there were a bound and one added a new attribute $alive$ which told whether an object is still in system or not, this would create the problem of having a large number of objects in the working set from the very beginning and since the size of the planning problem grows unbelievably large with the number of domain objects, it would be very slow. In this section we propose dynamic object relational MDPs (DORMDPs) as a model that caters to dynamic creation of an unbounded number of objects.

There are different possibilities for the dynamic universes in terms of the number of objects that could be created as a result of an action execution:

- **Unbounded Object Creation:** The most generic model would allow for an unbounded, variable number of new objects being created as a result of an action.
- **Bounded Object Creation:** An action producing variable number of objects subject to a maximum value.
- **Constant Object Creation:** An action producing only constant number of new objects.
- **Single Object Creation:** At most one new object being created per action execution.

For simplicity of presentation, we consider the last case in some detail. Note that, the following model can be easily extended to deal with any of above listed cases. For example, to model the first two cases, we could take the number of objects created as a probability distribution based on the current state.

Definition (DORMDP): A DORMDP is a tuple $<C, F, A, D, T, \Re>$ whose elements are the same as those of an RMDP except that D is infinite.

The state space S is $\wp(F_D)$ but since D is infinite, the set of relational fluents defined over D is also infinite, hence S is infinite. In particular, D is $D_0 \cup [\cup_{i=1}^{|C|} \{d_{i1}, d_{i2}, ...\}]$ where D_0 is the initial set of objects. We can shrink D by having the d_{ij}'s only for those types (c_i) for which some action creates an object of that type. Moreover, we also define two new functions over each action pc: $A \times S \rightarrow [0, 1]$ which denotes the probability that action A produces a new object in the state S and $tc : A \rightarrow C$ which denotes the class of the object created.

We maintain Assumption 1, in which we assume that relational fluents are independent and that an action affects only fluents instantiated by action parameters and the new object created. In this case, the transition function can be defined as follows:

Let the action being considered be $a^* = a(d_1, d_2, ..., d_{\alpha(a)})$ such that $a^* \in A_D$ and $a \in A$. Let D^* be $\cup_{i=1}^{\alpha(a)} \{d_i\}$. Moreover, let $D' = D^* \cup \{d_{new}\}$ where d_{new} is the new object created such that $type(d_{new}) = tc(a)$. Also, note there would be new relational fluents created (as a result of object creation) whose truth values need to be found out. The total set of relational fluents affected would be $F_{D'}$. Then our transition function for the action a^* is $T_{a^*}: \wp(F_D*) \times F_{D'} \rightarrow [0, 1]$.

Finally the observation model for full observability requires the agent to observe whether the new object was created or not and also the truth values of all the relational fluents from the set $F_{D'}$, if new object was created and F_{D^*}, if it was not created. The definitions of policy and reward model *etc.* can be inherited from the RMDP definition. And as usual, the goal of such a planning problem is to find an optimal policy ($S \rightarrow A_D$) such that discounted sum of expected rewards is maximised over an infinite horizon.

We know that DORMDPs can't be expanded into propositional MDPs directly since the state space is infinite. So, the only way to generate a value function, in this case, seems to be generating a first order one, partitioning the state space based on attributes of the states. Several methods may work:

- Adapt the symbolic regression approach in (Boutilier, Reiter, & Price 2001). The trick in this case would be developing efficient theorem proving techniques to simplify the partition boundaries.
- Use the heuristic search approach (Bonet & Geffner 2000) as follows: We incrementally expand DORMDP into a fraction of the unbounded MDP. We can then use Real Time Dynamic Programming (RTDP) (Barto, Bradtke, & Singh 1995) from the initial state to update the value function using Bellman's Equation only for the visited states until the goal is achieved. Several iterations of RTDP may give a suitable approximation to the value function of reachable states. We can then learn a first order value function based on our regression tree learning approach.
- Use reachability analysis with SPUDD for a dynamically expanding MDP (Feng & Hansen 2002). However, this approach explores the complete optimal reachable state space which can be potentially infinite. So the key here would be development of loop detection to stop the process. Perhaps, the techniques of (Smith & Peot 1996; Smith 1989) could be adopted.

Note that this work is still ongoing.

Conclusions

Important work has been done by neoclassical and UAI researchers in the field of planning under uncertainty. In this paper, we have combined the contributions of both communities. Specifically, our representation, output format and objective are similar to those of the UAI community and our schematised world view and dynamic object creation are neoclassist. While our work is ongoing, we have already made the following contributions:

1. We defined Relational MDPs. Although (Boutilier, Reiter, & Price 2001) has already done this in the context of situation calculus, we believe that our formalism is more practical.[1]
2. We presented a new solution method based on relational decision tree learning from the solution of expanded propositional MDPs.
3. We defined Dynamic Object Relational MDPs which allow one to model actions whose effects created objects.

[1]Although several planners may have used representations like ours, no one except Boutilier et al (2001), to our knowledge, has provided a formal definition of an RMDP or its equivalent.

In future, we will complete the implementation of our system and do experiments over it; these will answer our concerns on the convergence and scalability of the system. Further, we wish to relax the two assumptions we have made in the paper. For instance, we could deal with reward models which have rewards proportional to number of objects of a certain type and handle universally quantified effects. We could use First Order Regression System (FORS) (Karalic & Bratko 1997) since it has the ability to learn regression models over attributes. We can further incorporate the neoclassical objective of trying to achieve goals in our framework, instead of maximising rewards. We also wish to look for improved reachability analysis and better heuristics to speed up the system. Another direction of research is including temporal duration in uncertain actions. We will also continue to work on solving dynamic object RMDPs.

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